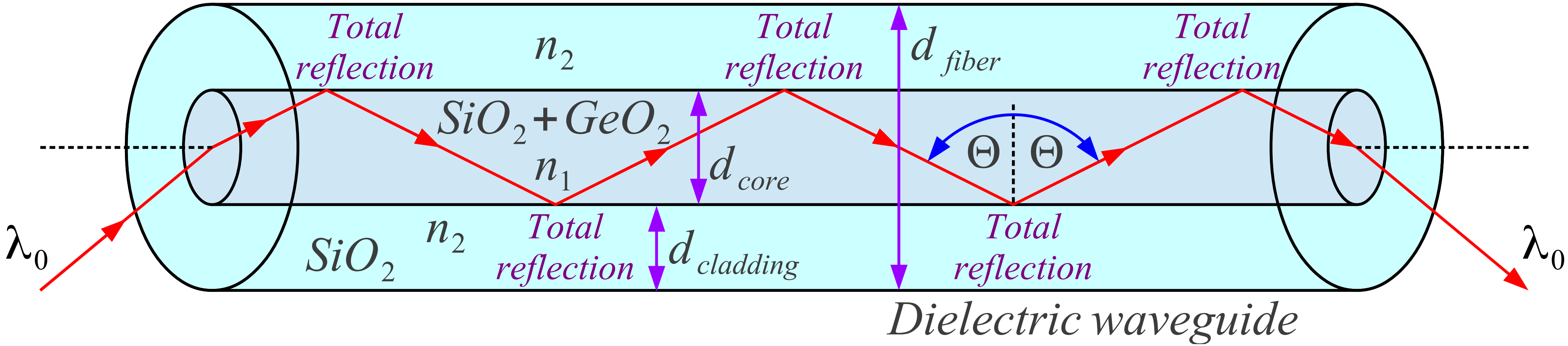
Communication Electronics

Lecture 3:

Optical-fiber communications

Glass optical fiber

Telecom standard: $d_{fiber} = 125\mu m$



Dielectric waveguide

Total reflection: $n_1(SiO_2+GeO_2) > n_2(SiO_2) \rightarrow \arcsin(n_2/n_1) < \Theta < \pi/2$

$3\mu m < d_{core} < 63\mu m$

$d_{cladding} \gg \lambda_0$

$780nm < \lambda_0 < 1630nm \Leftrightarrow 385THz > f_0 > 184THz$

Advantages in communications:

(1) *Low loss*: $a/l = -0.15 \ldots -2dB/km$

(2) *Large bandwidth*: B>4THz

(3) *Galvanic isolation!*

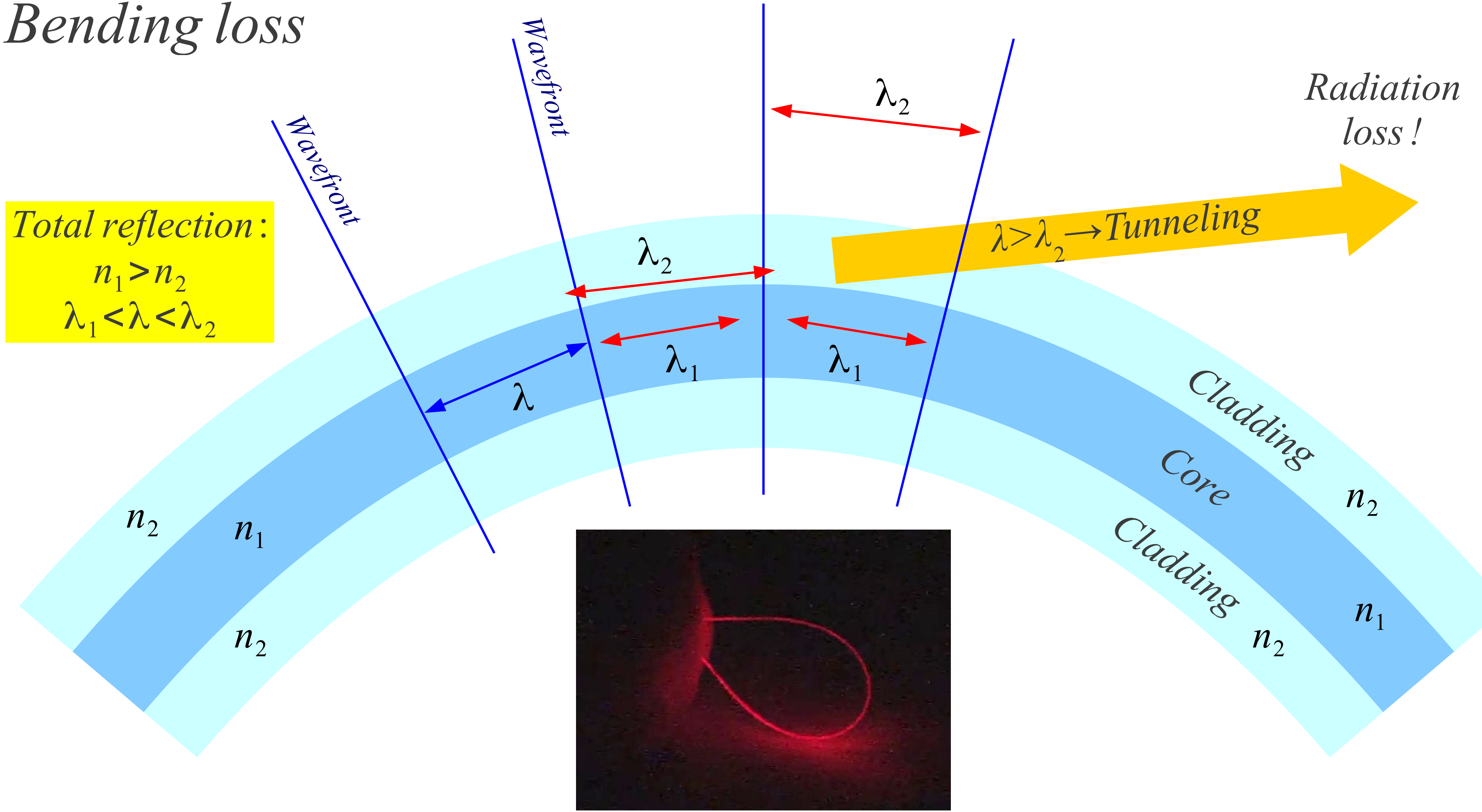
Drawbacks for signal processing:

(1) *Unidirectional amplifier for light-wave frequencies (optical trasistor) does not exist!*

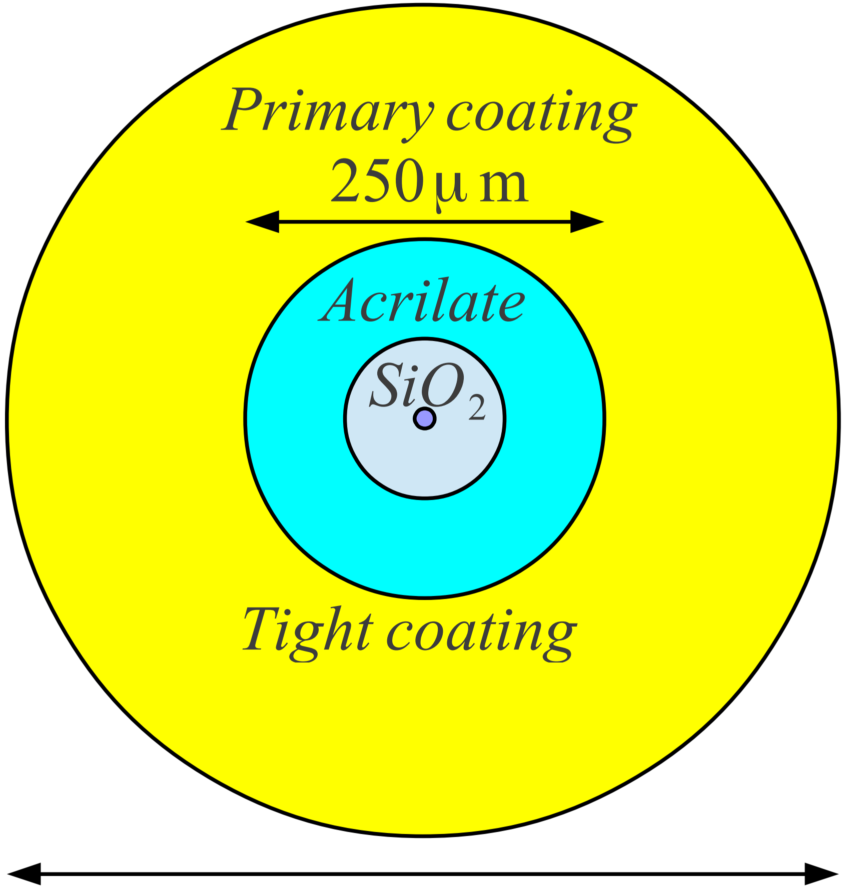
(2) *Passive components too large for integration!*

Bending loss

Total reflection :
$n_1 > n_2$
$\lambda_1 < \lambda < \lambda_2$



Fiber cabling



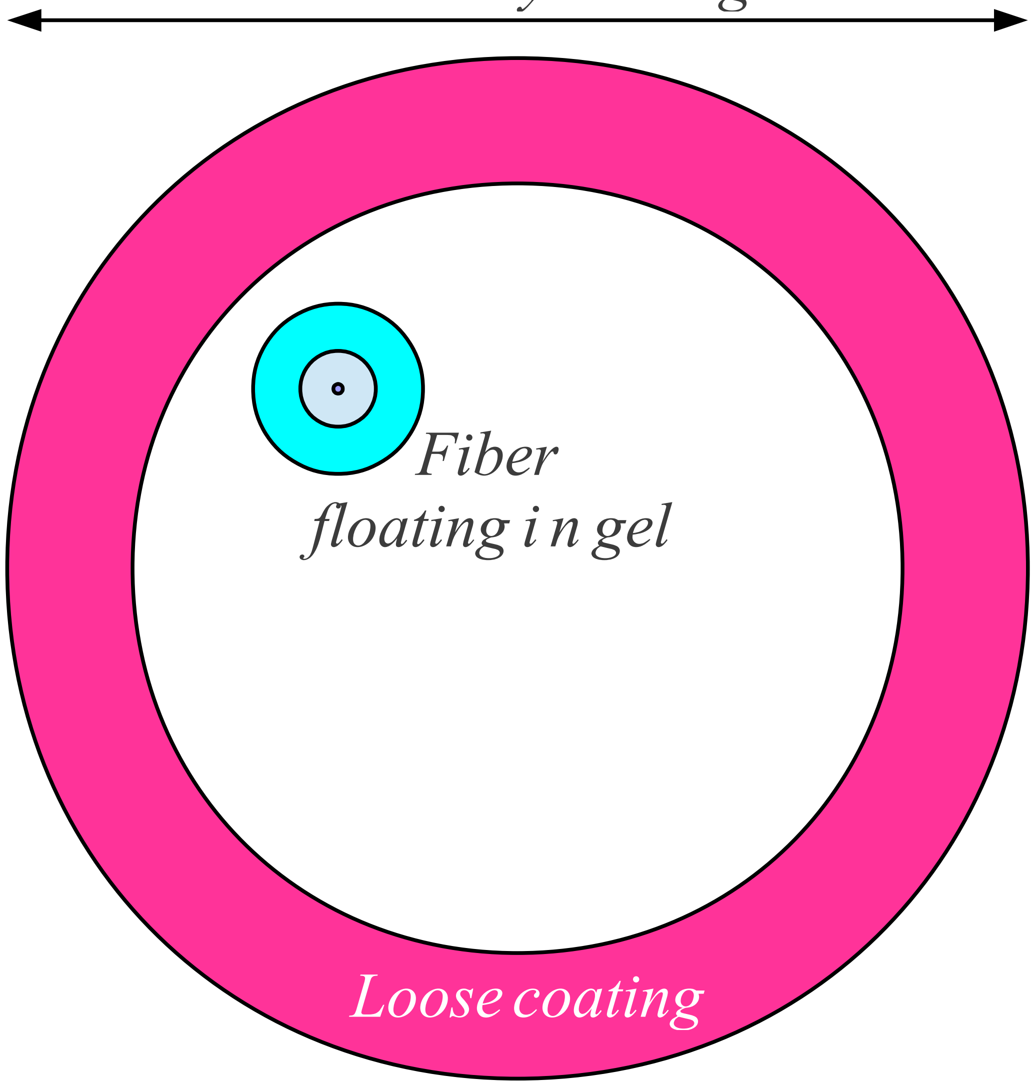
Primary coating
250 μm

Acrilate

SiO_2

Tight coating

Tight secondary coating
900 μm

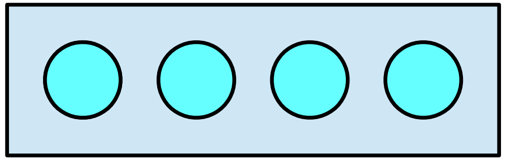


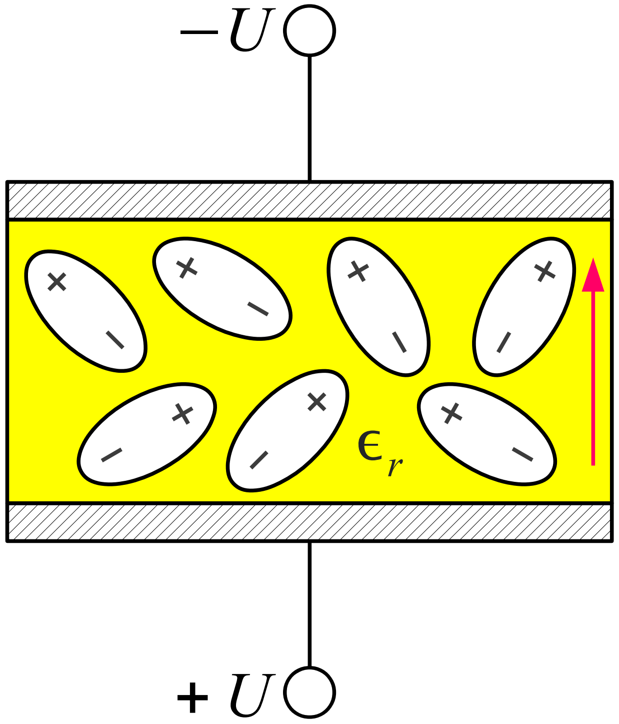
Loose secondary coating 3mm

Fiber floating in gel

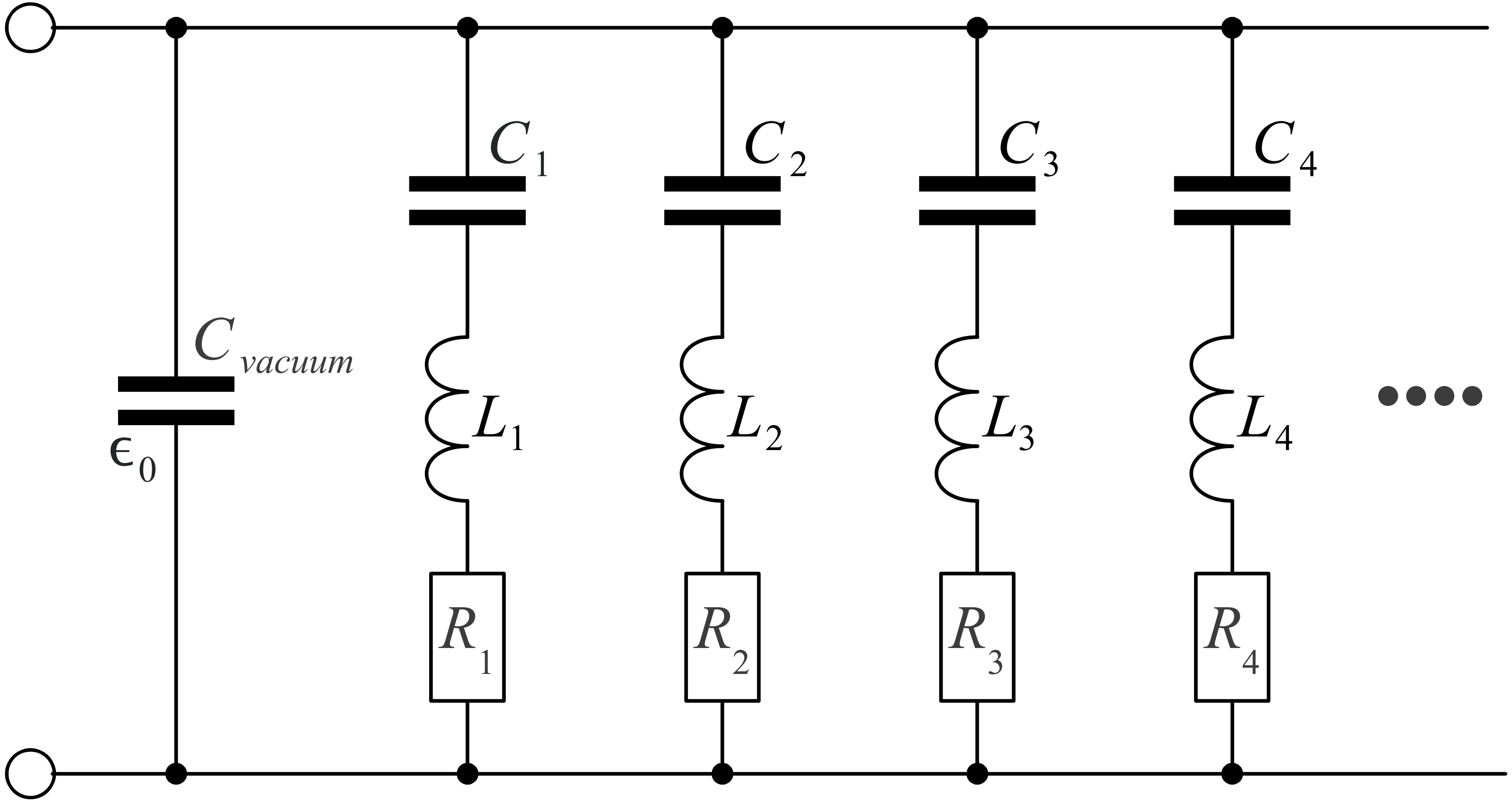
Loose coating

Ribbon 4 ... 24 *fibers*





$$\vec{D}=\epsilon_0\vec{E}+\vec{P}=\epsilon_0\epsilon_r\vec{E}$$

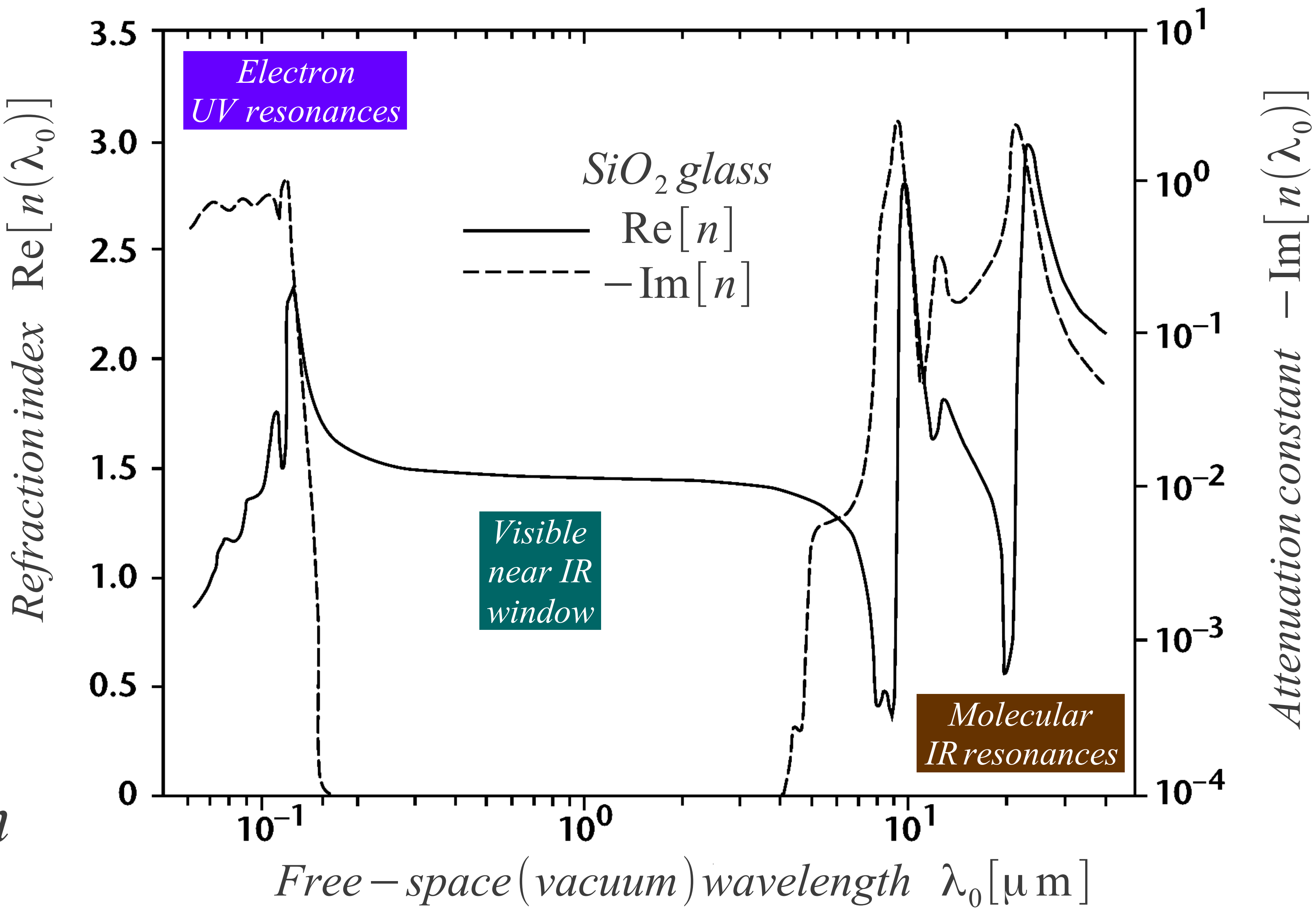


Electron UV resonances $f > 1500\text{THz}$

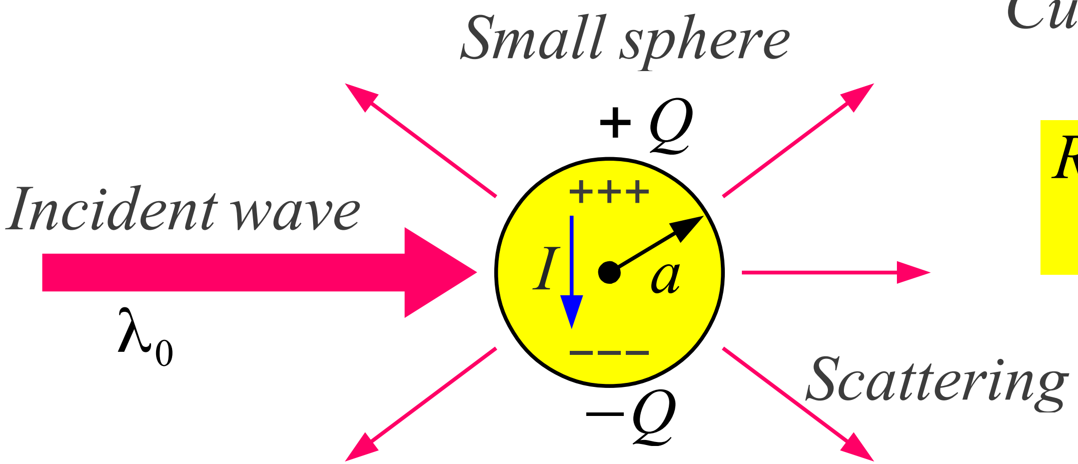
SiO_2 glass

Molecular IR resonances $f < 50\text{THz}$

Equivalent circuit of a dielectric



Complex refraction index



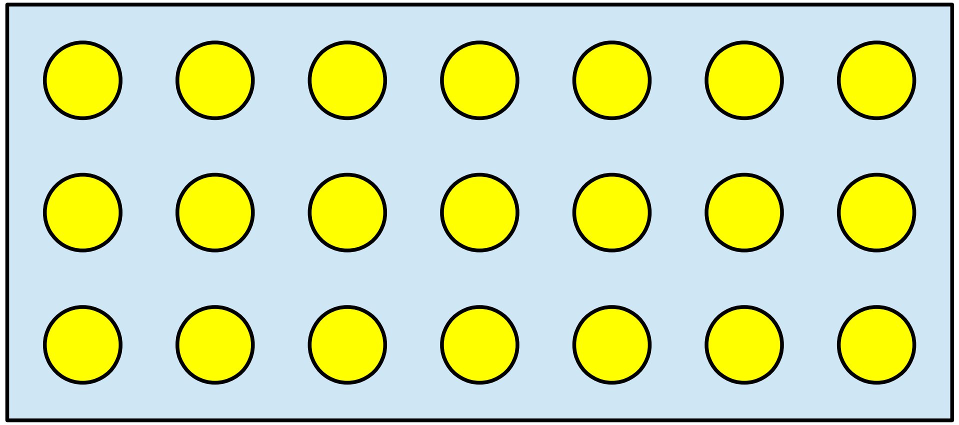
Current / charge continuity: $j\omega Q = I \rightarrow$ radiation!

Rayleigh
$a \ll \lambda_0$

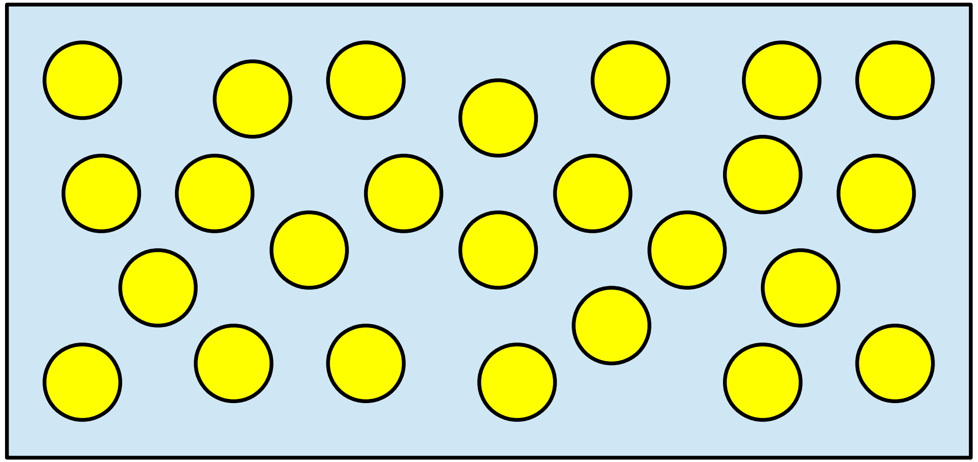
RADAR cross section of a metal sphere $\sigma = 64\pi^5 \dfrac{a^6}{\lambda_0^4}$

RADAR cross section of a dielectric sphere $\sigma = 64\pi^5 \dfrac{a^6}{\lambda_0^4} \left| \dfrac{\epsilon_r - 1}{\epsilon_r + 2} \right|^2$

SiO_2 crystal → Bragg diffraction

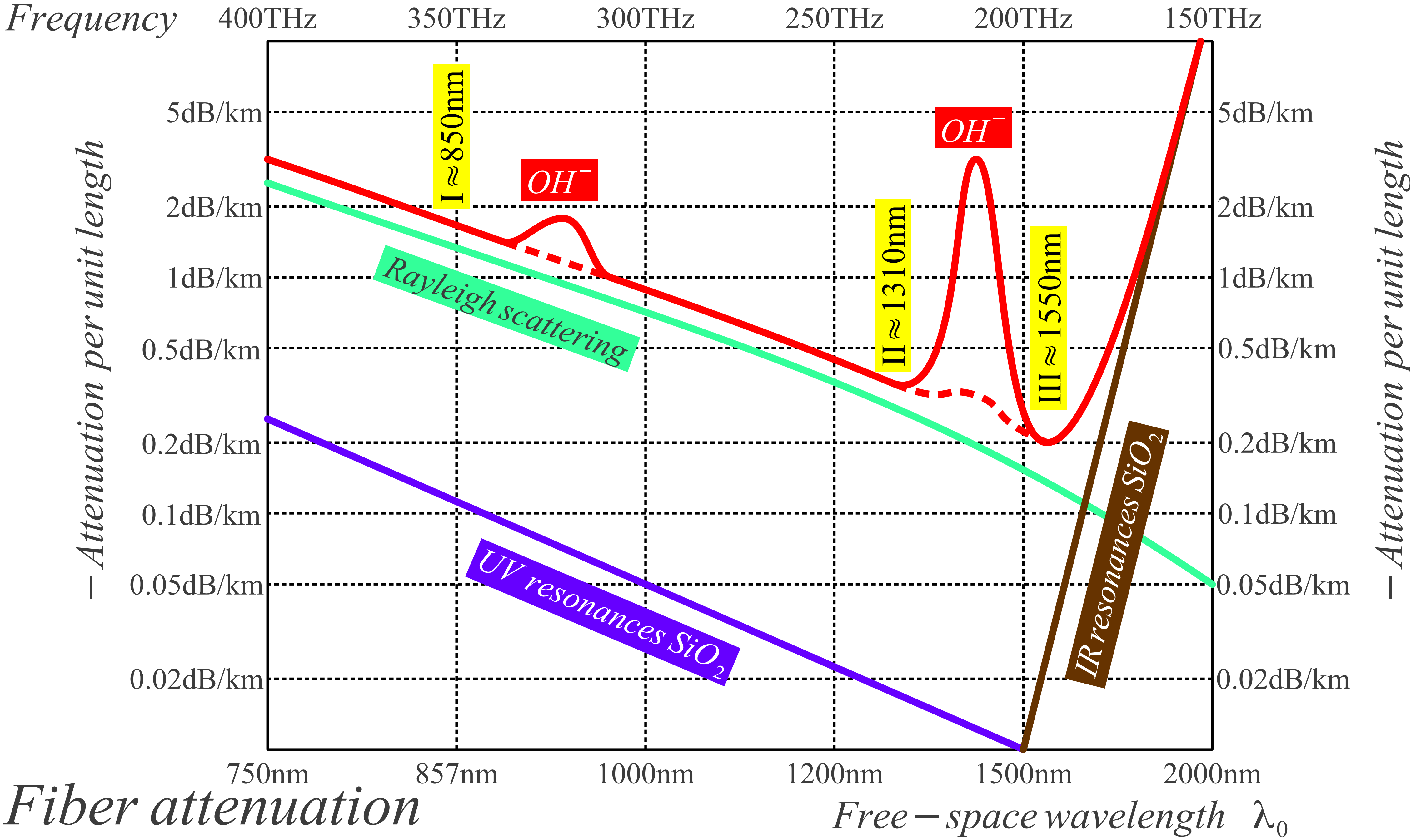


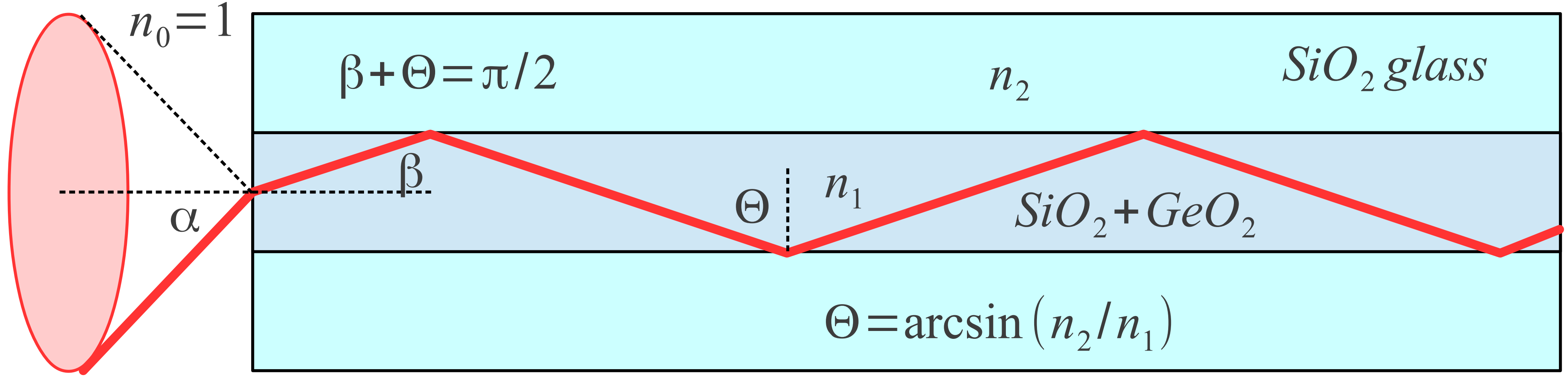
SiO_2 glass → Rayleigh scattering



Rayleigh scattering

Fiber attenuation





$n_2 \approx 1.463$

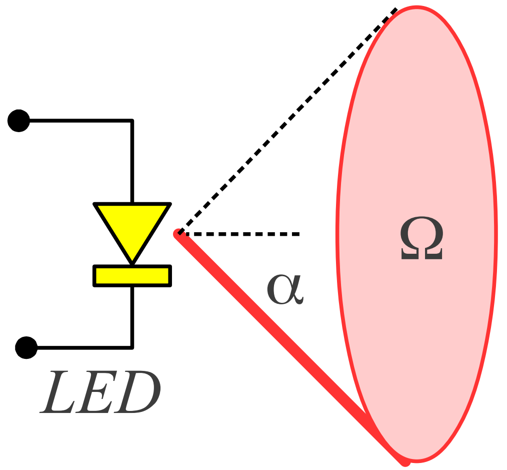
$n_1 \approx 1.476$

$\Delta \approx 0.009$

$NA = \sin\alpha$

$$\text{Relative index difference} \equiv \Delta = \frac{n_1 - n_2}{n_1} \ll 1 \;\rightarrow\; \text{weakly}-\text{guiding fiber}$$

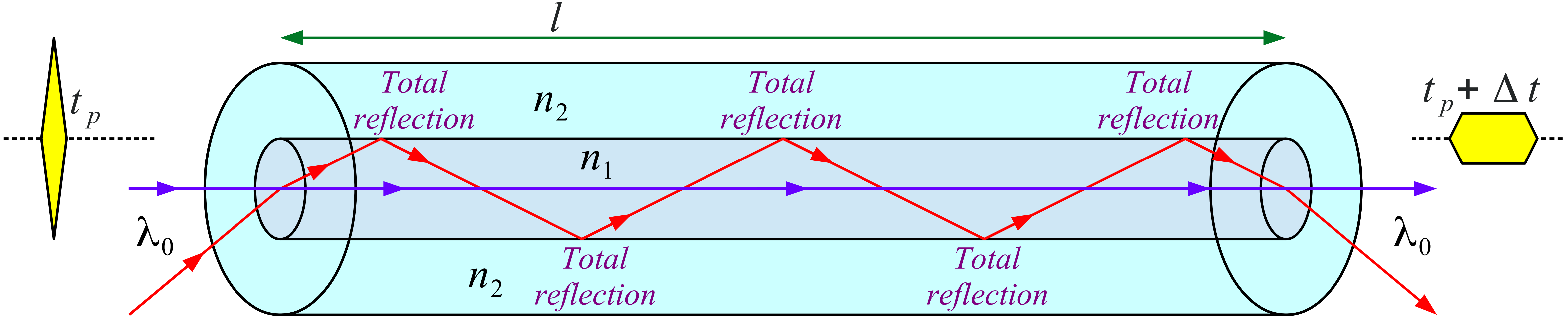
$$\text{Numerical aperture} \equiv NA = \sin\alpha = \frac{n_1}{n_0}\sin\beta = n_1\cos\Theta = \sqrt{n_1^2 - n_2^2} \approx n_1\sqrt{2\Delta} \approx 0.2$$



$$\text{Coupling efficiency} \equiv \eta = \frac{\Omega}{4\pi} = \frac{2\pi(1-\cos\alpha)}{4\pi} \approx \frac{NA^2}{4} \approx 0.01 = 1\%$$

$$\cos\alpha = \sqrt{1 - NA^2} \approx 1 - \frac{NA^2}{2}$$

Numerical aperture



Straight ray (blue): $t_1 = \frac{n_1 l}{c_0}$ *Zigzag ray* (red): $t_2 = \frac{n_1 l}{c_0 \sin\Theta_m} = \frac{n_1^2 l}{n_2 c_0}$

Multipath dispersion: $\Delta t = t_2 - t_1 = \frac{n_1 l}{c_0} = \left(\frac{n_1 - n_2}{n_2}\right)\frac{n_1 l}{c_0} \approx \Delta \cdot \frac{n_1 l}{c_0} \approx \frac{NA^2 l}{2 n_1 c_0}$

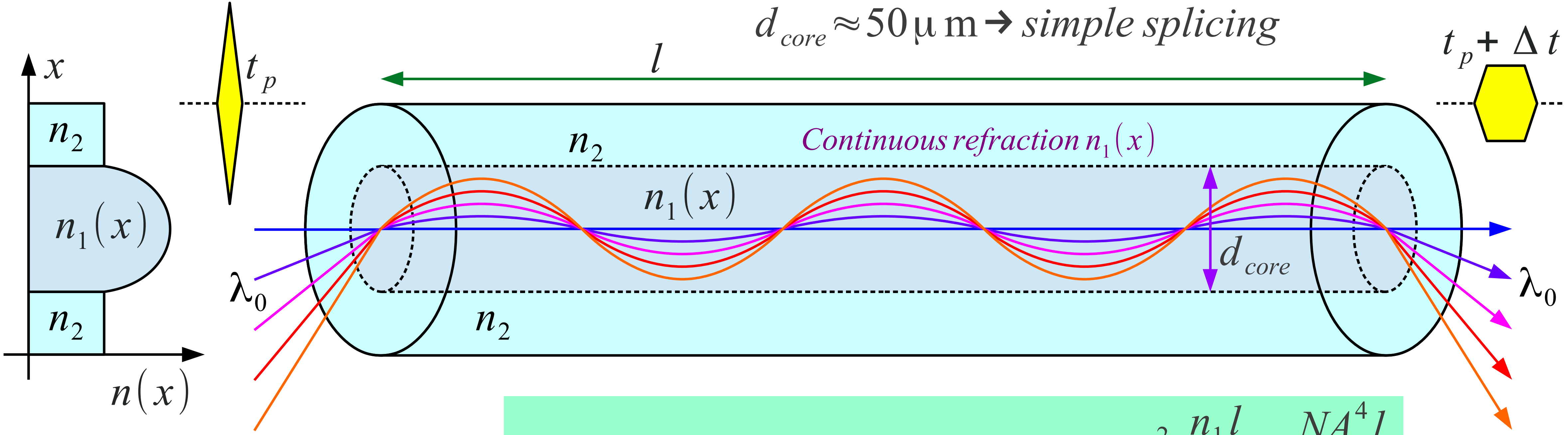
Example: $NA = 0.2$ $l = 10\text{km}$ $n_1 = 1.47$ → $\Delta t \approx 0.45\,\mu\text{s}$ → $C \approx \frac{1}{3\Delta t} \approx 740\text{kbit/s}$

Multipath (multimode) dispersion

Useless !

Graded index

$d_{core} \approx 50\,\mu\mathrm{m} \rightarrow$ *simple splicing*



$$NA = \sqrt{n_{1\,MAX}^2 - n_2^2}$$

Multipath dispersion: $\Delta t = t_2 - t_1 \approx \Delta^2 \cdot \frac{n_1 l}{c_0} \approx \frac{NA^4 l}{4 n_1^3 c_0}$

Example: $NA = 0.2$ $l = 10\mathrm{km}$ $n_1 = 1.47$ → $\Delta t \approx 4.2\mathrm{ns}$ → $C \approx \frac{1}{3\Delta t} \approx 80\mathrm{Mbit/s}$

$$\Delta_{MAX} \approx \frac{NA^2}{2 n_1^2} \approx 0.009$$

Standardized fiber GI 50/125 ≡ *ITU* G.651

First links ~1980 $C \sim 8\mathrm{Mbit/s}$ *LED* ~850nm

Use ~2020: *cheapest SFP modules* $C \le 1\mathrm{Gbit/s}$ @ $l \le 100\mathrm{m}$

Total reflection: $\Gamma = \frac{a - jb}{a + jb} = e^{j\phi}$

$$a_{TE} = \cos\Theta \qquad a_{TM} = (n_2/n_1)^2 \cos\Theta$$

$$b = \pm\sqrt{\sin^2\Theta - (n_2/n_1)^2}$$

$$\phi(\Theta) = 2\arctan(b/a) \qquad 0 \le \phi < \pi$$

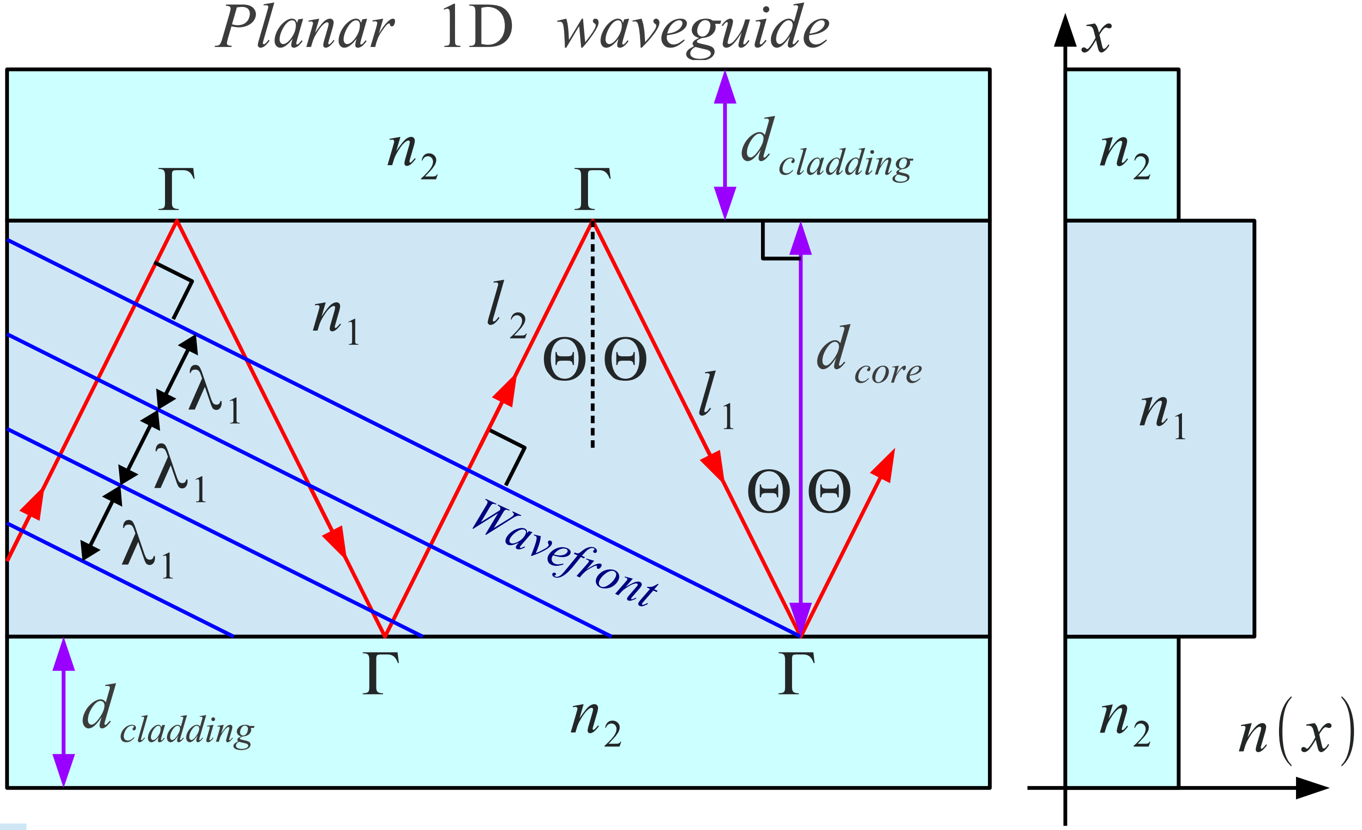
$$l_1 = \frac{d_{core}}{\cos\Theta} \qquad l_2 = l_1 \cdot \cos 2\Theta$$

$$l_1 + l_2 = \frac{d_{core}}{\cos\Theta}(1 + \cos 2\Theta) = 2 d_{core} \cos\Theta$$

$$\lambda_1 = \frac{\lambda_0}{n_1} \qquad k_1 = k_0 n_1 = \frac{2\pi}{\lambda_0} n_1 = \frac{\omega}{c_0} n_1$$

Transversal phase resonance

Planar 1D waveguide



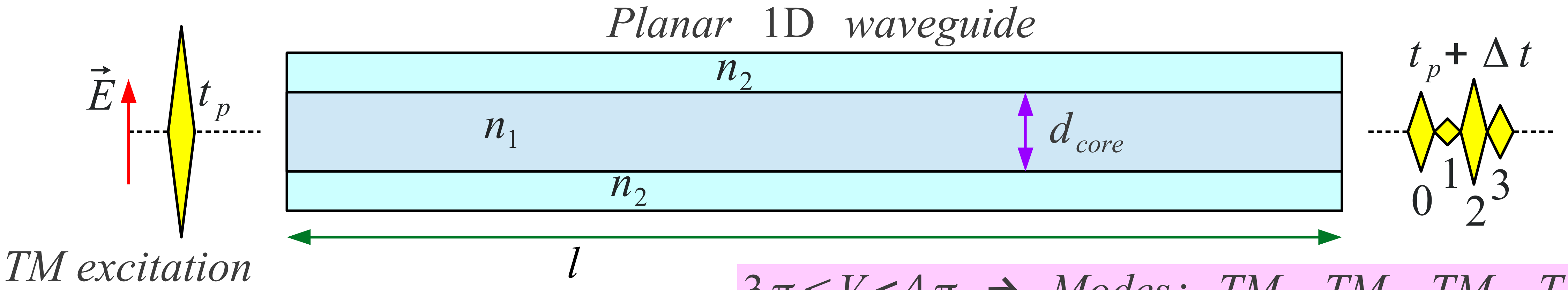
Step−index profile

$$(l_1 + l_2) k_1 - 2\phi(\Theta) = m \cdot 2\pi$$

$m = 0, 1, 2, 3, 4, 5 \ldots \equiv integer$

$$d_{core} \frac{\omega}{c_0} n_1 \cos\Theta - \phi(\Theta) = m \cdot \pi \rightarrow$$ $\Theta = ?$ TE or TM ?

1D waveguide modes

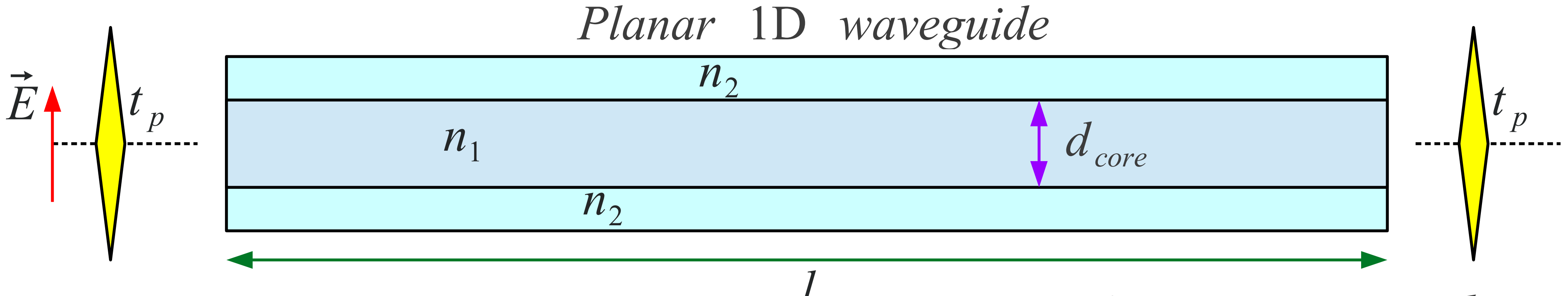


$3\pi \le V < 4\pi$ → $Modes:\ TM_0, TM_1, TM_2, TM_3$

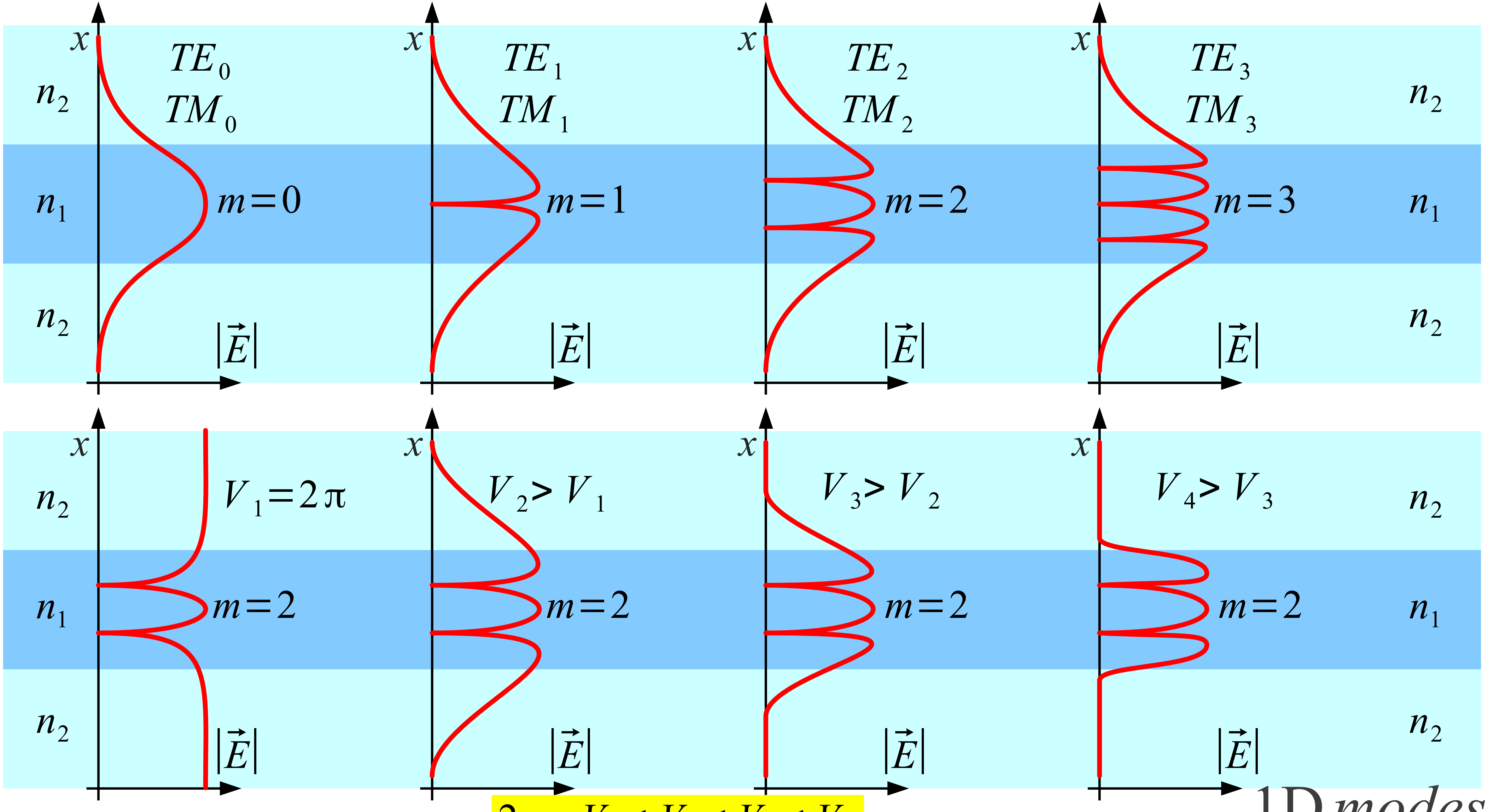
$$0 < n_1 \cos\Theta < n_1 \cos\Theta_m = n_1\sqrt{1-\sin^2\Theta_m} = n_1\sqrt{1-(n_2/n_1)^2} = \sqrt{n_1^2 - n_2^2} = NA$$

$$\text{Normalized frequency} \equiv V = d_{core}\frac{\omega}{c_0} n_1 \cos\Theta = d_{core}\frac{\omega}{c_0} NA$$

$0 \le V < \pi$ → $Mode:\ TM_0$

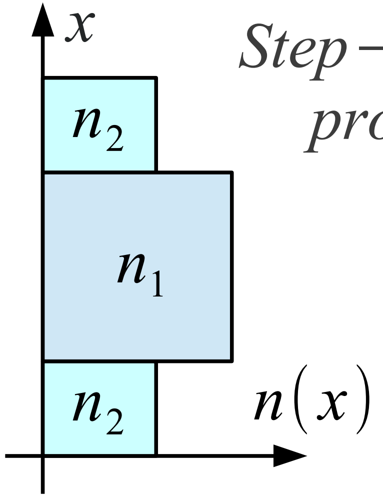


$V < 1$ → *Field escapes into cladding*

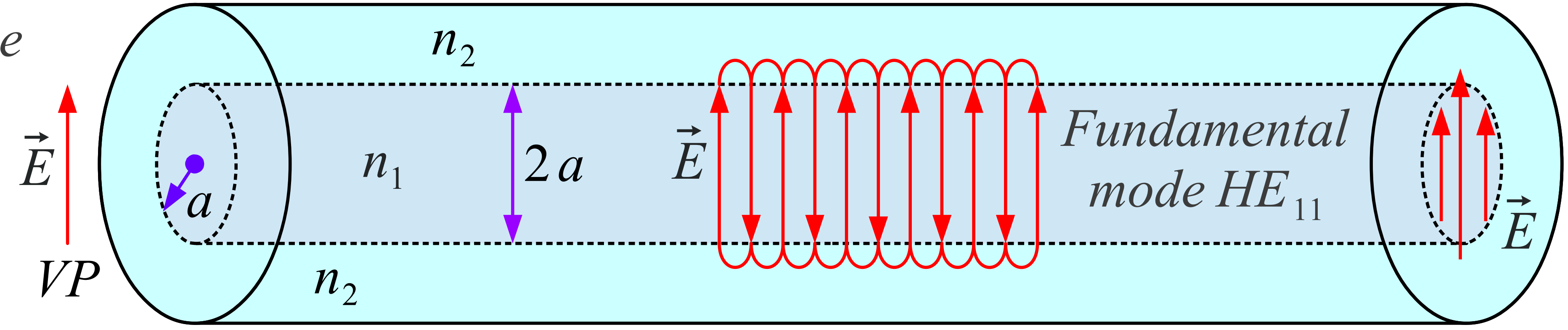


$2\pi=V_1< V_2< V_3< V_4$

1D *modes*



Fiber ≡ 2D cylindrical waveguide



HE_{11} has two variants HP + VP !

Normalized frequency $\equiv V = a\dfrac{2\pi}{\lambda_0}NA = a\dfrac{\omega}{c_0}NA$

$0 \le V < 2.4049 \rightarrow HE_{11}$ only

$V < 1.8 \rightarrow$ Field escapes into cladding

$2.4049 \le V < 3.8318 \rightarrow 4$ modes

$3.8318 \le V \rightarrow \ge 7$ modes

Bessel: $J_0(2.4049) = 0 \quad J_1(3.8318) = 0$

Standardized single-mode fiber
9/125 or 10/125 ≡ ITU G.652*

$2a \approx 9 \ldots 10\,\mu\mathrm{m} \qquad NA \approx 0.1$

$V(1.31\,\mu\mathrm{m}) \approx 2.16 \ldots 2.40$

$V(1.55\,\mu\mathrm{m}) \approx 1.82 \ldots 2.03$

G.652* used from 1984
Tolerance tightening* !

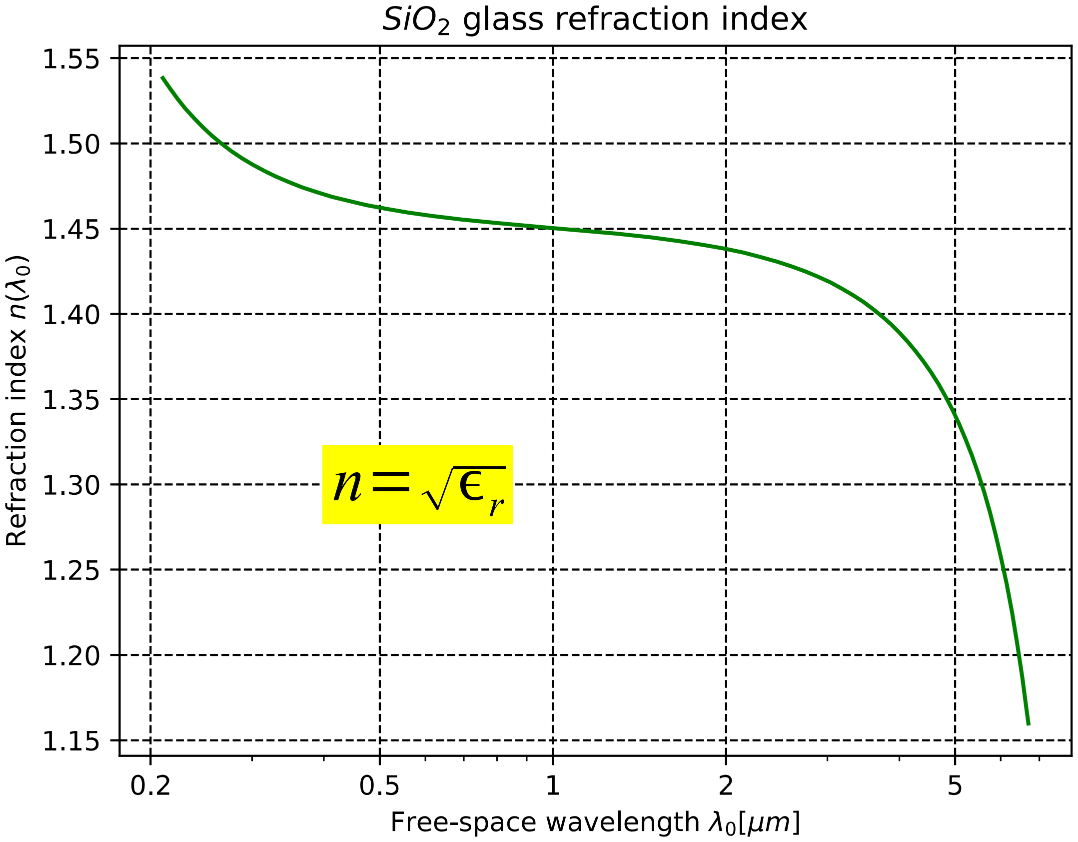
Fiber waveguide modes

Speed of light in SiO_2 glass

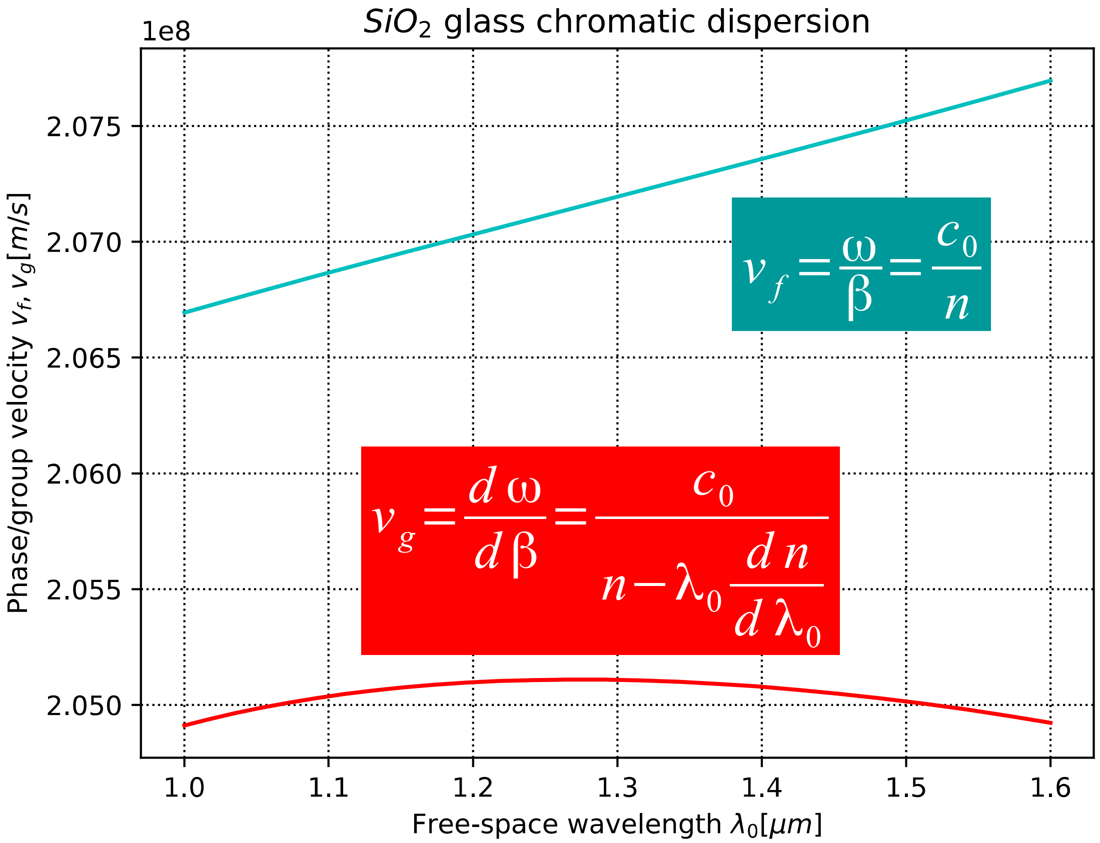
Sellmeier for SiO_2 glass $\lambda_0 = 0.21 \ldots 6.7\,\mu\mathrm{m}$ *Loss-less!*

$$\epsilon_r = 1 + \frac{0.6961663 \cdot \lambda_0^2}{\lambda_0^2 - 0.00467914826} + \frac{0.4079426 \cdot \lambda_0^2}{\lambda_0^2 - 0.0135120631} + \frac{0.8974794 \cdot \lambda_0^2}{\lambda_0^2 - 97.9340025}$$

Group velocity v_g slowed down by the reactive power (stored energy) of oscillating SiO_2 electrons/molecules



$$n = \sqrt{\epsilon_r}$$



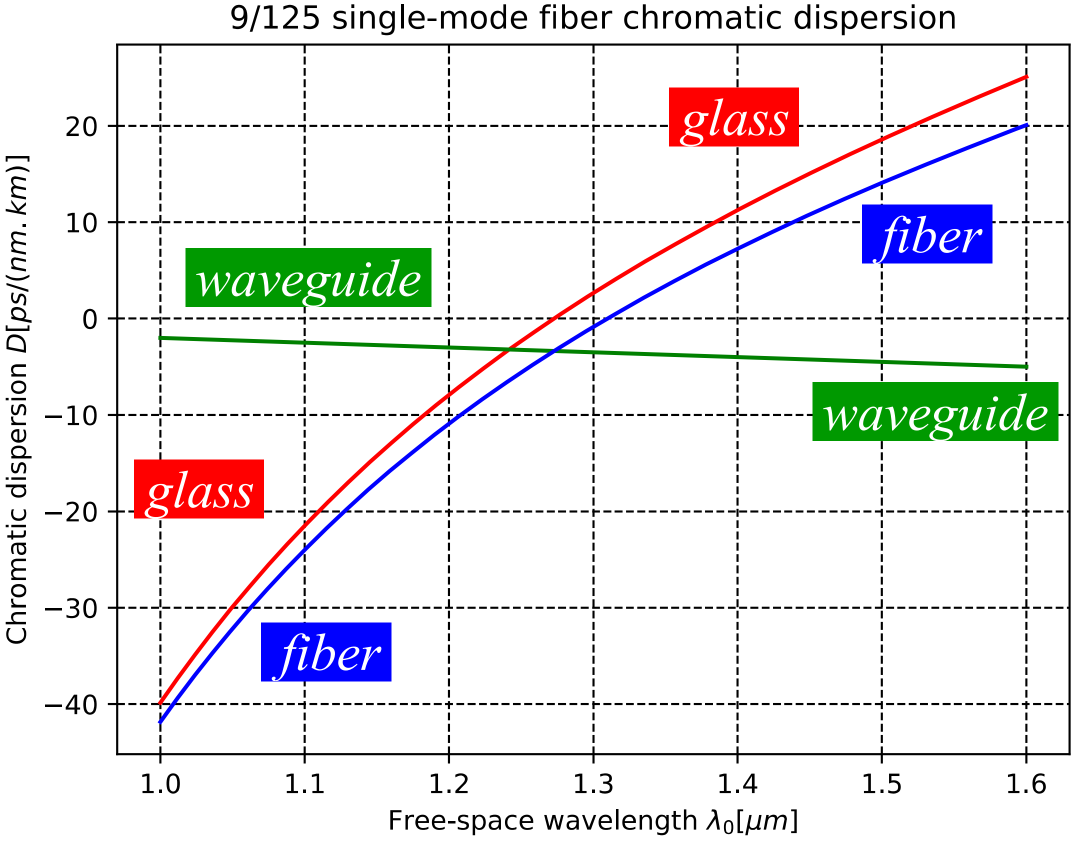
$$v_f = \frac{\omega}{\beta} = \frac{c_0}{n}$$

$$v_g = \frac{d\,\omega}{d\,\beta} = \frac{c_0}{n - \lambda_0 \dfrac{d\,n}{d\,\lambda_0}}$$

$$t_g = \frac{l}{v_g} \qquad \text{Chromatic dispersion:} \quad \Delta t = D\cdot l\cdot \Delta\lambda_0$$

Group velocity v_g slowed down by the reactive power (stored energy) of
(1) oscillating SiO_2 electrons/molecules
(2) waveguide transversal phase resonance

9/125 single-mode fiber chromatic dispersion



Inexpensive FP laser: $\Delta\lambda_0 = 3\text{nm}$ $\quad l = 50\text{km} \quad C \approx \frac{1}{3\Delta t}$

$$|D(1.31\,\mu\text{m})| < 2\frac{\text{ps}}{\text{nm}\cdot\text{km}} \rightarrow \Delta t < 300\text{ps} \rightarrow C > 1.1\text{Gbit/s}$$

$$D(1.55\,\mu\text{m}) \approx 17\frac{\text{ps}}{nm\cdot km} \rightarrow \Delta t \approx 2.55\text{ns} \rightarrow C \approx 130\text{Mbit/s}$$

$$DFB\ laser + LiNbO_3\ EOM \rightarrow \frac{1}{\Delta t} \approx C \approx \Delta f = \Delta\lambda_0 \frac{c_0}{\lambda_0^2}$$

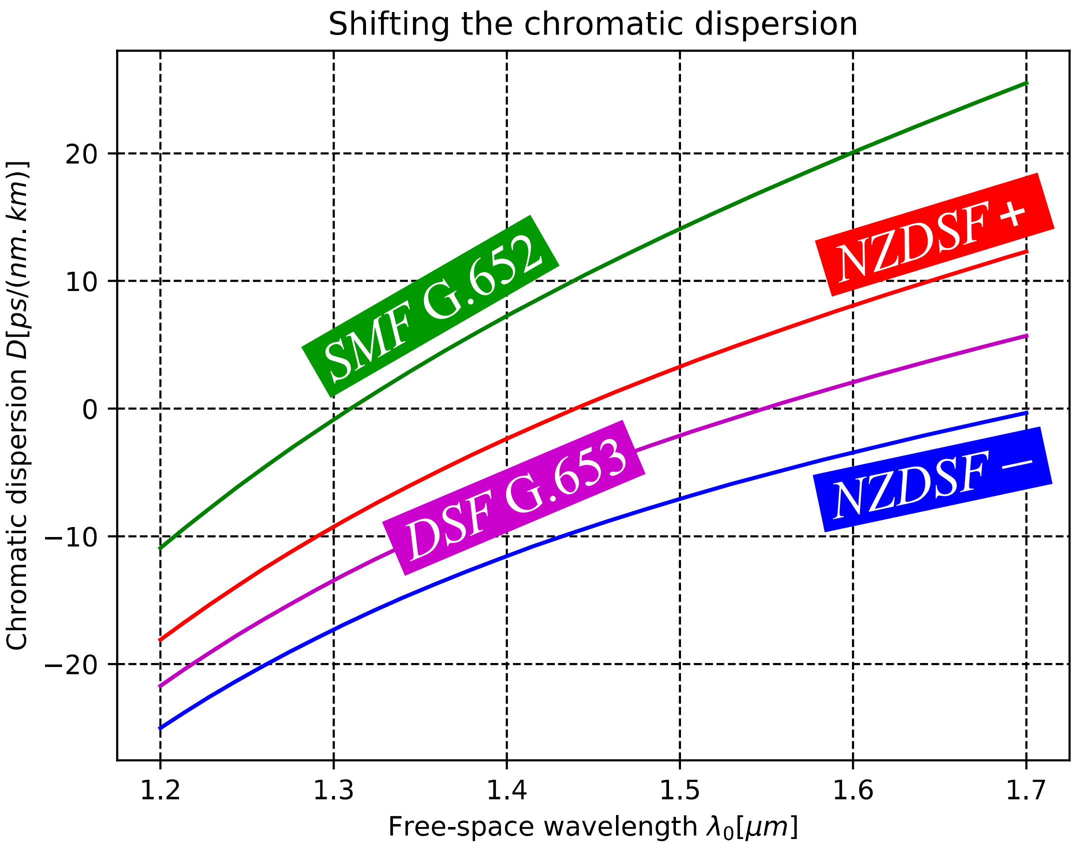
$$D\left[\frac{\text{ps}}{\text{nm}\cdot\text{km}} = 10^{-6}\frac{\text{s}}{\text{m}^2}\right] = \frac{1}{l}\frac{d\,t_g}{d\,\lambda_0} = -\frac{1}{v_g^2}\frac{d\,v_g}{d\,\lambda_0}$$

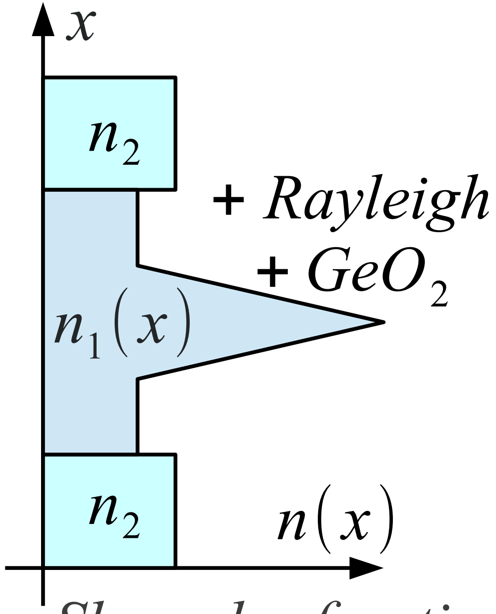
$$l = \frac{\Delta t}{D\cdot\Delta\lambda_0} \approx \frac{c_0}{D\lambda_0^2 C^2} \qquad \lambda_0 = 1550\text{nm} \quad D \approx 17\ \text{ps}/(nm\cdot km)$$

C	2.5Gbit/s	10Gbit/s	40Gbit/s	100Gbit/s
l	1175km	73km	4.59km	0.73km

Fiber chromatic dispersion

$Dispersion-shifted\ fibers$





$Shaped\ refraction$
$index\ profile$

$NZDSF \equiv Non-Zero$
$Dispersion-Shifted\ Fiber$

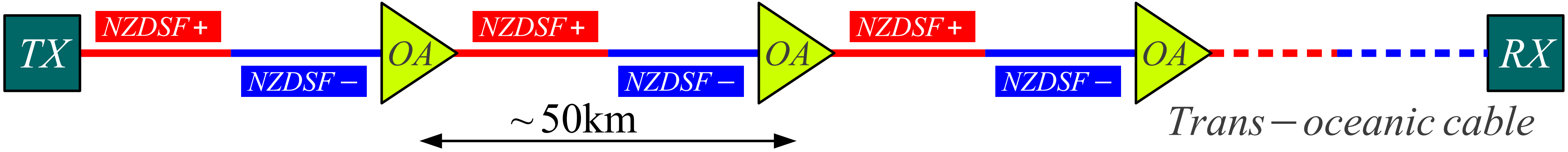
$$NZDSF+\ \equiv ITU\ \mathrm{G.655}$$
$$D(1550\mathrm{nm})\approx +4\ldots+7\frac{\mathrm{ps}}{\mathrm{nm.km}}$$
$$A_{core}\to 80\,\mu\,\mathrm{m}^2(LEAF)$$

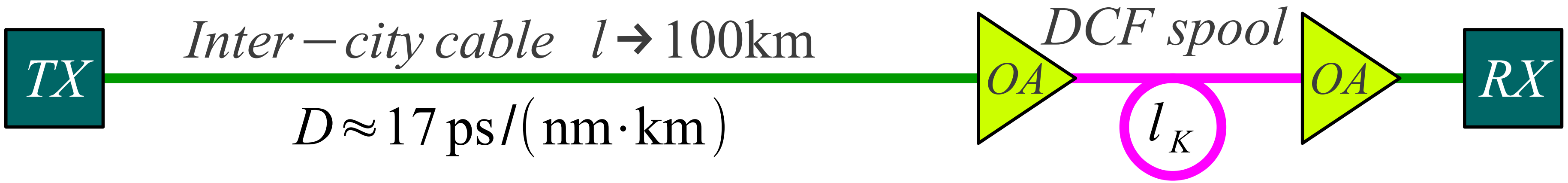
$$NZDSF-$$
$$D(1550\mathrm{nm})\approx -4\ldots-7\frac{\mathrm{ps}}{\mathrm{nm.km}}$$

$$DSF\ \ (8/125)\ \equiv ITU\ \mathrm{G.653}$$
$$D(1550\mathrm{nm})\approx 0\frac{\mathrm{ps}}{\mathrm{nm.km}}$$
$$a/l\approx 0.5\mathrm{dB/km}\to 0.25\mathrm{dB/km}$$
$$A_{core}\approx 30\,\mu\,\mathrm{m}^2$$
$Useless\ due\ to\ nonlinearity$

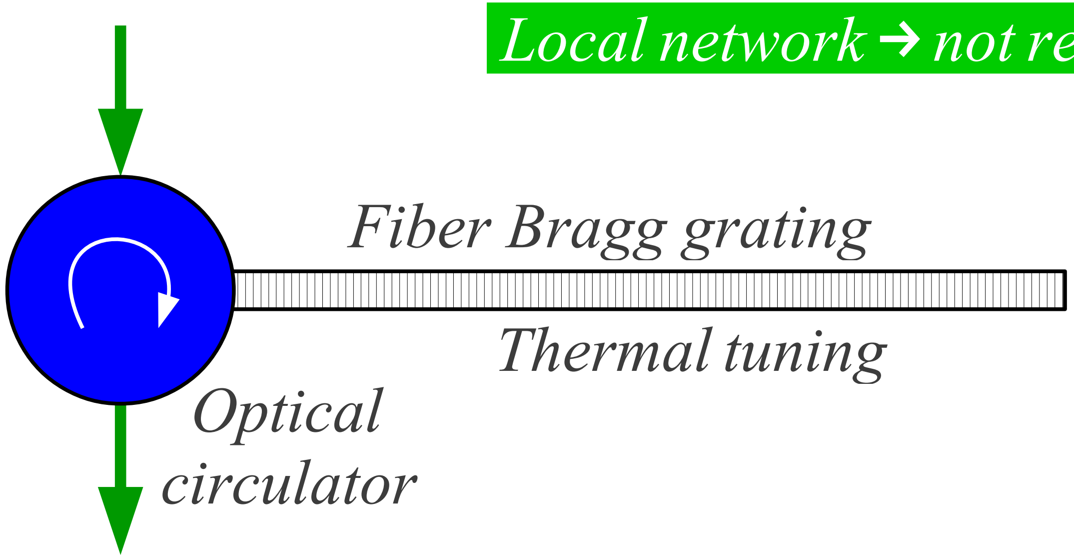
$$9/125\ \ (10/125)\equiv ITU\ \mathrm{G.652}$$
$$D(1550\mathrm{nm})\approx +17\frac{\mathrm{ps}}{\mathrm{nm\cdot km}}$$
$$a/l\approx 0.2\mathrm{dB/km}\to 0.15\mathrm{dB/km}$$
$$A_{core}\approx 70\,\mu\,\mathrm{m}^2$$

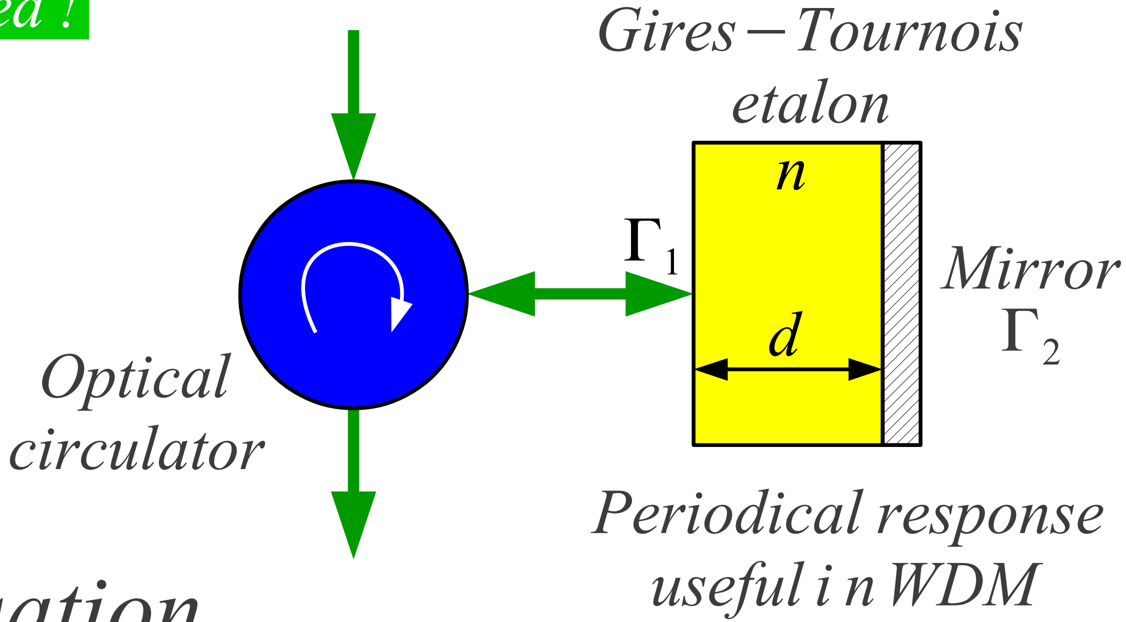
$Dispersion-compensating\ fiber$
$$D(1550\mathrm{nm})\approx -80\frac{\mathrm{ps}}{\mathrm{nm\cdot km}}$$
$$a/l\approx 1\mathrm{dB/km}$$





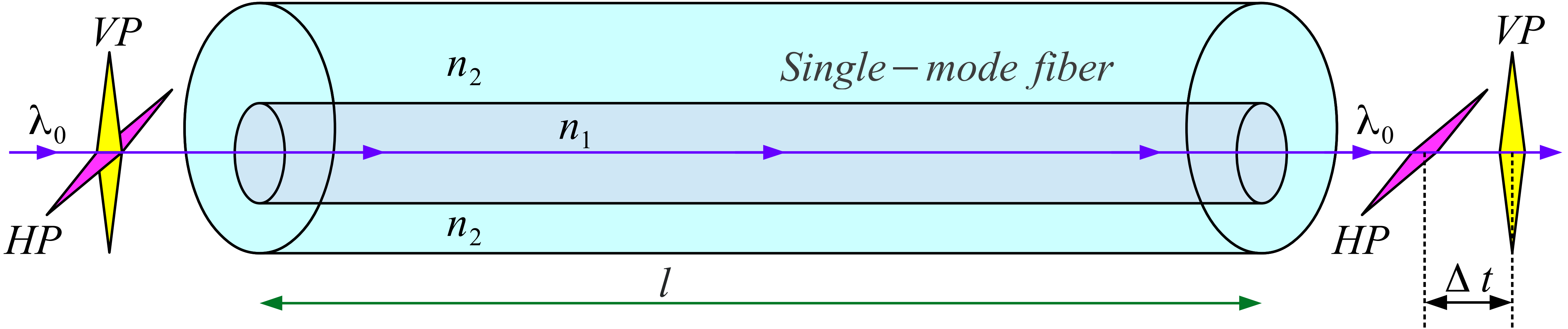
Local network → not required !





Chromatic − dispersion compensation

Eccentric core → Mechanical forces → Birefringence → PMD



HE_{11} has two variants HP + VP, that are not perfectly identical (manufacturing tolerances!)

PMD ≡ Polarization Mode Dispersion

Birefringence is distributed randomly!
Optical fiber does not maintain polarization!

$\Delta t \approx D_{PMD} \cdot \sqrt{l}$ @ $l > 1\text{km}$

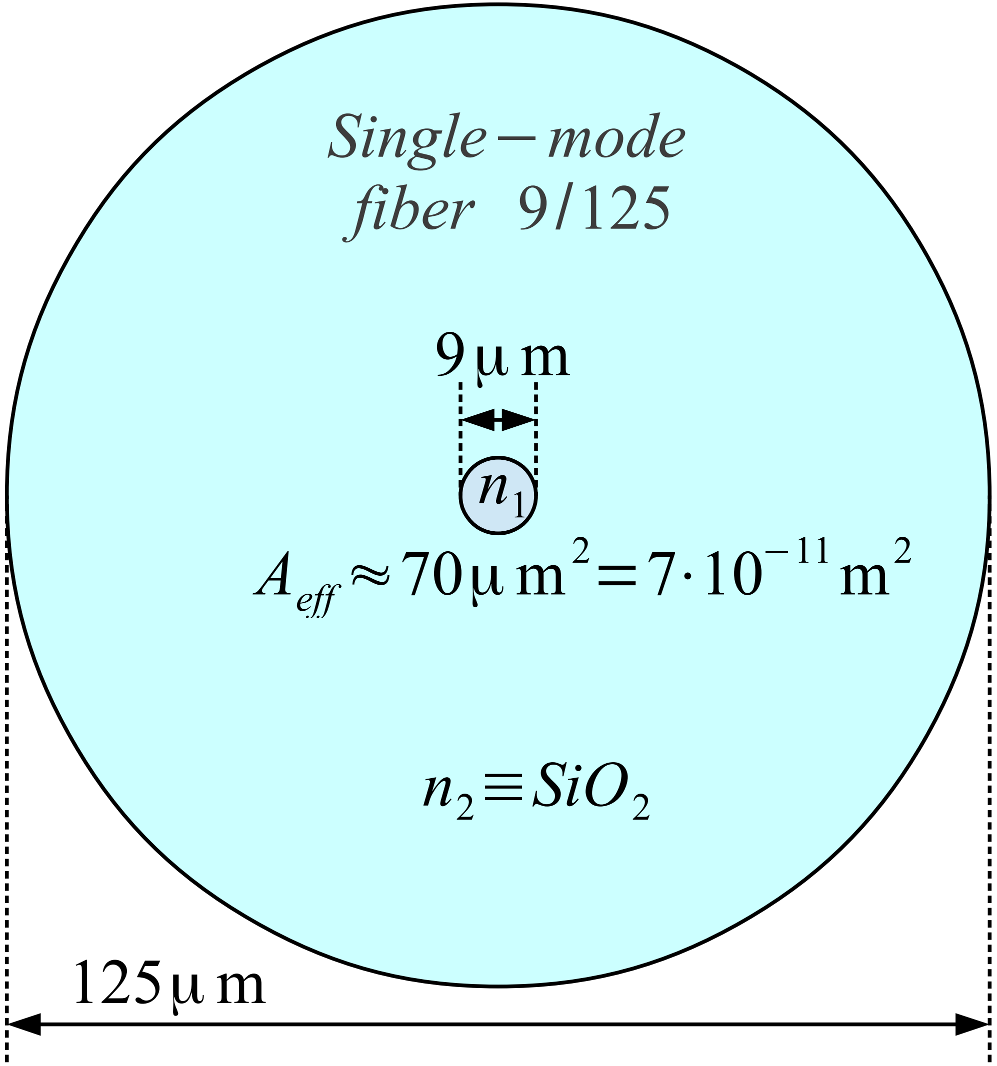
	D_{PMD}	$\Delta t(100\text{km})$	$\Delta t(10000\text{km})$
Old fibers <2000	~10ps/√km	~100ps	~1ns
Spun fibers >2000	~0.1ps/√km	~1ps	~10ps

Δt ≡ random!
Installed cable changing slowly: weeks, months ...

Spinning during fiber drawing ~5 *turns* / m

PMD

Fiber − material loading



Single − mode fiber 9/125

9 µm

n_1

$A_{eff} \approx 70\,\mu\mathrm{m}^2 = 7\cdot10^{-11}\,\mathrm{m}^2$

$n_2 \equiv SiO_2$

125 µm

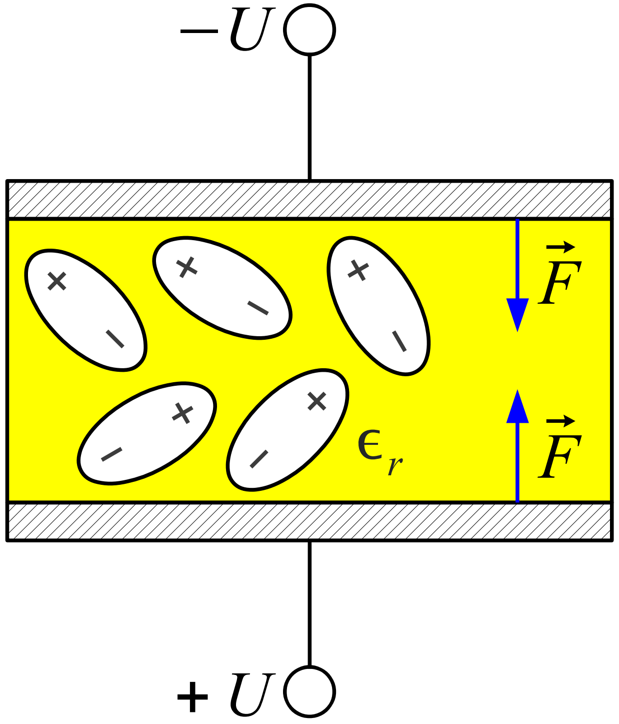
$n_1 \equiv SiO_2 + GeO_2$

Example: $P \approx 100\,\mathrm{mW}$ $n_1 \approx 1.46$ $Z_0 \approx 377\,\Omega$

$$S = \frac{P}{A_{eff}} \approx \frac{100\,\mathrm{mW}}{7\cdot10^{-11}\,\mathrm{m}^2} \approx 1.4\,\mathrm{GW/m^2} = 140\,\mathrm{kW/cm^2}$$

$$|\vec{E}| = \sqrt{\frac{2Z_0S}{n_1}} \approx 850\,\mathrm{kV/m} = 8.5\,\mathrm{kV/cm}$$

P	*Effect*
1mW	*Nonlinear response!*
10mW	*Connector burn-out!*
100mW	*Max P for connectors!*
1W	*Max P in fiber!*
10W	*Fiber-core melting!*



Electrostriction → *increase* $n=\sqrt{\epsilon_r}\uparrow$

$$n(E)=n_0+n_1'\cdot E+n_2'\cdot E^2+n_3'\cdot E^3+\dots$$

$n_2'\equiv$ *Kerr effect* (*electrostriction*)

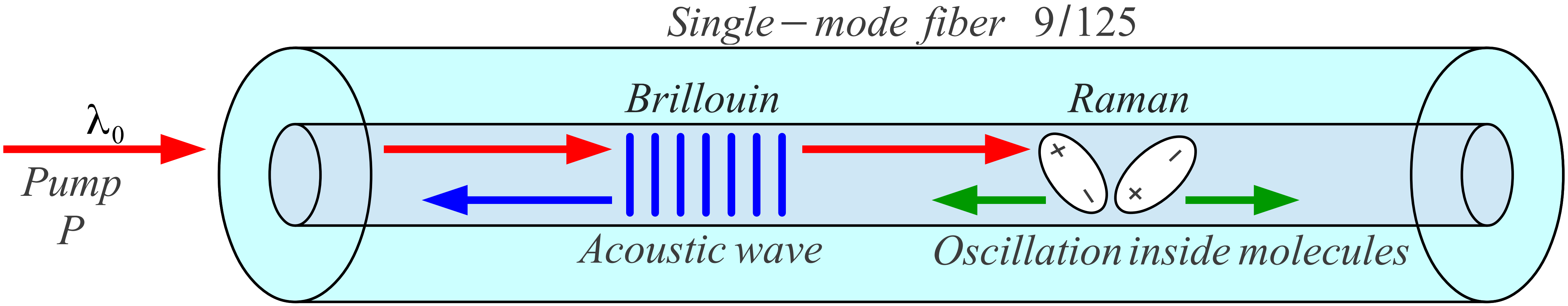
$n_1'\equiv$ *Pockels effect* (*electrostriction* + *internal* E_0)

$SiO_2(+GeO_2)$ *glass* → *internal* $E_0=0$ → $n_1'=0$

$$S=\frac{E^2}{2Z} \rightarrow n(S)=n_0+n_2\cdot S$$

$SiO_2(+GeO_2)$ *glass*: $n_0\approx 1.46$ $n_2\approx 2.5\cdot 10^{-20}\,\frac{\mathrm{m}^2}{\mathrm{W}}\dots 3.2\cdot 10^{-20}\,\frac{\mathrm{m}^2}{\mathrm{W}}$

Electrostriction in glass



$v \approx 5.9\,\text{km/s}$

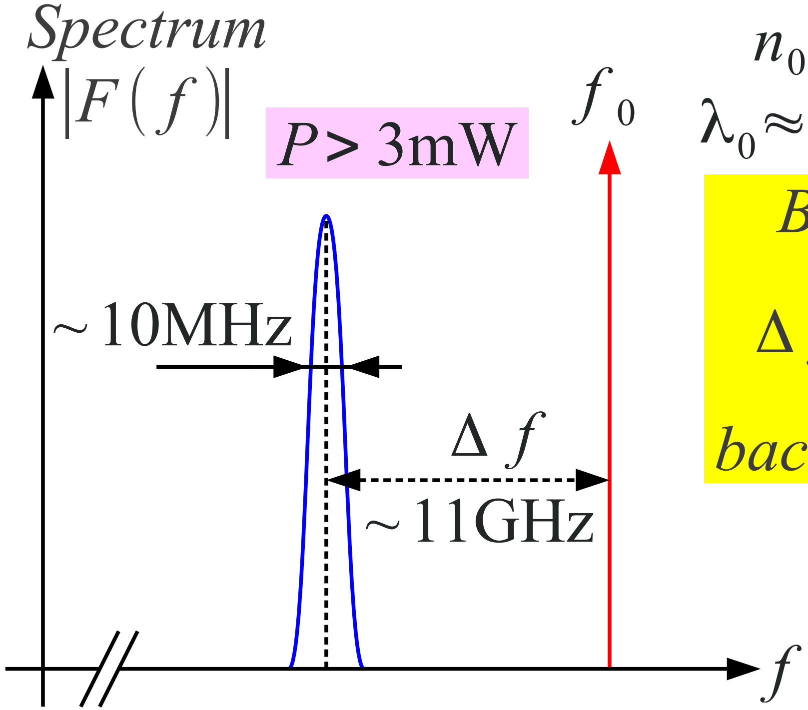
$n_0 \approx 1.46$

$\lambda_0 \approx 1.55\,\mu\text{m}$

Brillouin:

$$\Delta f = \frac{2 v n_0}{\lambda_0}$$

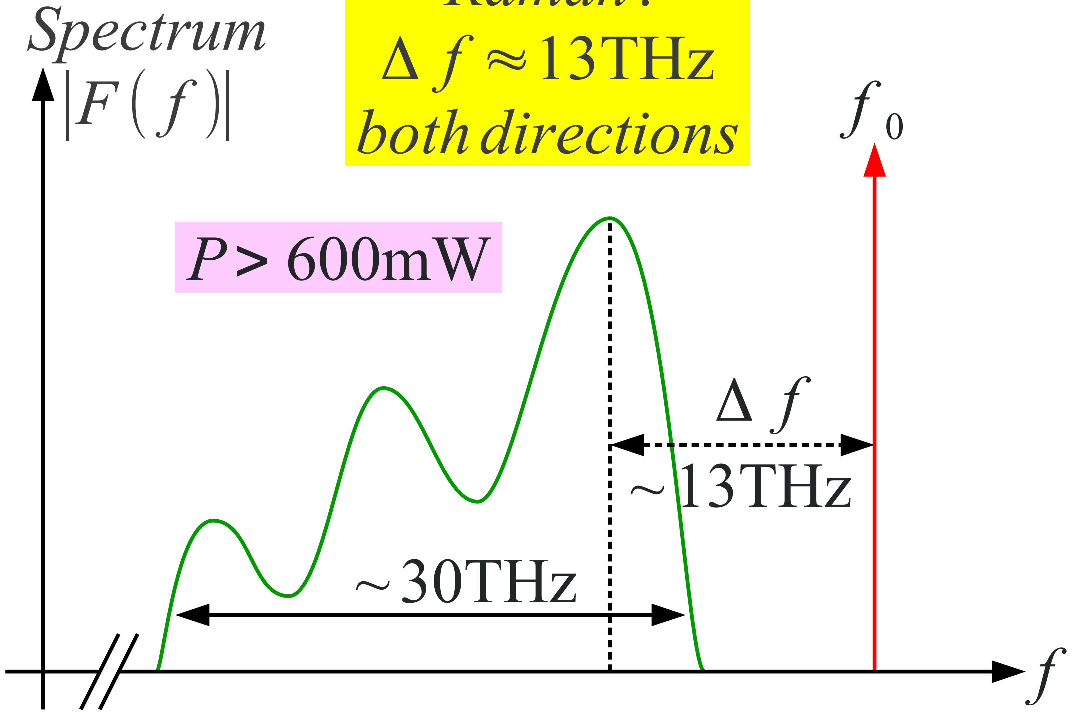
backward only



Brillouin scattering

Nuisance?
Amplifier?
Sensor?

Raman:
$\Delta f \approx 13\,\text{THz}$
both directions



Raman scattering

Self phase modulation: $\Delta\phi = \Delta k \cdot l = \frac{2\pi n_2 P}{\lambda_0 A_{eff}} \cdot l \approx 0.15\,\mathrm{rd}$

$P = 100\,\mathrm{mW} \quad A_{eff} = 70\,\mu\mathrm{m}^2$

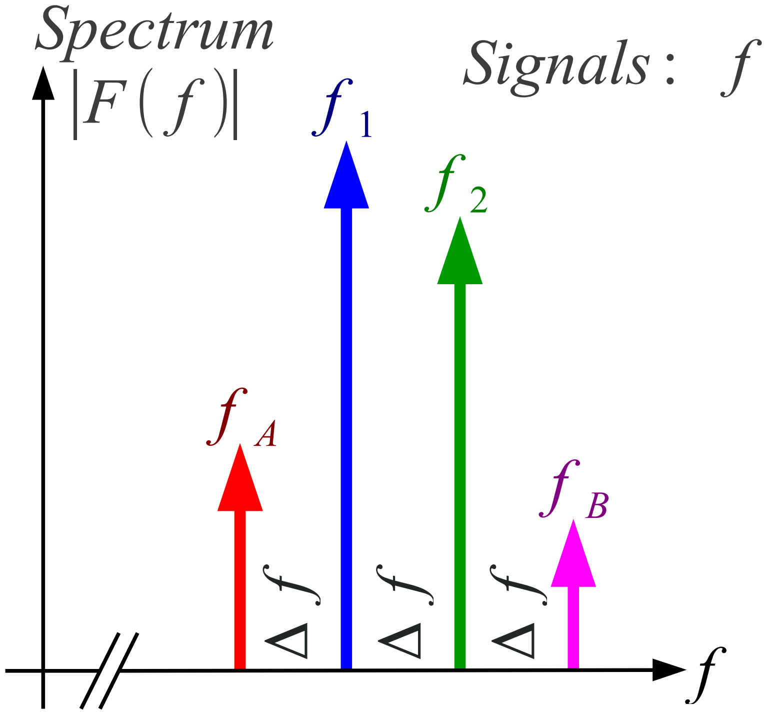
$n_2 \approx 2.5 \cdot 10^{-20}\,\mathrm{m}^2/\mathrm{W}$

$\lambda_0 = 1550\,\mathrm{nm} \quad l = 1\,\mathrm{km}$

Linear D – Nonlinear n → *Soliton transmission ~ 1995 ?*

Cross phase modulation → *Four – Wave Mixing (FWM)*

Radio engineers 50 years earlier than optics : Inter – Modulation Distortion (IMD)



Signals : $f_1 \quad f_2$ → *Mixing products :* $f_A = 2f_1 - f_2 \quad f_B = 2f_2 - f_1$

$$P_A = \frac{P_1^2 P_2}{P_{IP3}^2}$$

$$P_B = \frac{P_1 P_2^2}{P_{IP3}^2}$$

$$P_{IP3}[\mathrm{W}] = \frac{\lambda_0 A_{eff}}{2\pi n_2 l_{eff}}$$

$$\alpha\left[\frac{\mathrm{Np}}{\mathrm{m}}\right] = \frac{-\ln 10}{20} a/l \left[\frac{\mathrm{dB}}{\mathrm{m}}\right]$$

Long fiber : $l \gg l_{eff}[\mathrm{m}] = \frac{1}{\sqrt{(2\alpha)^2 + (\Delta\beta)^2}}$

Four – Wave Mixing

Phase mismatch : $\Delta\beta\left[\frac{\mathrm{rd}}{\mathrm{m}}\right] = \beta_2 + \beta_A - 2\beta_1 \approx -\frac{2\pi\lambda_0^2 D}{c_0} \cdot (\Delta f)^2$

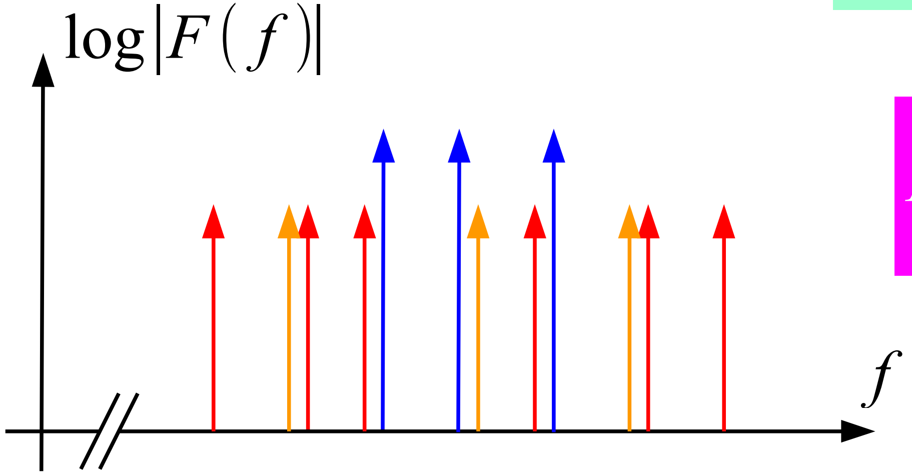
$\sim 1995 \rightarrow \mathit{DSF}\ \mathrm{G.653}$

$a/l \approx -0.3\mathrm{dB/km} \qquad A_{\mathit{eff}} \approx 30\mu m^2$

$D \approx 0 \rightarrow \Delta\beta \approx 0$

$$l_{\mathit{eff}} \approx \frac{1}{2\alpha} \approx 14.5\mathrm{km}$$

$$P_{IP3} \approx 20\mathrm{mW} = +\ 13\mathrm{dBm}$$



$4\ \mathit{channels}\ \mathit{WDM} \times 2.5\mathrm{Gb/s} = 10\mathrm{Gb/s}$

$\mathit{Trans-oceanic\ cable}$

$$\lambda_0 \approx 1550\mathrm{nm}$$

$$n_2 = 2.5 \cdot 10^{-20} \frac{\mathrm{m}^2}{\mathrm{W}}$$

$$B \approx 4\mathrm{THz}$$

$$P_{FWM} = \frac{P^3}{P_{IP3}^2}$$

$\sim 2015 \rightarrow \mathit{NZDSF}\ \mathrm{G.655}$

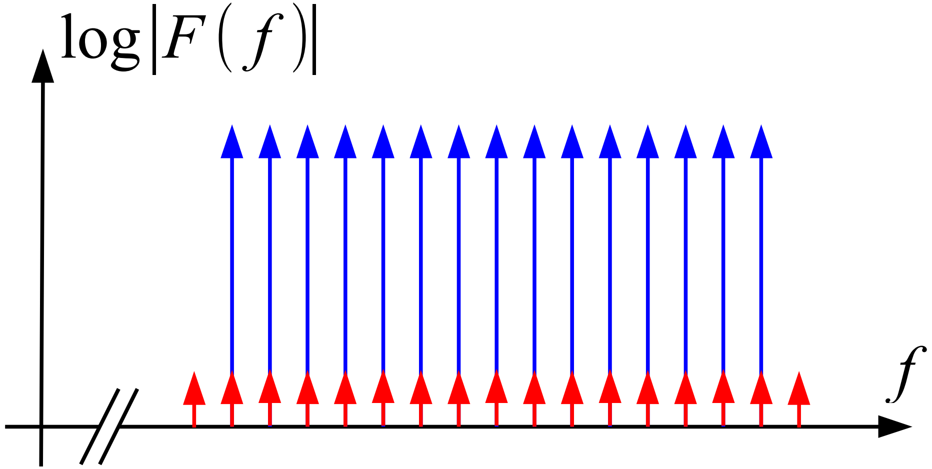
$a/l \approx -0.2\mathrm{dB/km} \qquad A_{\mathit{eff}} \approx 80\mu m^2$

$D \approx +5\mathrm{ps/(nm.km)} \qquad \Delta f = 100\mathrm{GHz}$

$$\Delta\beta \approx -2.52\mathrm{rd/km} \gg 2\alpha$$

$$l_{\mathit{eff}} \approx \frac{1}{|\Delta\beta|} \approx 0.4\mathrm{km}$$

$$P_{IP3} \approx 2\mathrm{W} = +\ 33\mathrm{dBm}$$



$40\ \mathit{channels}\ \mathit{WDM} \times 100\mathrm{Gb/s} = 4\mathrm{Tb/s}$