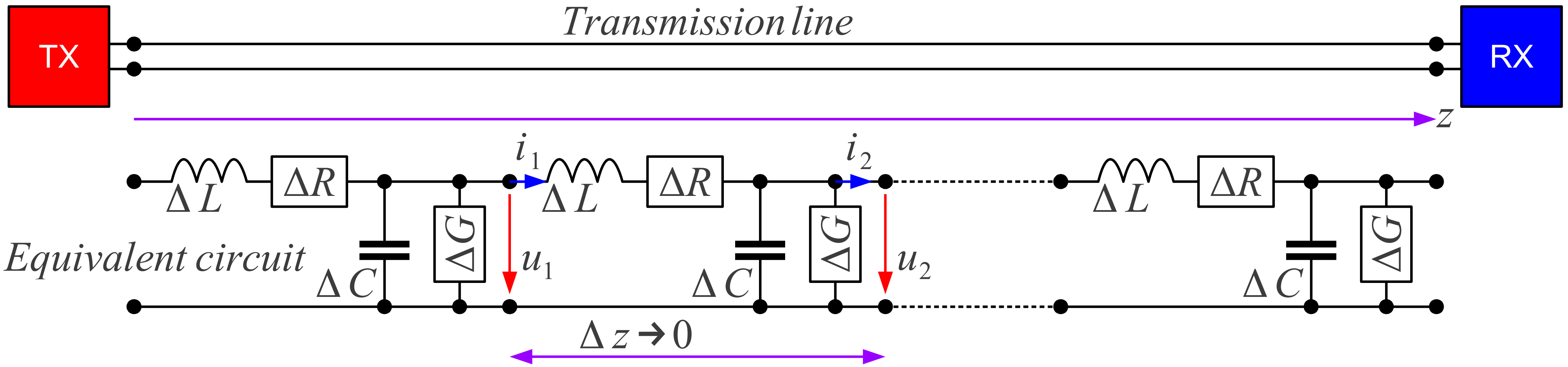
Communication Electronics

Lecture 2:

Telegrapher's equations



$$\Delta u = u_2 - u_1 = -\Delta L \frac{di_1}{dt} - \Delta R i_1$$

$$\Delta i = i_2 - i_1 = -\Delta C \frac{du_2}{dt} - \Delta G u_2$$

Distributed inductance	Distributed resistance	Distributed capacitance	Distributed conductance
$\frac{\Delta L}{\Delta z} \approx L/l$	$\frac{\Delta R}{\Delta z} \approx R/l$	$\frac{\Delta C}{\Delta z} \approx C/l$	$\frac{\Delta G}{\Delta z} \approx G/l$

$$\lim_{\Delta z \to 0} \frac{\Delta u}{\Delta z} \approx \frac{\partial u(t,z)}{\partial z} \qquad \lim_{\Delta z \to 0} u_2 \approx u(t,z)$$

$$\lim_{\Delta z \to 0} \frac{\Delta i}{\Delta z} \approx \frac{\partial i(t,z)}{\partial z} \qquad \lim_{\Delta z \to 0} i_1 \approx i(t,z)$$

$$\frac{\partial u(t,z)}{\partial z} = -L/l \cdot \frac{\partial i(t,z)}{\partial t} - R/l \cdot i(t,z)$$

$$\frac{\partial i(t,z)}{\partial z} = -C/l \cdot \frac{\partial u(t,z)}{\partial t} - G/l \cdot u(t,z)$$

Telegrapher's equations

$$R/l \to 0,\quad G/l \to 0 \;\to\; \frac{\partial u(t,z)}{\partial z} = -L/l \cdot \frac{\partial i(t,z)}{\partial t},\quad \frac{\partial i(t,z)}{\partial z} = -C/l \cdot \frac{\partial u(t,z)}{\partial t}$$

$$\to \frac{\partial}{\partial z} \to \frac{\partial^2 u(t,z)}{\partial z^2} = -L/l \cdot \frac{\partial^2 i(t,z)}{\partial z \partial t}$$

$$\to \frac{\partial}{\partial t} \to \frac{\partial^2 i(t,z)}{\partial t \partial z} = -C/l \cdot \frac{\partial^2 u(t,z)}{\partial t^2}$$

Change the sequence of differentiation & combine $\to \dfrac{\partial^2 u(t,z)}{\partial z^2} = L/l \cdot C/l \cdot \dfrac{\partial^2 u(t,z)}{\partial t^2}$

Guess the solution $u(t,z) = f(x) = f\left(t \pm \dfrac{z}{v}\right)$ *where* $v\left[\dfrac{\mathrm{m}}{\mathrm{s}}\right] \equiv constant$

$$\frac{\partial^2 u(t,z)}{\partial z^2} = \frac{1}{v^2} f''\left(t \pm \frac{z}{v}\right),\quad \frac{\partial^2 u(t,z)}{\partial t^2} = f''\left(t \pm \frac{z}{v}\right) \to v = \frac{1}{\sqrt{L/l \cdot C/l}}$$

Reflected wave | *Forward wave*

$$u(t,z) = u_R\left(t + \frac{z}{v}\right) + u_F\left(t - \frac{z}{v}\right)$$

Dielectric − loaded TEM lines $\to v = \dfrac{c_0}{\sqrt{\epsilon_r}} \approx \dfrac{3 \cdot 10^8\ \mathrm{m/s}}{\sqrt{\epsilon_r}}$

Loss − less time domain

$$\frac{\partial}{\partial z}u\left(t\pm\frac{z}{v}\right)=-L/l\cdot\frac{\partial}{\partial t}i\left(t\pm\frac{z}{v}\right)$$

$$\pm\frac{1}{v}\cdot u'\left(t\pm\frac{z}{v}\right)=-L/l\cdot i'\left(t\pm\frac{z}{v}\right)$$

$$\frac{u'}{i'}=\mp v\cdot L/l=\mp\sqrt{\frac{L/l}{C/l}}=\mp R_K=\frac{u}{i}$$

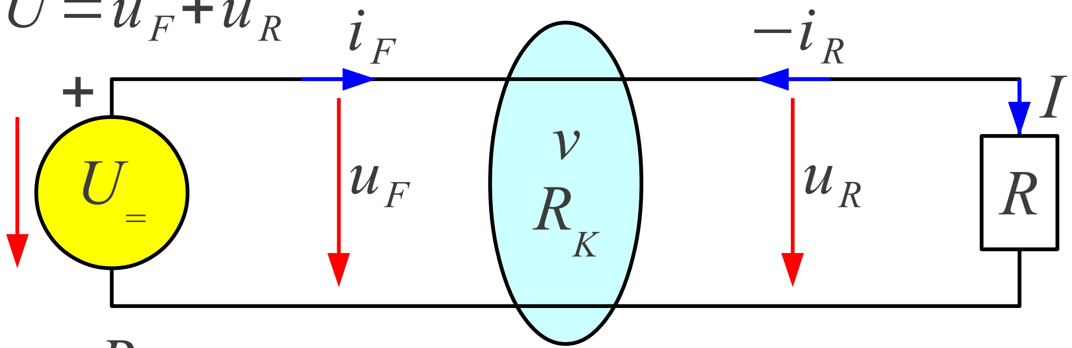
$$R_K=\sqrt{\frac{L/l}{C/l}}=\frac{u_F}{i_F}=-\frac{u_R}{i_R}$$

$$i(t,z)=\frac{-u_R\left(t+\frac{z}{v}\right)}{R_K}+\frac{u_F\left(t-\frac{z}{v}\right)}{R_K}$$

Characteristic resistance

Reflection coefficient

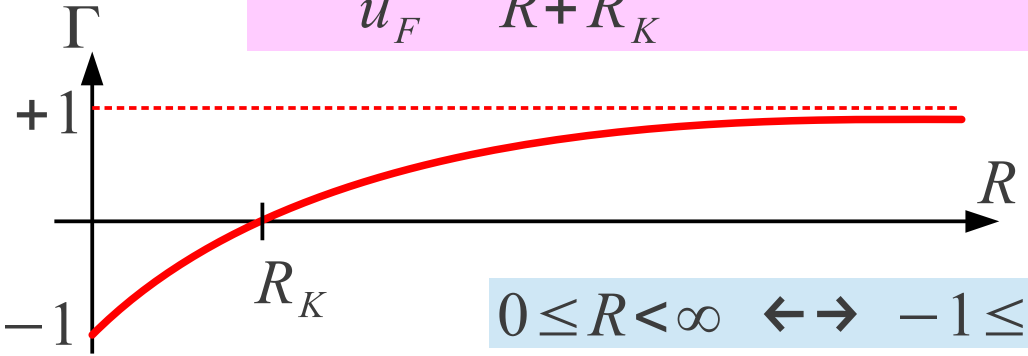
$$U=u_F+u_R$$



$$U\frac{R_K}{R}=u_F-u_R \leftarrow I=\frac{U}{R}=i_F+i_R=\frac{u_F-u_R}{R_K}$$

$$u_F=\frac{U}{2}\cdot\left(1+\frac{R_K}{R}\right)\qquad u_R=\frac{U}{2}\cdot\left(1-\frac{R_K}{R}\right)$$

$$\Gamma=\frac{u_R}{u_F}=\frac{R-R_K}{R+R_K}\leftrightarrow R=R_K\cdot\frac{1+\Gamma}{1-\Gamma}$$

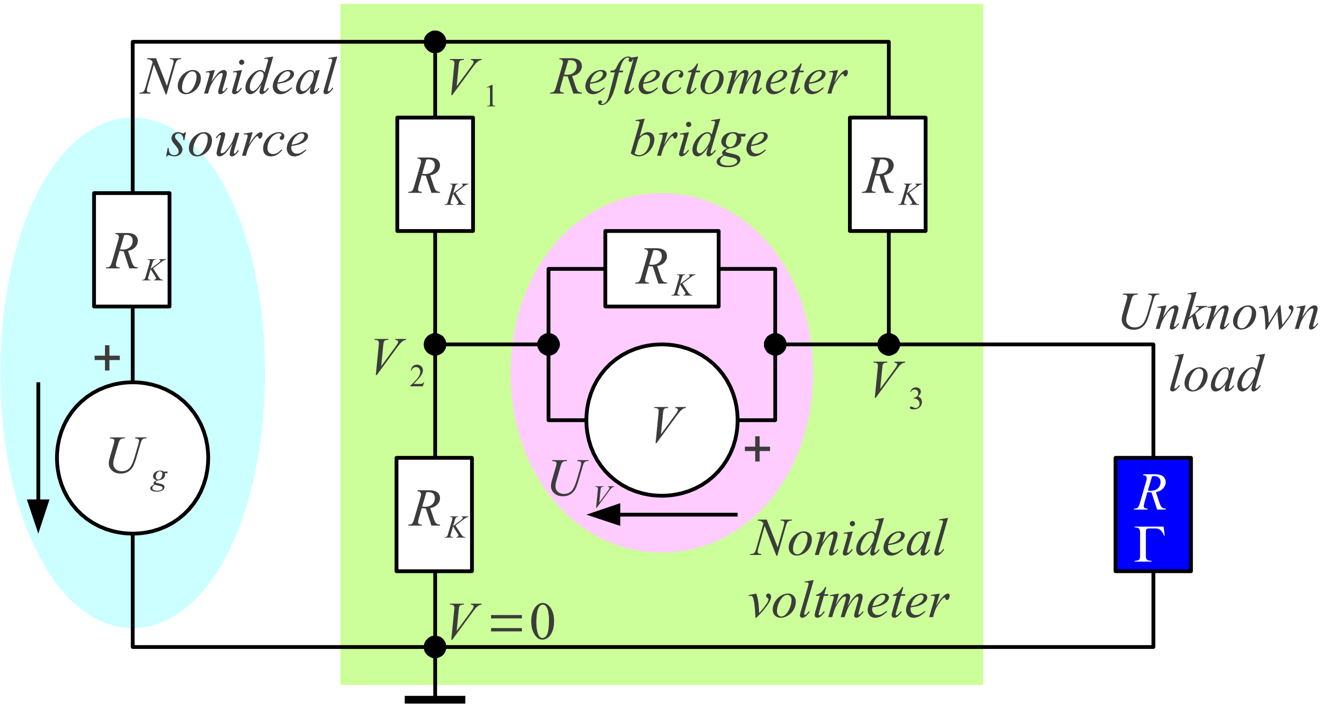


$$0\le R<\infty \leftrightarrow -1\le\Gamma<+1$$

$$\text{①}\quad \frac{V_1-U_g}{R_K}+\frac{V_1-V_2}{R_K}+\frac{V_1-V_3}{R_K}=0 \rightarrow 3V_1=U_g+V_2+V_3 \rightarrow 8V_2=U_g+4V_3 \rightarrow V_3=\frac{U_g}{2\left(1+\frac{R_K}{R}\right)}$$

$$\text{②}\quad \frac{V_2-V_1}{R_K}+\frac{V_2-V_3}{R_K}+\frac{V_2}{R_K}=0 \rightarrow 3V_2=V_1+V_3 \rightarrow V_1=3V_2-V_3$$

$$\text{③}\quad \frac{V_3-V_2}{R_K}+\frac{V_3-V_1}{R_K}+\frac{V_3}{R}=0 \rightarrow \left(2+\frac{R_K}{R}\right)V_3=V_1+V_2 \rightarrow \left(3+\frac{R_K}{R}\right)V_3=4V_2 \rightarrow V_2=\frac{U_g}{8}\cdot\frac{3+\frac{R_K}{R}}{1+\frac{R_K}{R}}$$

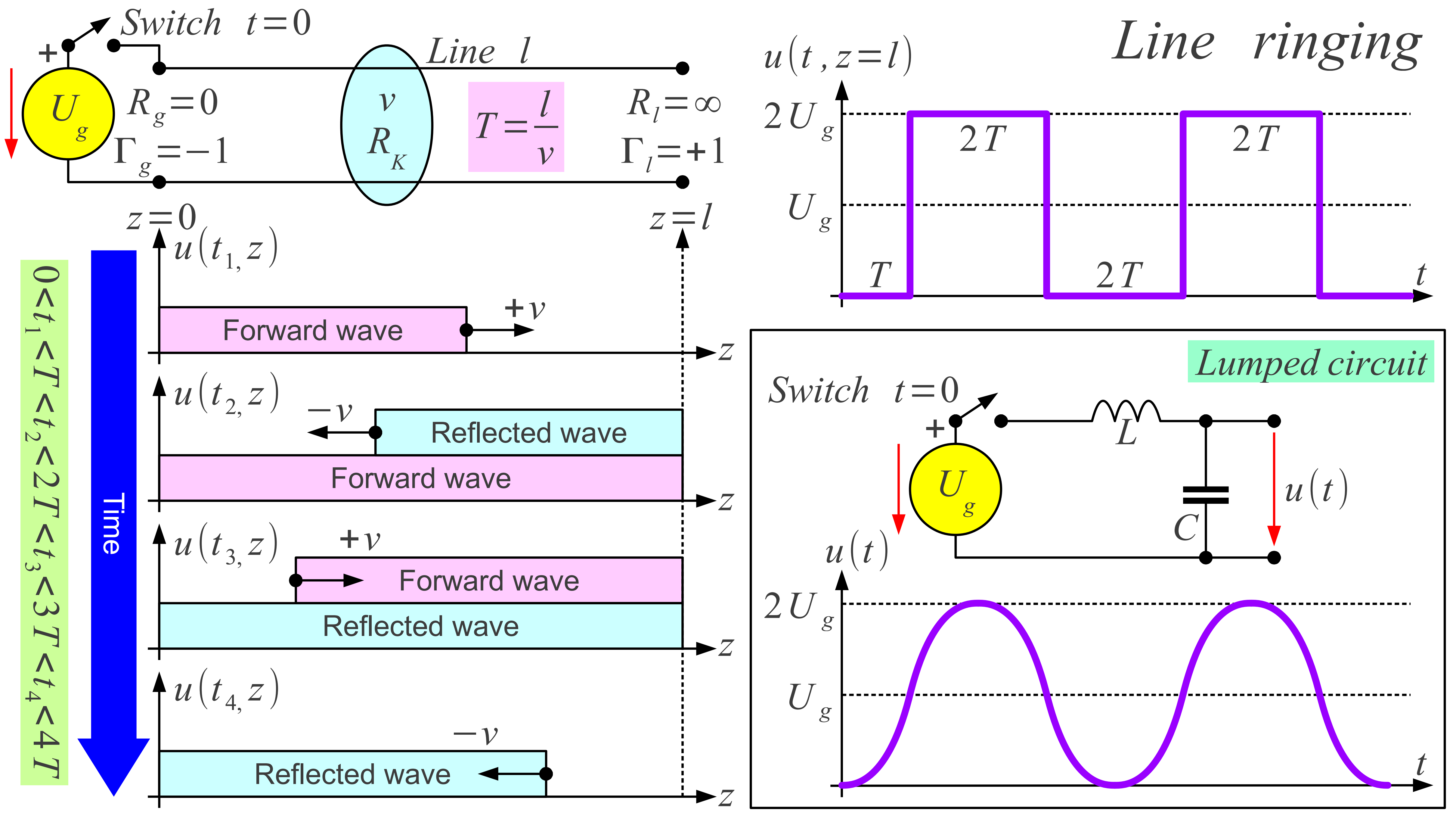


$$U_V=V_3-V_2=\frac{U_g}{2\left(1+\frac{R_K}{R}\right)}-\frac{U_g}{8}\cdot\frac{3+\frac{R_K}{R}}{1+\frac{R_K}{R}}\,R_K$$

$$U_V=\frac{U_g}{8}\cdot\frac{R-R_K}{R+R_K}$$

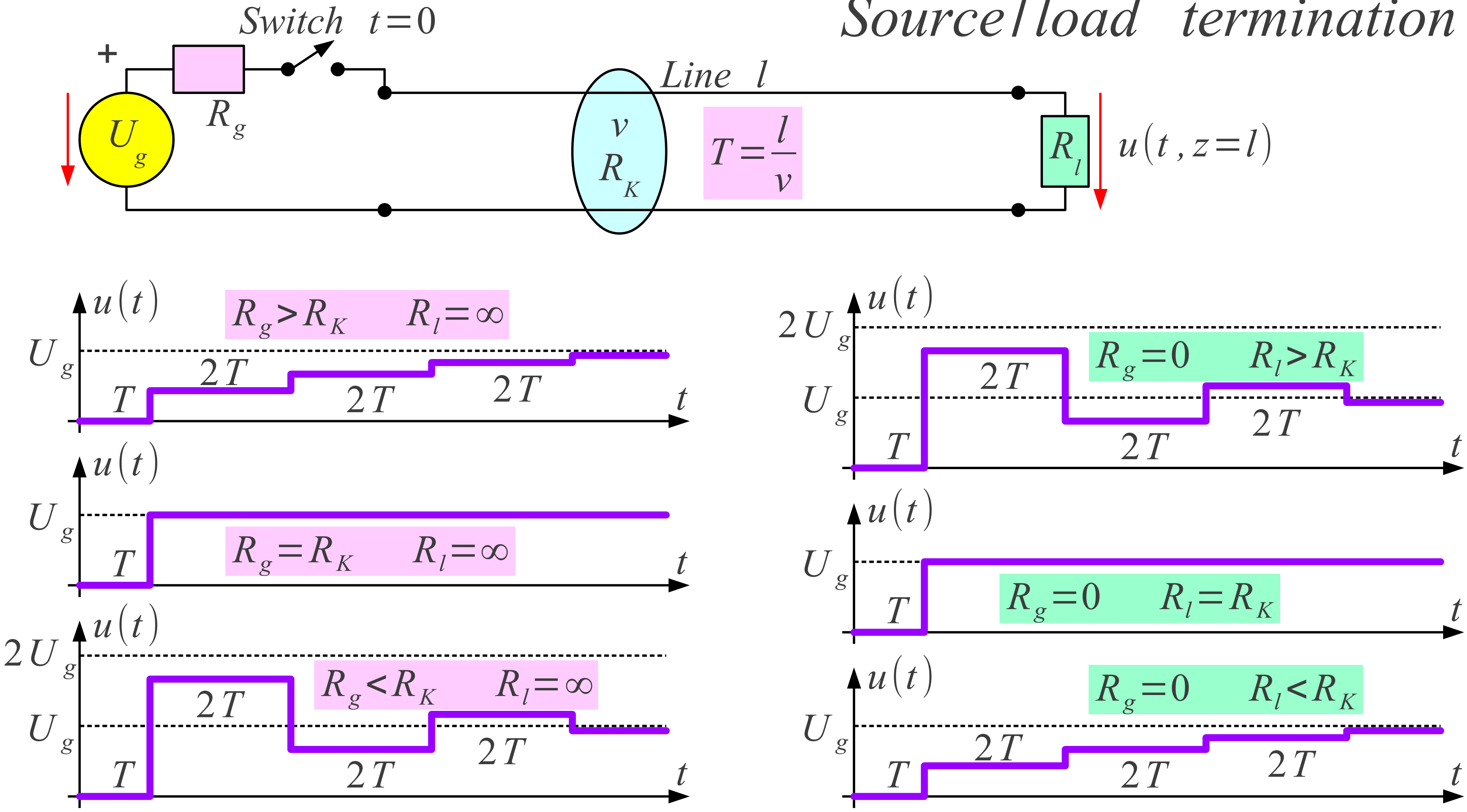
$$\Gamma=8\cdot\frac{U_V}{U_g}$$

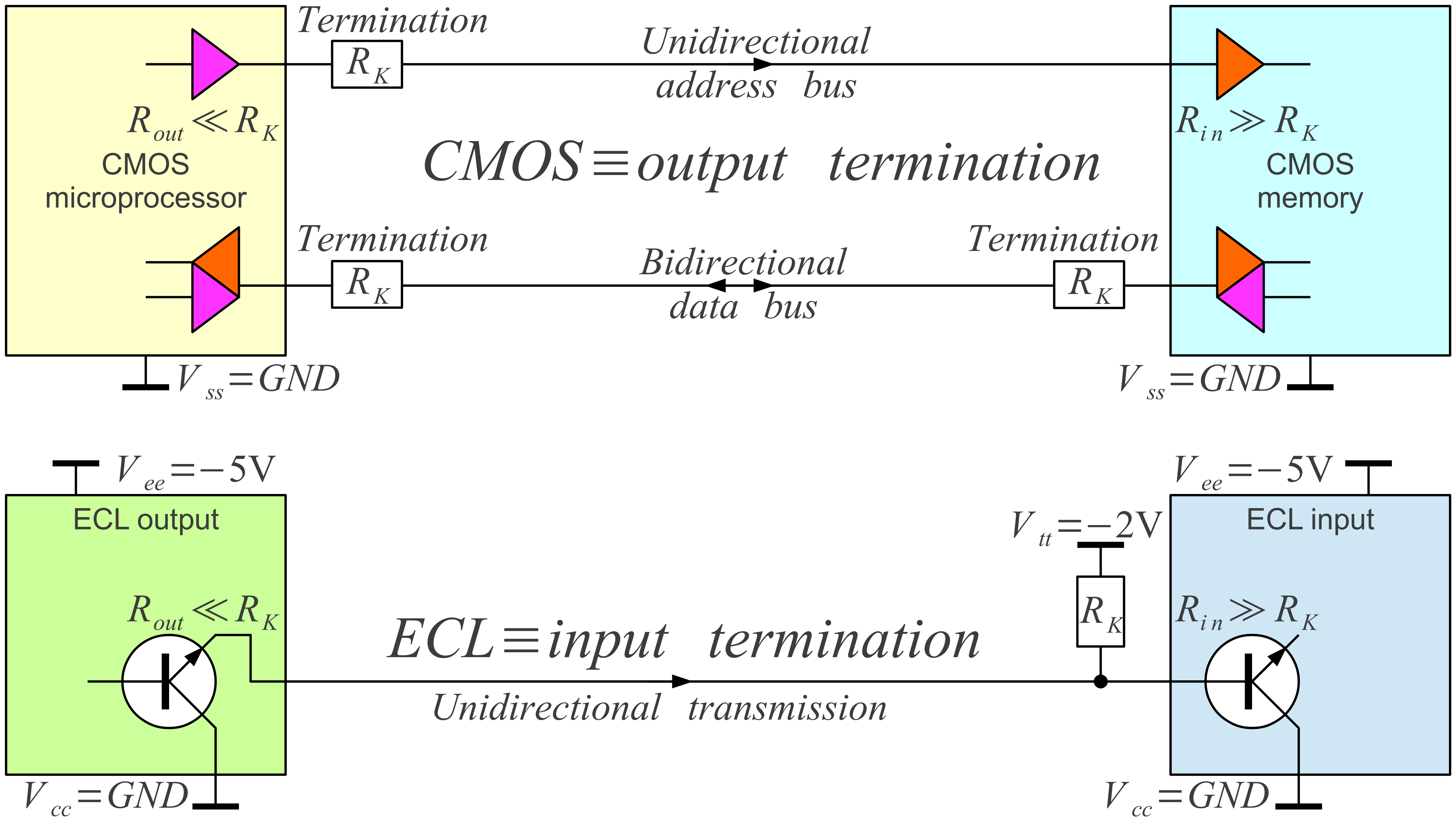
Reflectometer bridge



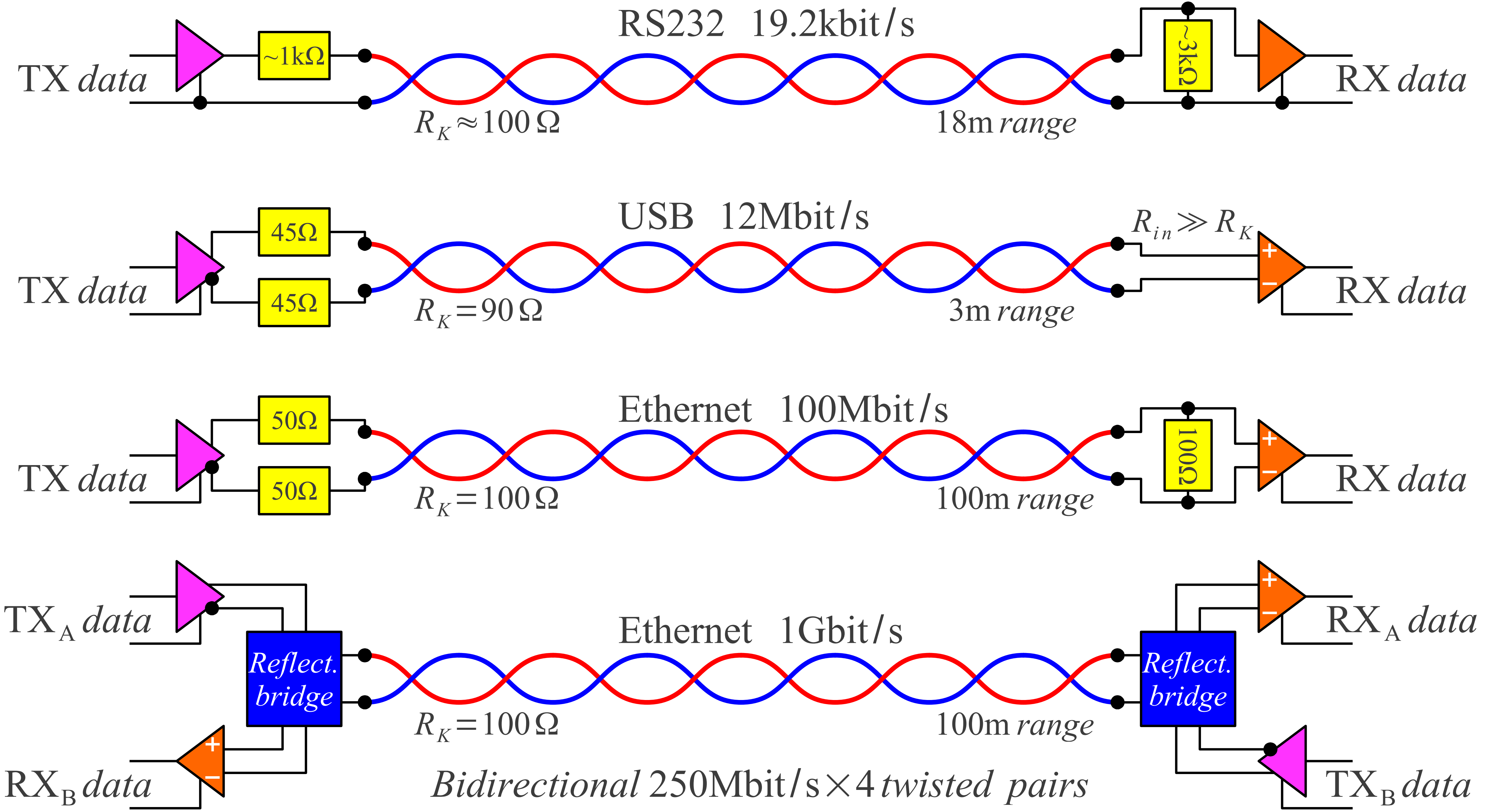
Line ringing
Switch t=0
+
U g
R g =0
Γ g =−1
Line l
v
R K
T = l / v
R l =∞
Γ l =+1
z=0
z=l
0<t1<T<t2<2T<t3<3T<t4<4T
Time
u(t1, z)
Forward wave
+v
z
u(t2, z)
−v
Reflected wave
Forward wave
z
u(t3, z)
+v
Forward wave
Reflected wave
z
u(t4, z)
−v
Reflected wave
z
u(t, z=l)
2U g
U g
T
2T
2T
2T
t
Lumped circuit
Switch t=0
+
U g
L
C
u(t)
u(t)
2U g
U g
t

Source/load termination





Termination
R_K
Unidirectional
address bus
$R_{out} \ll R_K$
CMOS
microprocessor
$R_{in} \gg R_K$
CMOS
memory
CMOS ≡ output termination
Termination
R_K
Bidirectional
data bus
Termination
R_K
$V_{ss} = GND$
$V_{ss} = GND$
$V_{ee} = -5V$
ECL output
$R_{out} \ll R_K$
$V_{ee} = -5V$
ECL input
$V_{tt} = -2V$
R_K
$R_{in} \gg R_K$
ECL ≡ input termination
Unidirectional transmission
$V_{cc} = GND$
$V_{cc} = GND$


RS232 19.2kbit/s
TX data
~1kΩ
~3kΩ
RX data
$R_K \approx 100\,\Omega$
18m range
USB 12Mbit/s
45Ω
45Ω
$R_{in} \gg R_K$
TX data
RX data
$R_K = 90\,\Omega$
3m range
Ethernet 100Mbit/s
50Ω
50Ω
100Ω
TX data
RX data
$R_K = 100\,\Omega$
100m range
Ethernet 1Gbit/s
TX_A data
RX_A data
Reflect. bridge
Reflect. bridge
$R_K = 100\,\Omega$
100m range
RX_B data
TX_B data
Bidirectional 250Mbit/s×4 twisted pairs

Guess the solution $u(t,z)=\mathrm{Re}\left[U\cdot e^{j(\omega t\pm kz)}\right]$ $i(t,z)=\mathrm{Re}\left[I\cdot e^{j(\omega t\pm kz)}\right]$

$$\omega\left[\frac{\mathrm{rd}}{\mathrm{s}}\right]=2\pi f$$

$$\frac{\partial u(t,z)}{\partial z}=-L/l\cdot\frac{\partial i(t,z)}{\partial t}-R/l\cdot i(t,z)$$

$$\frac{\partial i(t,z)}{\partial z}=-C/l\cdot\frac{\partial u(t,z)}{\partial t}-G/l\cdot u(t,z)$$

→

$$\pm jk\cdot U=(-j\omega L/l-R/l)\cdot I$$

$$\pm jk\cdot I=(-j\omega C/l-G/l)\cdot U$$

$$k\left[\frac{\mathrm{rd}}{\mathrm{m}}\right]=\sqrt{-(j\omega L/l+R/l)\cdot(j\omega C/l+G/l)}=\beta-j\alpha\equiv(angular)\,wavenumber$$

$$\beta\left[\frac{\mathrm{rd}}{\mathrm{m}}\right]\equiv phase\ constant$$

Reflected wave | *Forward wave*

$$u(t,z)=\mathrm{Re}\left[U_R\cdot e^{j(\omega t+kz)}+U_F\cdot e^{j(\omega t-kz)}\right]$$

$$\alpha\left[\frac{\mathrm{Np}}{\mathrm{m}}\right]\equiv attenuation\ constant$$

$$Z_K[\Omega]=\frac{U_F}{I_F}=-\frac{U_R}{I_R}=\frac{j\omega L/l+R/l}{jk}=\sqrt{\frac{j\omega L/l+R/l}{j\omega C/l+G/l}}\equiv characteristic\ impedance$$

Frequency domain

Low − loss frequency domain

Copper wire + excellent dielectric
$R/l \ll \omega L/l \qquad G/l \rightarrow 0$

$$Z_K = \sqrt{\frac{j\omega L/l + R/l}{j\omega C/l}} \approx \sqrt{\frac{L/l}{C/l}}$$

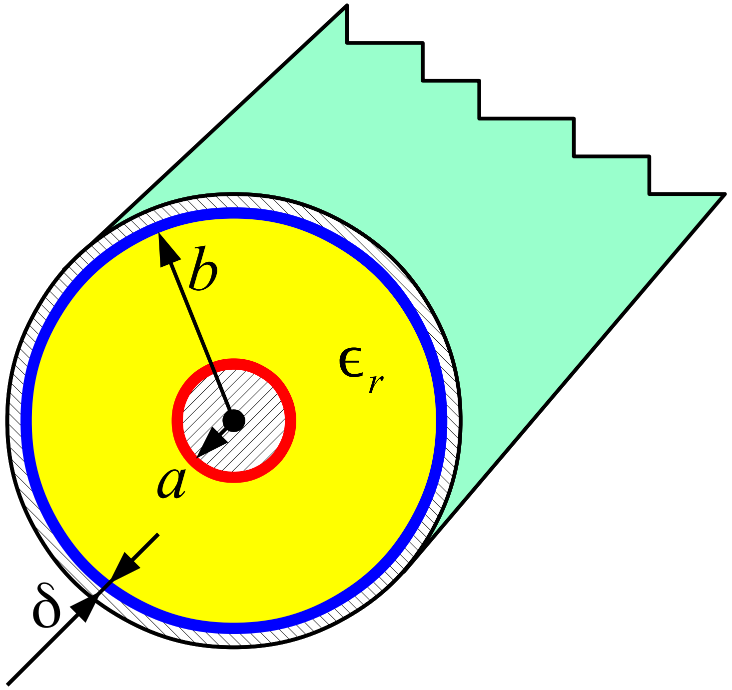
$$k = \sqrt{-(j\omega L/l + R/l) \cdot j\omega C/l} = \sqrt{\omega^2 L/l \cdot C/l \cdot \left(1 + \frac{R/l}{j\omega L/l}\right)} = \omega\sqrt{L/l \cdot C/l} \cdot \sqrt{1 - j\frac{R/l}{\omega L/l}}$$

$$k \approx \omega\sqrt{L/l \cdot C/l} \cdot \left(1 - j\frac{R/l}{2\omega L/l}\right) = \omega\sqrt{L/l \cdot C/l} - j\frac{R/l}{2}\sqrt{\frac{C/l}{L/l}} \approx \omega\sqrt{L/l \cdot C/l} - j\frac{R/l}{2Z_K}$$

$$k = \beta - j\alpha \rightarrow \quad \beta \approx \omega\sqrt{L/l \cdot C/l} = \frac{\omega}{v} \qquad \alpha \approx \frac{R/l}{2Z_K}$$

Reflected wave | Forward wave

$$u(t,z) = \mathrm{Re}\left[U_R \cdot e^{j(\omega t + \beta z)} e^{\alpha z} + U_F \cdot e^{j(\omega t - \beta z)} e^{-\alpha z}\right]$$



$$L/l = \frac{\mu_0}{2\pi}\ln\frac{b}{a} \qquad C/l = \frac{2\pi\epsilon_0\epsilon_r}{\ln\frac{b}{a}} \qquad Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$Z_K = \frac{Z_0}{2\pi\sqrt{\epsilon_r}}\ln\frac{b}{a} \approx \frac{60\,\Omega}{\sqrt{\epsilon_r}}\ln\frac{b}{a} \equiv \textit{characteristic impedance}$$

$$\textit{Example}: \quad f = 100\text{MHz} \quad \textit{Copper}: \quad \mu \approx \mu_0 \quad \gamma = 56\cdot 10^6 \frac{\text{S}}{\text{m}}$$

$$\delta = \sqrt{\frac{2}{\omega\mu_0\gamma}} \approx 6.73\,\mu\text{m} \equiv \textit{penetration depth into copper}$$

$$R/l = R_{wire}/l + R_{shield}/l = \frac{1}{2\pi a\delta\gamma} + \frac{1}{2\pi b\delta\gamma} = \frac{1}{2\pi\delta\gamma}\left(\frac{1}{a} + \frac{1}{b}\right)$$

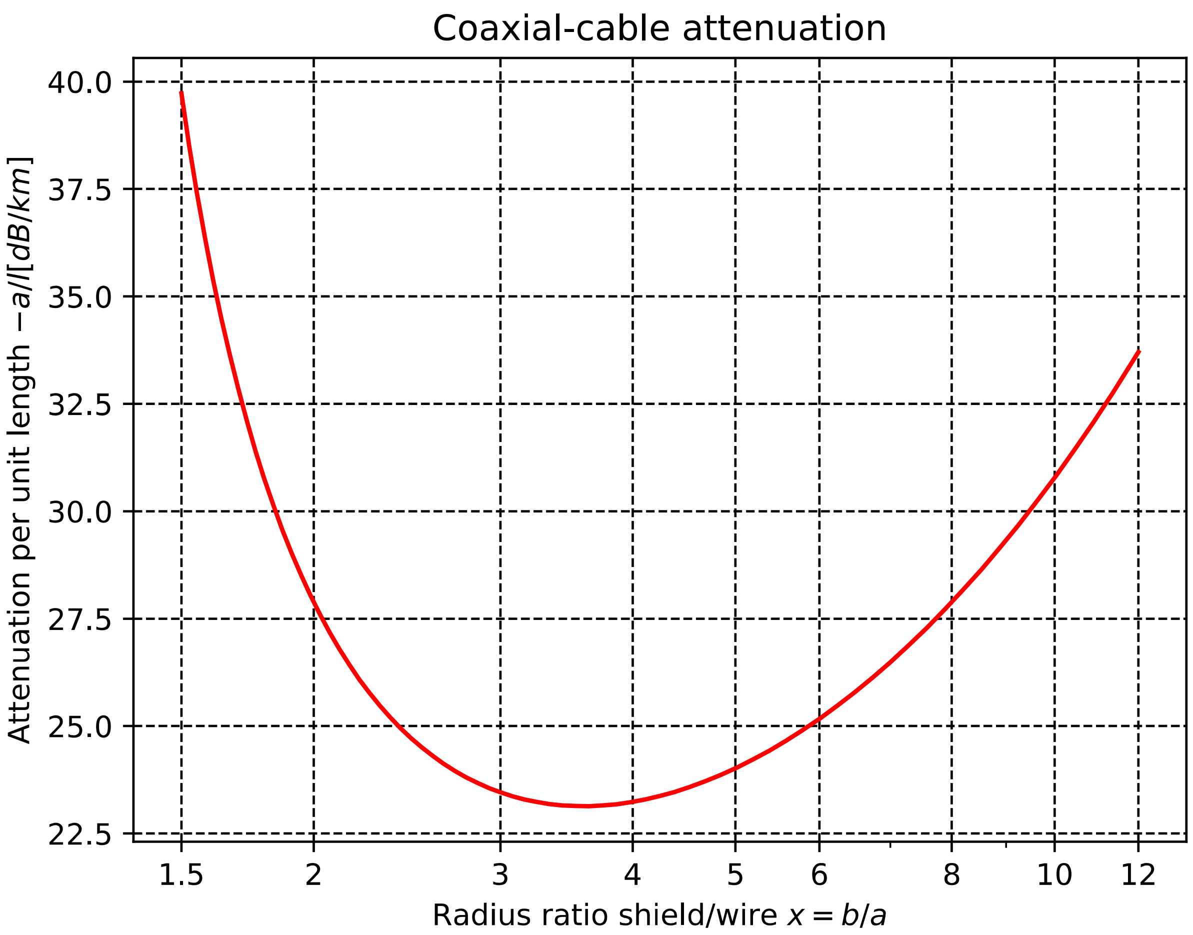
$x = 3.6$	ϵ_r	Z_{Kopt}
air	1	76.9Ω
foam	1.5	62.8Ω
teflon	2.1	53.0Ω
PE	2.3	50.7Ω

$$\alpha\left[\frac{\text{Np}}{\text{m}}\right] = \frac{R/l}{2Z_K} = \frac{\sqrt{\epsilon_r}}{2b\delta\gamma Z_0}\left(\frac{b}{a} + 1\right)/\ln\frac{b}{a} \qquad x = \frac{b}{a}$$

$$a/l\left[\frac{\text{dB}}{\text{m}}\right] = \frac{-20}{\ln 10}\alpha = -\left(\frac{10\sqrt{\pi\epsilon_0}}{\ln 10}\right)\frac{1}{b}\sqrt{\frac{\epsilon_r f}{\gamma}}\left(\frac{x+1}{\ln x}\right)$$

Coax calculation

$$a/l\left[\frac{\mathrm{dB}}{\mathrm{m}}\right]=-\left(\frac{10\sqrt{\pi\epsilon_0}}{\ln 10}\right)\frac{1}{b}\sqrt{\frac{\epsilon_r f}{\gamma}}\left(\frac{x+1}{\ln x}\right)$$



$$\textit{Example}: \quad a=2\mathrm{mm} \quad b=7.2\mathrm{mm}$$
$$\epsilon_r=2.3 \quad f=100\mathrm{MHz} \quad \gamma=56\cdot 10^6\,\mathrm{S/m}$$

$$x=3.6$$

$$a/l\left[\frac{\mathrm{dB}}{\mathrm{m}}\right]\approx -0.023\mathrm{dB/m}=-23\mathrm{dB/km}$$

$$x_{MIN}=3.59112147666862\approx 3.6$$

Single TEM mode limit:

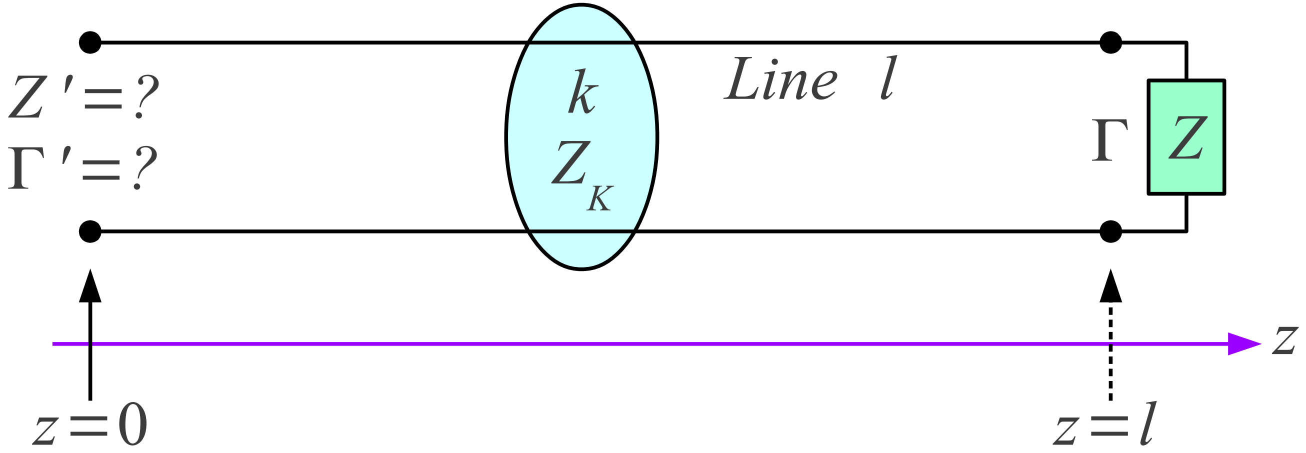
$$f_{MAX}\approx\frac{c_0}{\pi(a+b)\sqrt{\epsilon_r}}\approx 6.84\mathrm{GHz}$$

Coax design

Load transformation

$$\Gamma=\frac{Z-Z_K}{Z+Z_K}$$

$$k=\beta-j\alpha \;\rightarrow\; u(t,z)=\mathrm{Re}\left[U_R\cdot e^{j(\omega t+\beta z)}e^{\alpha z}+U_F\cdot e^{j(\omega t-\beta z)}e^{-\alpha z}\right]$$



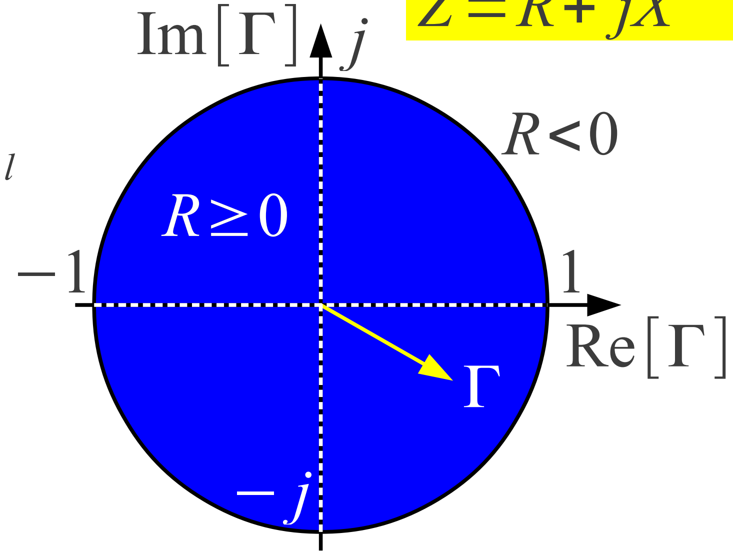
$$\Gamma=\frac{U_R\cdot e^{j(\omega t+\beta l)}e^{\alpha l}}{U_F\cdot e^{j(\omega t-\beta l)}e^{-\alpha l}}$$

$$\Gamma=\frac{U_R}{U_F}\cdot e^{j2\beta l}e^{2\alpha l}$$

Passive load

$Z=R+jX \quad R\geq 0 \;\leftrightarrow\; |\Gamma|\leq 1$

$$\Gamma'=\frac{U_R}{U_F}=\Gamma\cdot e^{-j2\beta l}e^{-2\alpha l}$$

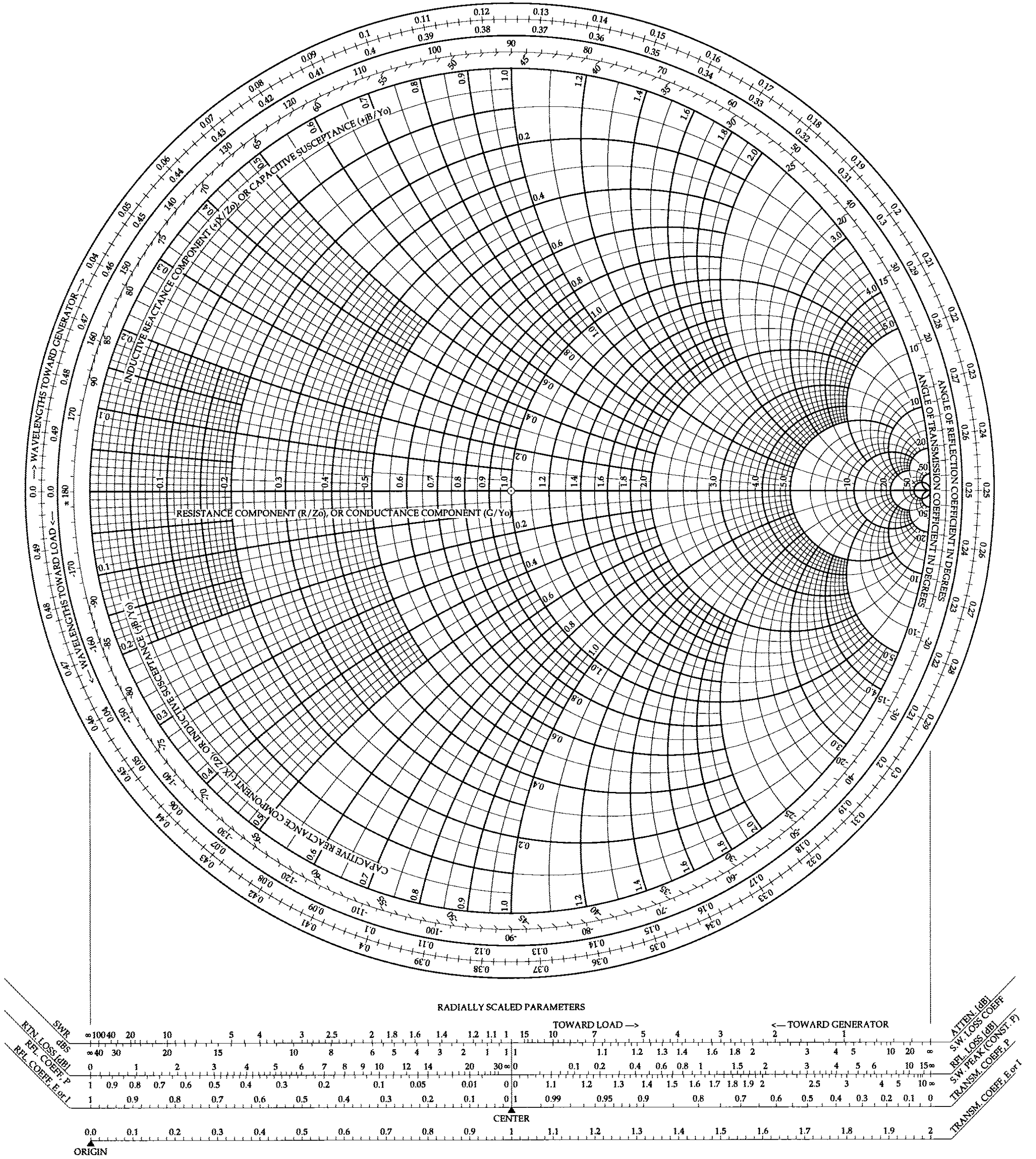


$$Z'=Z_K\cdot\frac{1+\Gamma'}{1-\Gamma'}$$

Loss-less impedance inverter

$$\beta l=\frac{\pi}{2},\quad \alpha=0 \;\rightarrow\; \Gamma'=-\Gamma,\quad Z'=\frac{Z_K^2}{Z}$$

Smith chart



RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo) OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo) OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
—> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—
RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF, P
RFL. COEFF, E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF, P
TRANSM. COEFF, E or I
CENTER
ORIGIN

$$P=\frac{1}{2}U\cdot I^*=\frac{1}{2}\left[U_R\cdot e^{j(\omega t+\beta z)}e^{\alpha z}+U_F\cdot e^{j(\omega t-\beta z)}e^{-\alpha z}\right]\cdot\left[\frac{-U_R}{Z_K}\cdot e^{j(\omega t+\beta z)}e^{\alpha z}+\frac{U_F}{Z_K}\cdot e^{j(\omega t-\beta z)}e^{-\alpha z}\right]^*$$

Forward power — *Reflected power* — *Standing wave energy (reactive power)*

$$P=\frac{|U_F|^2}{2Z_K}\cdot e^{-2\alpha z}-\frac{|U_R|^2}{2Z_K}\cdot e^{2\alpha z}+j\frac{|U_F\cdot U_R|}{Z_K}\cdot\sin(2\beta z+\phi)$$

$$\mathrm{Re}[P(z)]=P_F(z)-P_R(z)=\frac{|U_F|^2}{2Z_K}\cdot e^{-2\alpha z}-\frac{|U_R|^2}{2Z_K}\cdot e^{2\alpha z}$$

$$P_R(z)=P_F(z)\cdot|\Gamma(z)|^2$$

$$\mathrm{Re}[P(z)]=P_F(z)\cdot\left(1-|\Gamma(z)|^2\right)$$

(Voltage) Standing − Wave Ratio
(loss − less case only: $\alpha=0$*)*

$$(V)SWR=\rho=\frac{U_{MAX}}{U_{MIN}}=\frac{1+|\Gamma|}{1-|\Gamma|}$$

$$1\leq SWR=\rho<\infty$$

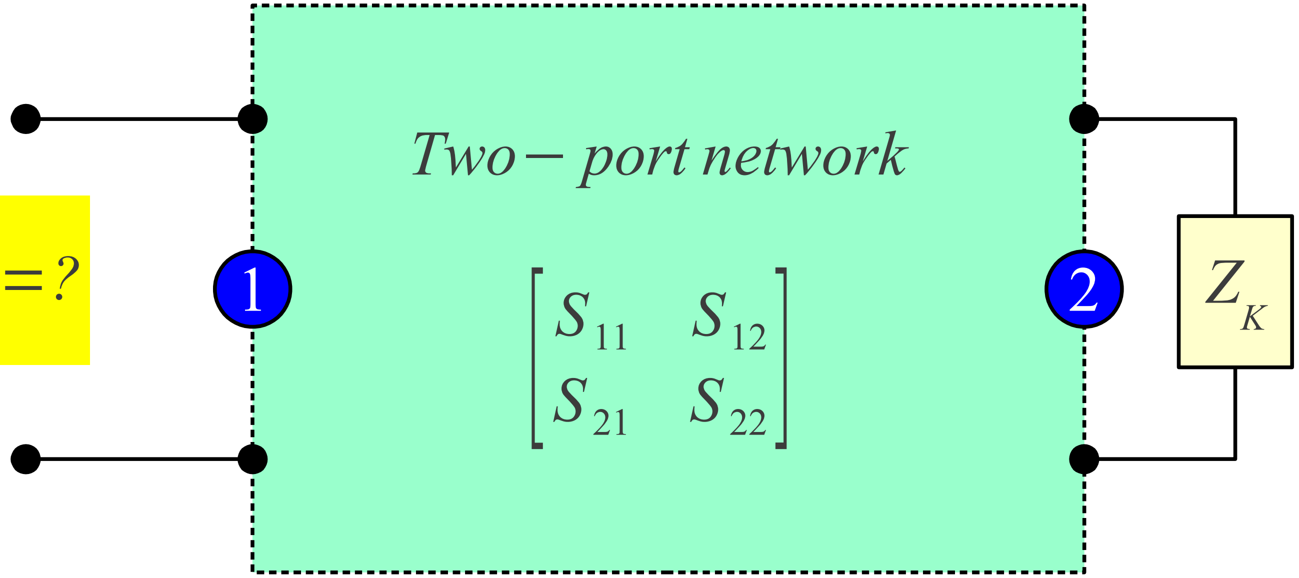
$$|\Gamma|=\frac{\rho-1}{\rho+1}\leq 1 \qquad \textit{Passive load}$$

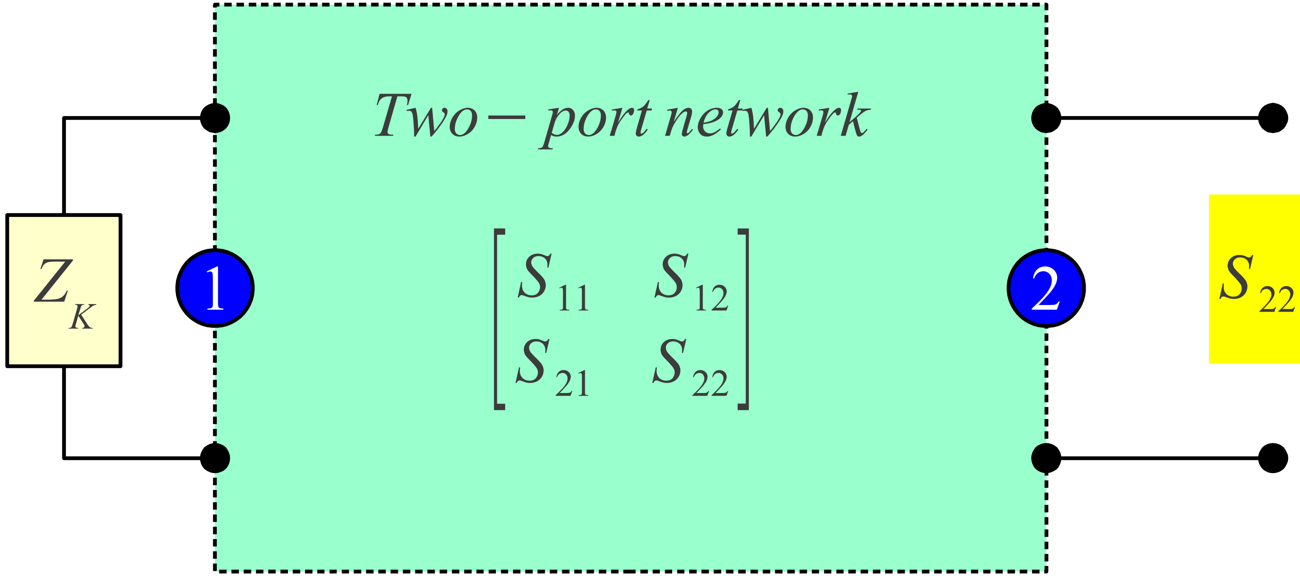
$$|\Gamma|=\frac{\rho+1}{\rho-1}>1 \qquad \textit{Active load}$$

Power & energy

Reflection parameters

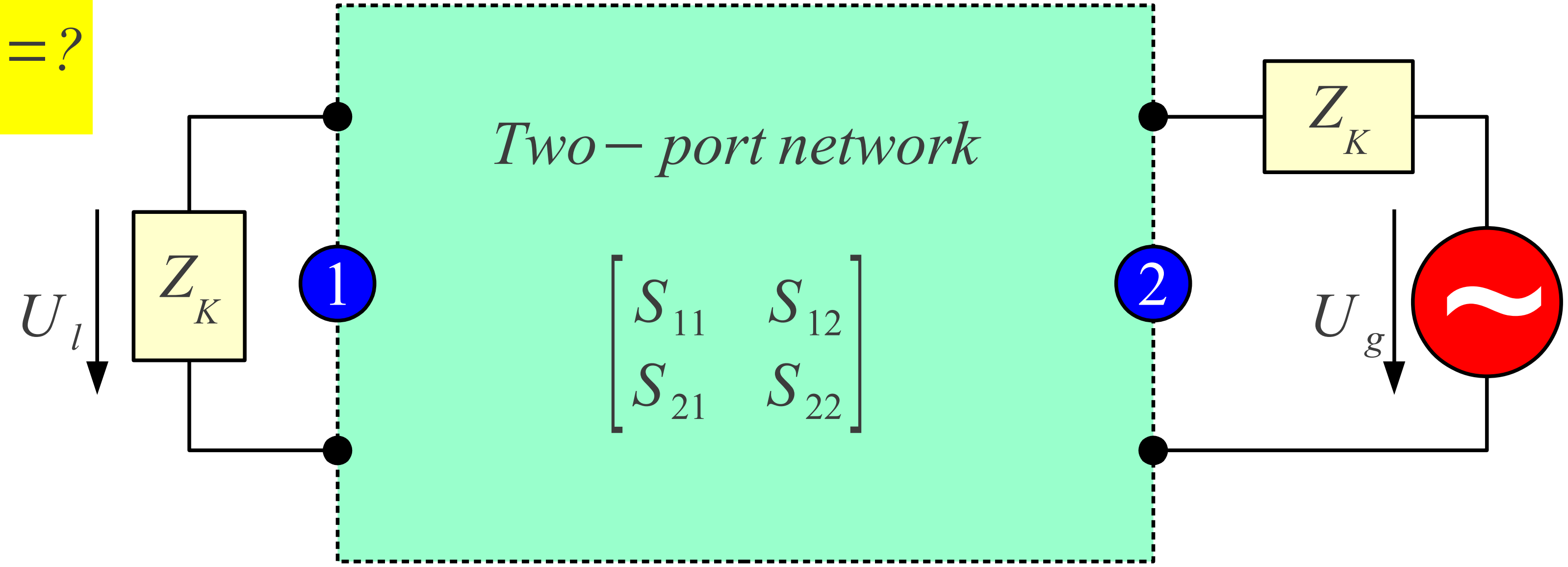
$$S_{11}=\frac{Z_1-Z_K}{Z_1+Z_K}=?$$

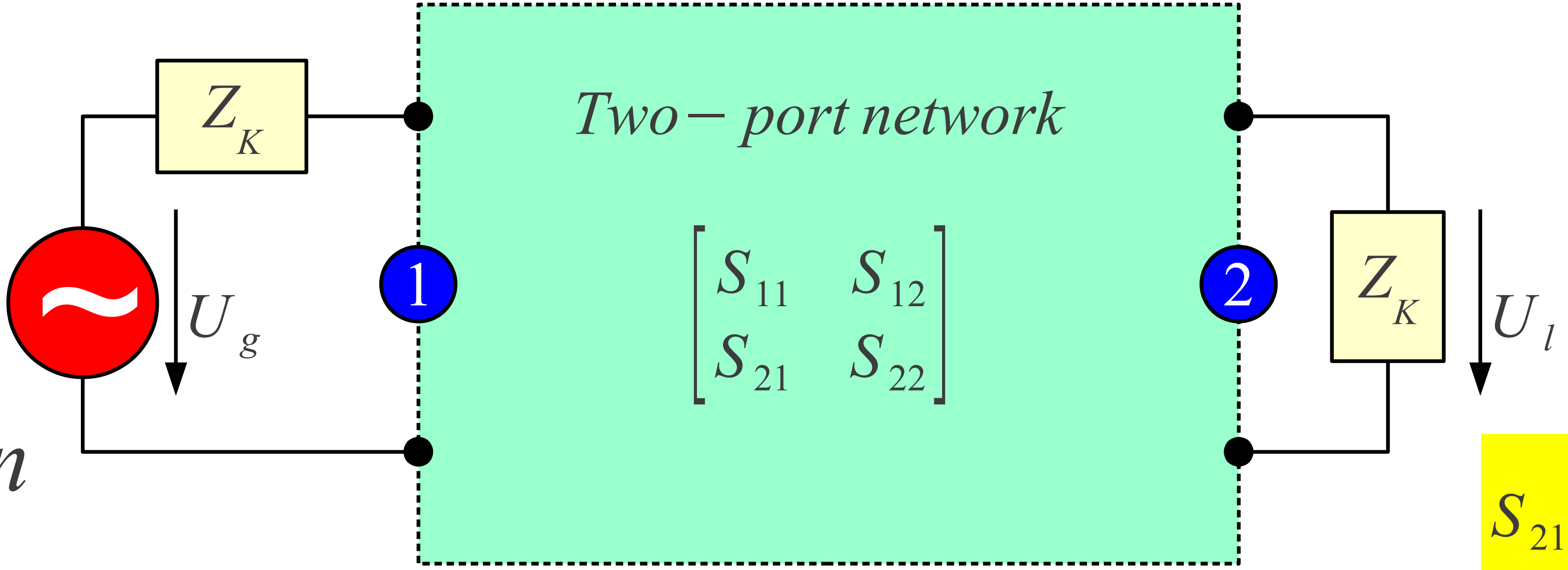




$$S_{22}=\frac{Z_2-Z_K}{Z_2+Z_K}=?$$

$$S_{12} = \frac{2U_l}{U_g} = ?$$





Transmission parameters

$$S_{21} = \frac{2U_l}{U_g} = ?$$