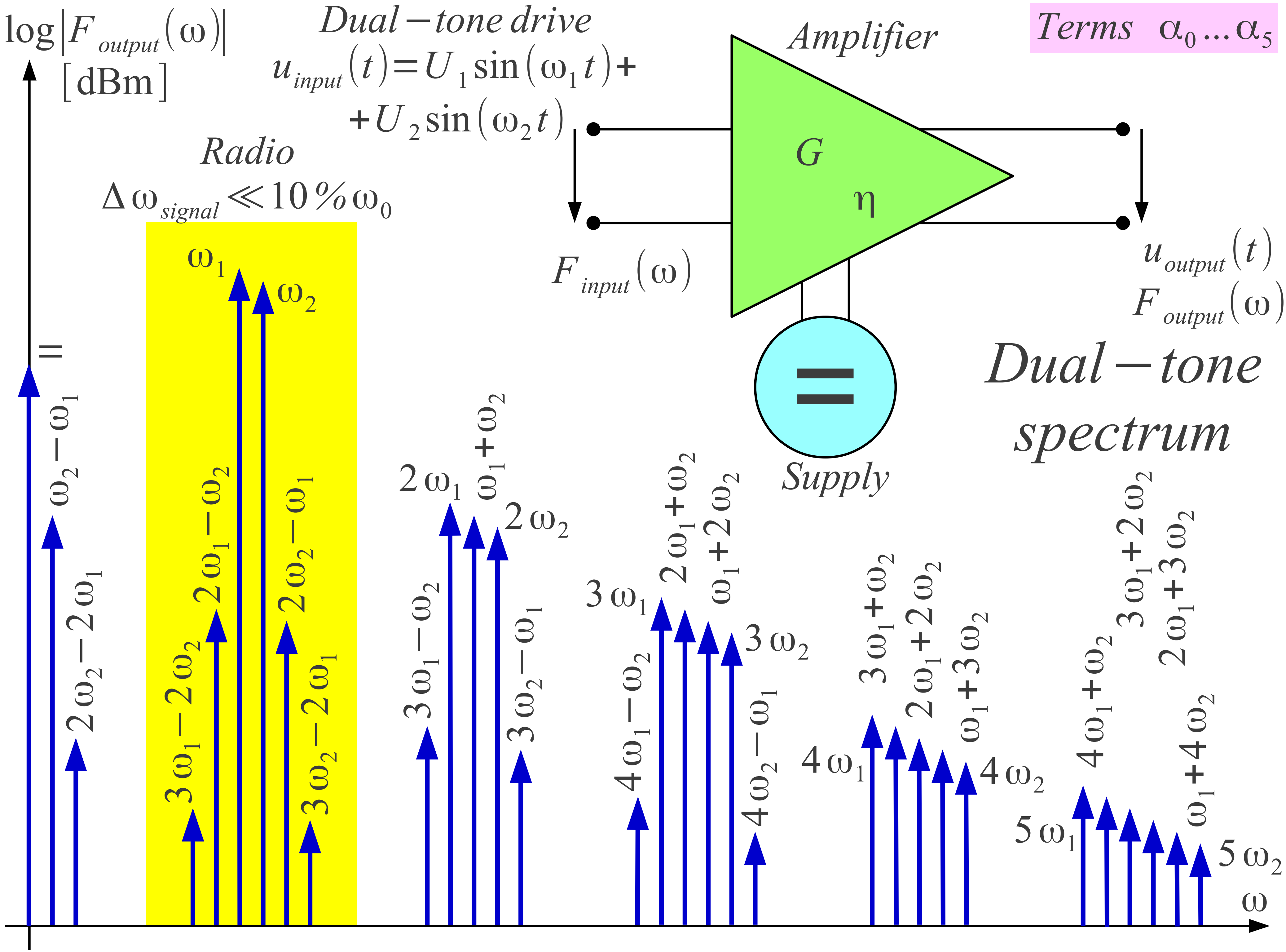
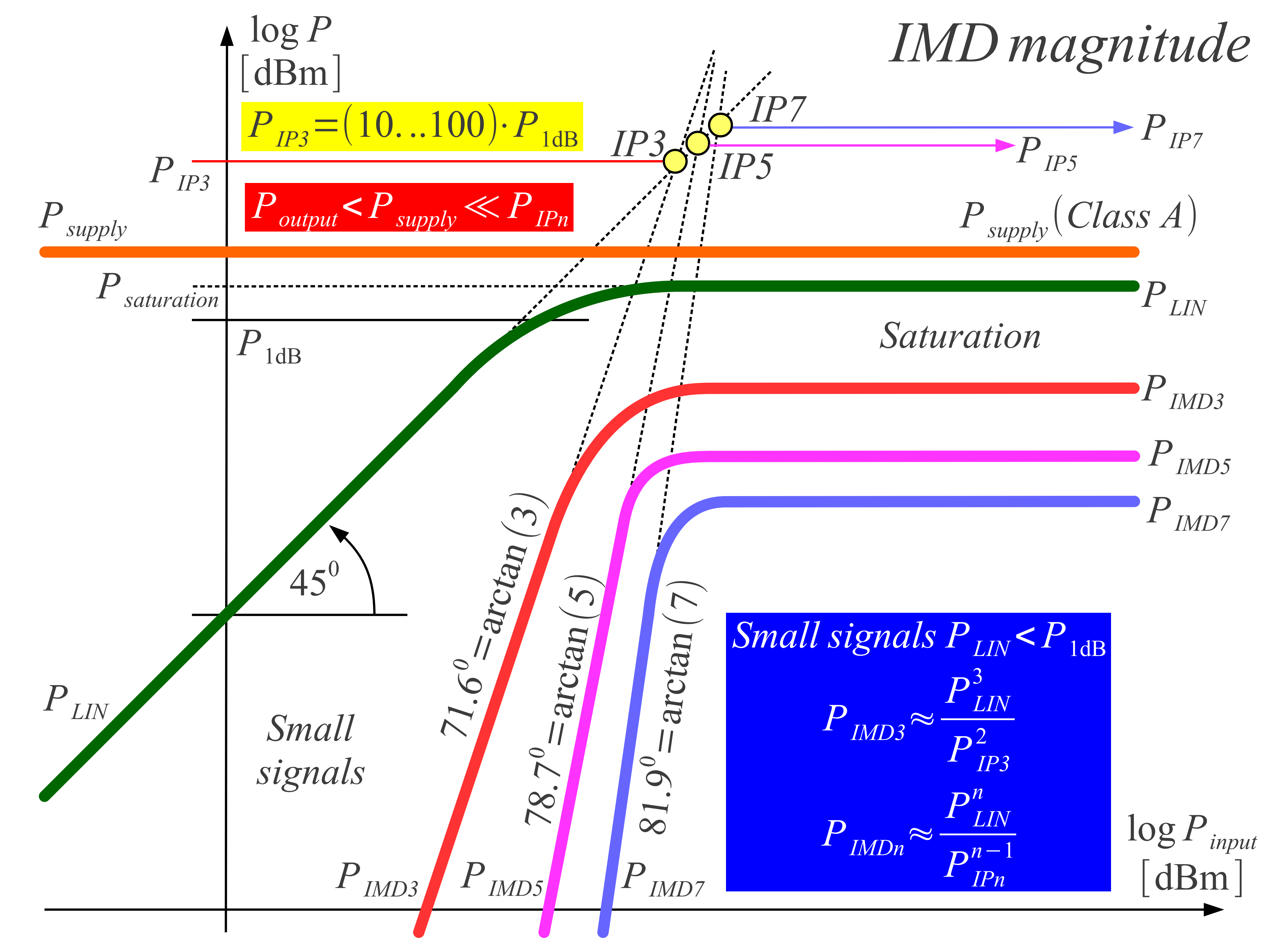
# Communication Electronics

## Lecture 13:

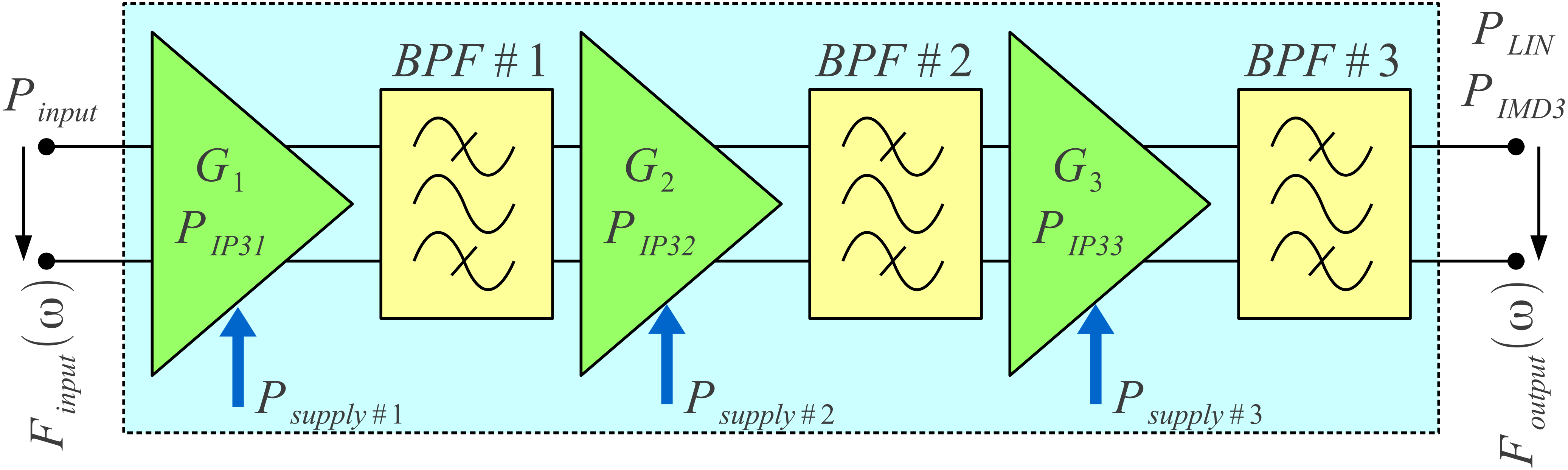
## Distortion mitigation



$\log|F_{output}(\omega)|$ [dBm]
Dual−tone drive
$u_{input}(t)=U_1\sin(\omega_1 t)+$
$+U_2\sin(\omega_2 t)$
Amplifier
Terms $\alpha_0\ldots\alpha_5$
G
η
$F_{input}(\omega)$
$u_{output}(t)$
$F_{output}(\omega)$
Supply
Dual−tone spectrum
Radio
$\Delta\omega_{signal}\ll 10\%\,\omega_0$
=
$\omega_2-\omega_1$
$2\omega_2-2\omega_1$
$3\omega_1-2\omega_2$
$2\omega_1-\omega_2$
$\omega_1$
$\omega_2$
$2\omega_2-\omega_1$
$3\omega_2-2\omega_1$
$3\omega_1-\omega_2$
$2\omega_1$
$\omega_1+\omega_2$
$2\omega_2$
$3\omega_2-\omega_1$
$4\omega_1-\omega_2$
$3\omega_1$
$2\omega_1+\omega_2$
$\omega_1+2\omega_2$
$3\omega_2$
$4\omega_2-\omega_1$
$4\omega_1$
$3\omega_1+\omega_2$
$2\omega_1+2\omega_2$
$\omega_1+3\omega_2$
$4\omega_2$
$5\omega_1$
$4\omega_1+\omega_2$
$3\omega_1+2\omega_2$
$2\omega_1+3\omega_2$
$\omega_1+4\omega_2$
$5\omega_2$
ω



IMD magnitude
log P [dBm]
$P_{IP3} = (10...100) \cdot P_{1dB}$
$P_{IP3}$
$P_{output} < P_{supply} \ll P_{IPn}$
$P_{supply}$
IP3
IP5
IP7
$P_{IP7}$
$P_{IP5}$
$P_{supply}$ (Class A)
$P_{saturation}$
$P_{LIN}$
$P_{1dB}$
Saturation
$P_{IMD3}$
$P_{IMD5}$
$P_{IMD7}$
$45^0$
$71.6^0 = \arctan(3)$
$78.7^0 = \arctan(5)$
$81.9^0 = \arctan(7)$
$P_{LIN}$
Small signals
Small signals $P_{LIN} < P_{1dB}$
$P_{IMD3} \approx \frac{P_{LIN}^3}{P_{IP3}^2}$
$P_{IMDn} \approx \frac{P_{LIN}^n}{P_{IPn}^{n-1}}$
$P_{IMD3}$
$P_{IMD5}$
$P_{IMD7}$
log $P_{input}$ [dBm]



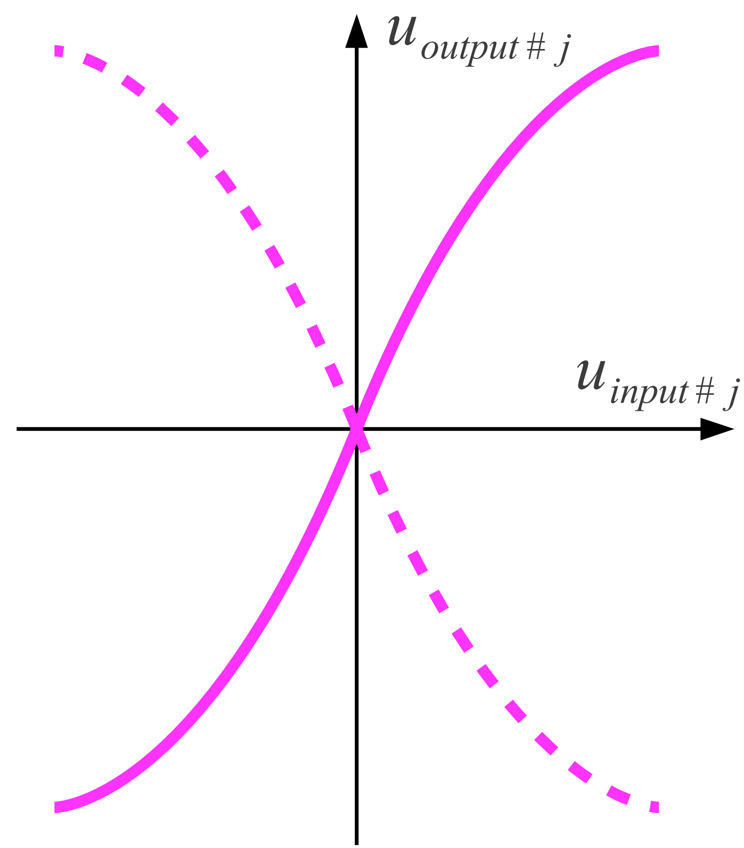

$$Z_{input} = Z_{output} = Z_K = 50\,\Omega$$

$$G_{chain} = \frac{P_{output}}{P_{input}} = \left|\frac{U_{output}}{U_{input}}\right|^2 = G_1 \cdot G_2 \cdot G_3$$

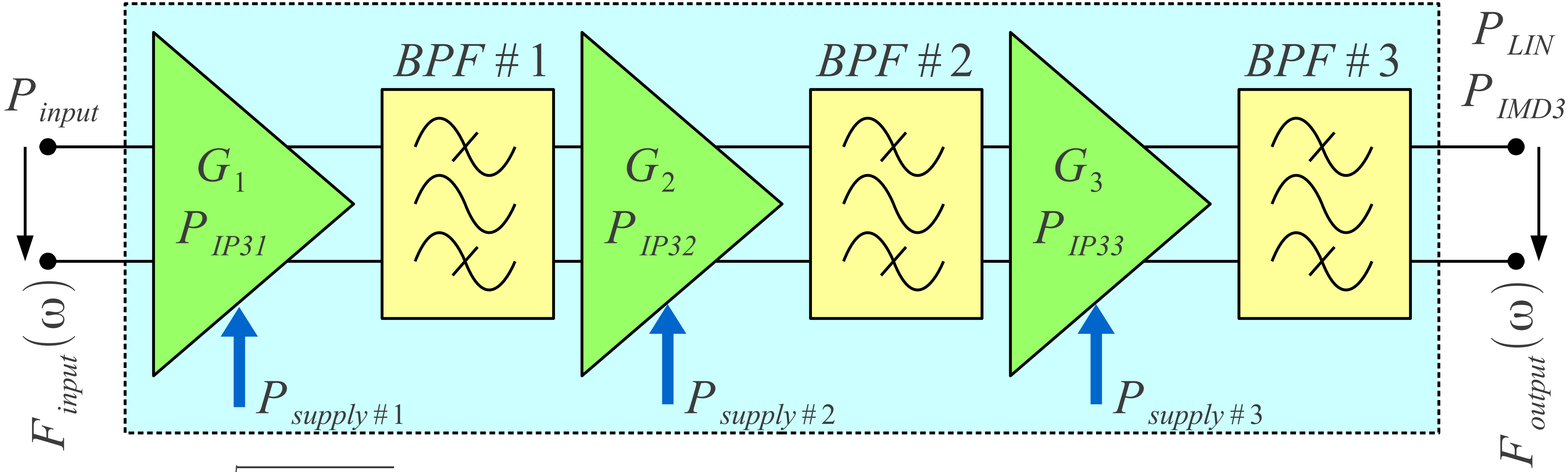
*Neglecting distortion of distortion:*

$$U_{IMD3chain} = U_{IMD33} + U_{IMD32} \cdot \sqrt{G_3} + U_{IMD31} \cdot \sqrt{G_2 G_3}$$

*Saturation* $\alpha_{1j} \cdot \alpha_{3j} < 0$ → *inphase phasor sum!*




# *IMD inside an amplifier chain*




$$\sqrt{P_{IMD3}} = \sqrt{\frac{P_{LIN}^3}{P_{IP3chain}^2}} = \frac{\sqrt{P_{LIN}^3}}{P_{IP3chain}} = \sqrt{P_{IMD33}} + \sqrt{G_3 \cdot P_{IMD32}} + \sqrt{G_3 \cdot G_2 \cdot P_{IMD31}}$$

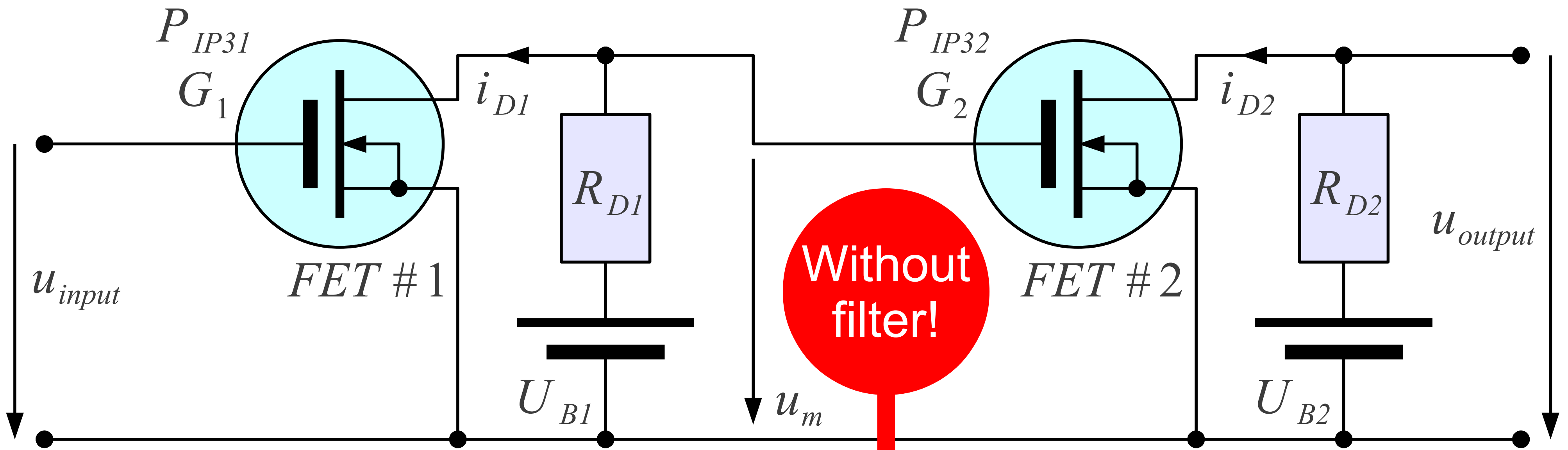
*Neglecting distortion of distortion!*

$$P_{IMD33} = \frac{P_{LIN}^3}{P_{IP33}^2} \qquad P_{IMD32} = \frac{\left(P_{LIN}/G_3\right)^3}{P_{IP32}^2} \qquad P_{IMD31} = \frac{\left(P_{LIN}/(G_2 \cdot G_3)\right)^3}{P_{IP31}^2}$$

$$\frac{\sqrt{P_{LIN}^3}}{P_{IP3chain}} = \frac{\sqrt{P_{LIN}^3}}{P_{IP33}} + \frac{\sqrt{P_{LIN}^3}}{P_{IP32} \cdot G_3} + \frac{\sqrt{P_{LIN}^3}}{P_{IP31} \cdot G_2 \cdot G_3}$$

$$P_{IP3chain} = \frac{1}{\dfrac{1}{P_{IP33}} + \dfrac{1}{P_{IP32} \cdot G_3} + \dfrac{1}{P_{IP31} \cdot G_2 \cdot G_3}}$$

# *Amplifier − chain $P_{IP3}$*




$$u_m = \alpha_{01} + \alpha_{11} \cdot u_{input} + \alpha_{21} \cdot u_{input}^2$$

$$u_{output} = \alpha_{02} + \alpha_{12} \cdot u_m + \alpha_{22} \cdot u_m^2$$

$$u_{IZH} = \alpha_0 + \alpha_1 \cdot u_{VH} + \alpha_2 \cdot u_{VH}^2 + \alpha_3 \cdot u_{VH}^3 + \alpha_4 \cdot u_{VH}^4$$

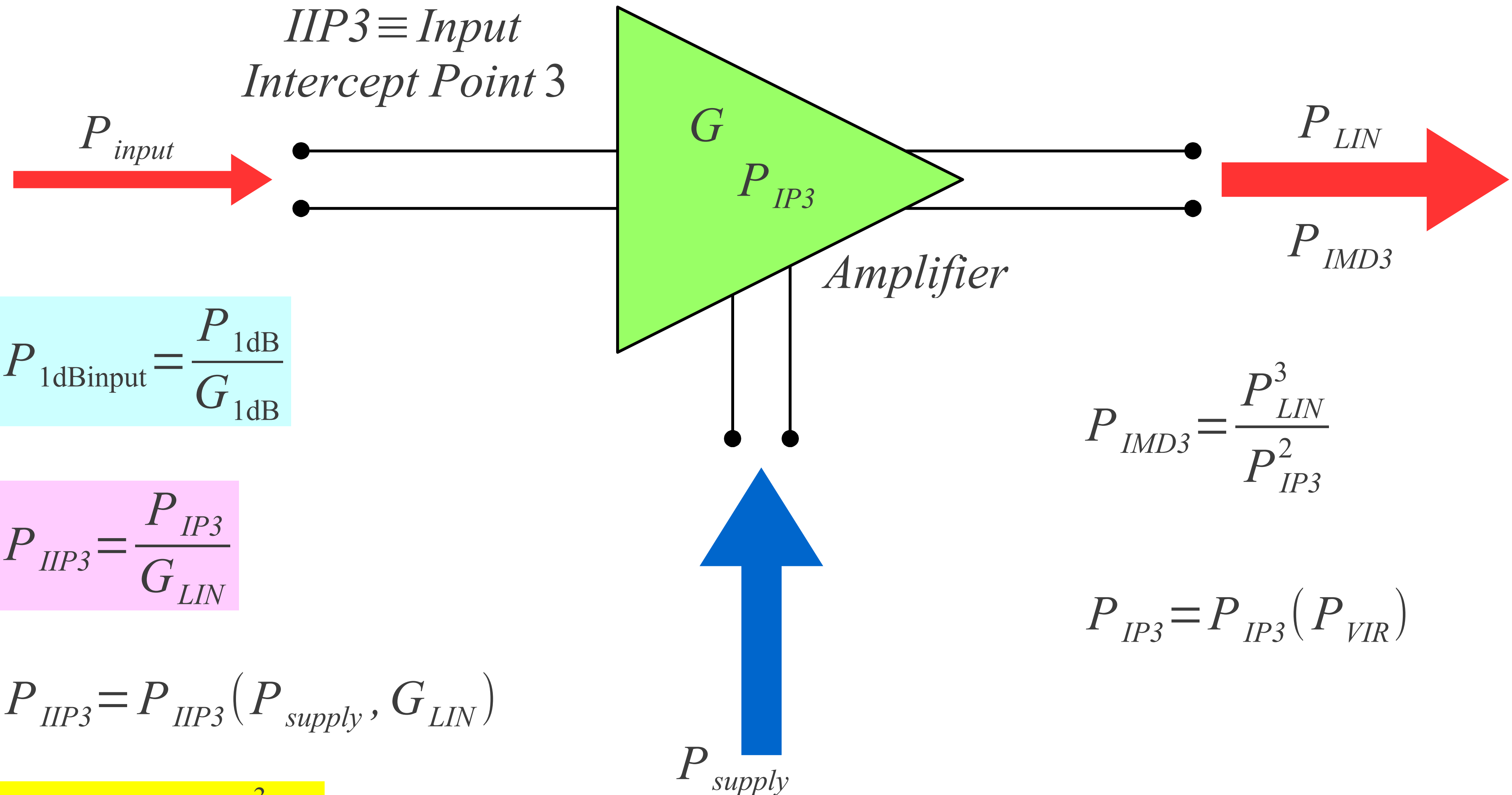
*Distortion of distortion causes IMD !*

$$P_{IP3chain} = \frac{1}{\frac{1}{P_{IP32}} + \frac{1}{P_{IP31} \cdot G_2}}$$

Equation not valid without interstage filter!

*Improper chain design*

*Receiver IMD → all quantities referenced t o the receiver input !*




$$P_{1dBinput} = \frac{P_{1dB}}{G_{1dB}}$$

$$P_{IIP3} = \frac{P_{IP3}}{G_{LIN}}$$

$$P_{IIP3} = P_{IIP3}\left(P_{supply}, G_{LIN}\right)$$

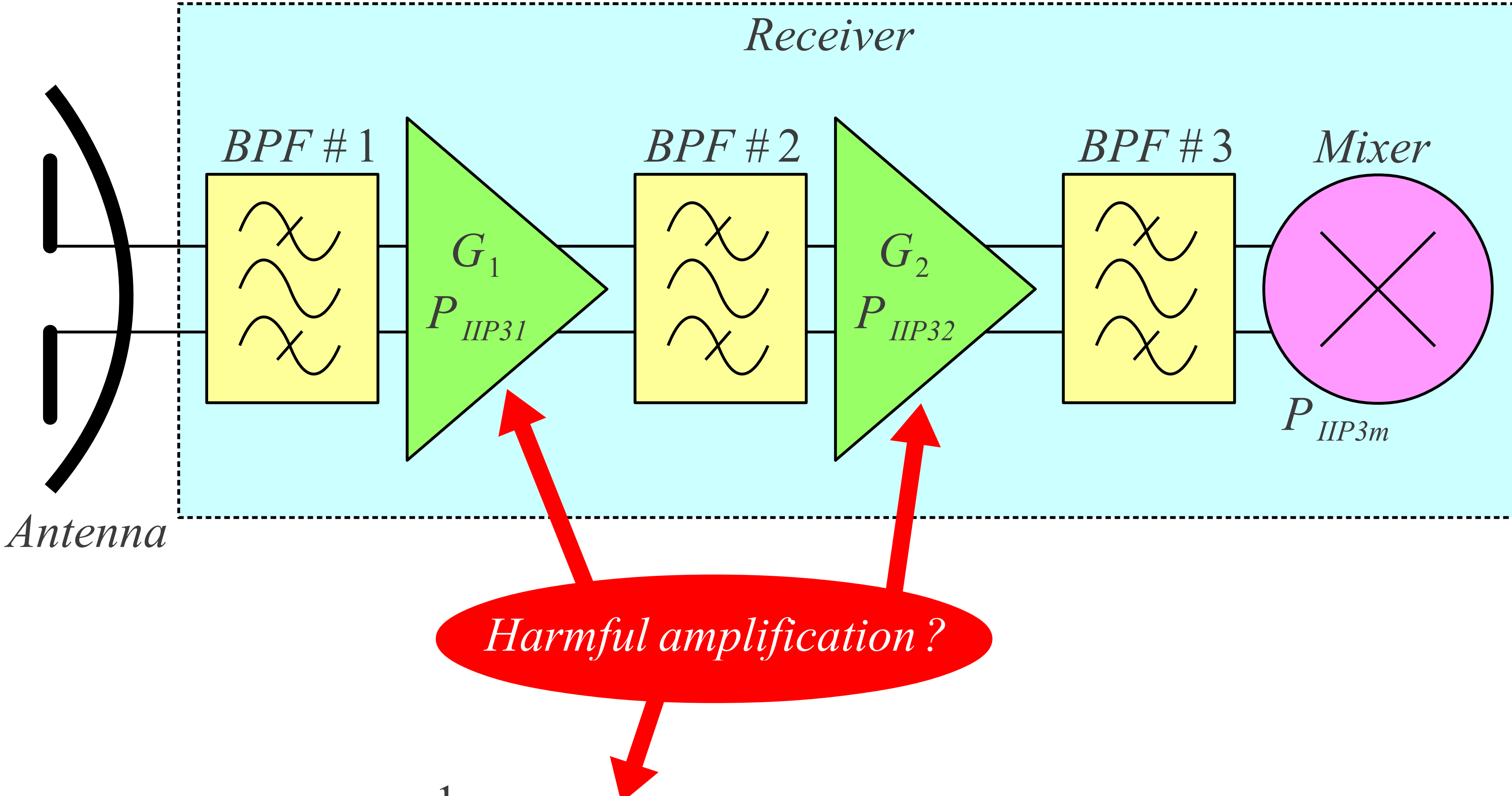
$$P_{IMD3input} = \frac{P^3_{input}}{P^2_{IIP3}}$$

$$P_{IMD3} = \frac{P^3_{LIN}}{P^2_{IP3}}$$

$$P_{IP3} = P_{IP3}\left(P_{VIR}\right)$$

*A receiver may be heavily overdriven !*
*The input signal may exceed*
$P_{input} > P_{IIP3}$ *o r* $P_{input} > P_{1dBinput}$
*Large signals may damage the receiver !*

# Receiver IMD




$$P_{IIP3chain} = \frac{1}{\frac{1}{P_{IIP31}} + \frac{G_1}{P_{IIP32}} + \frac{G_1 \cdot G_2}{P_{IIP3m}}}$$

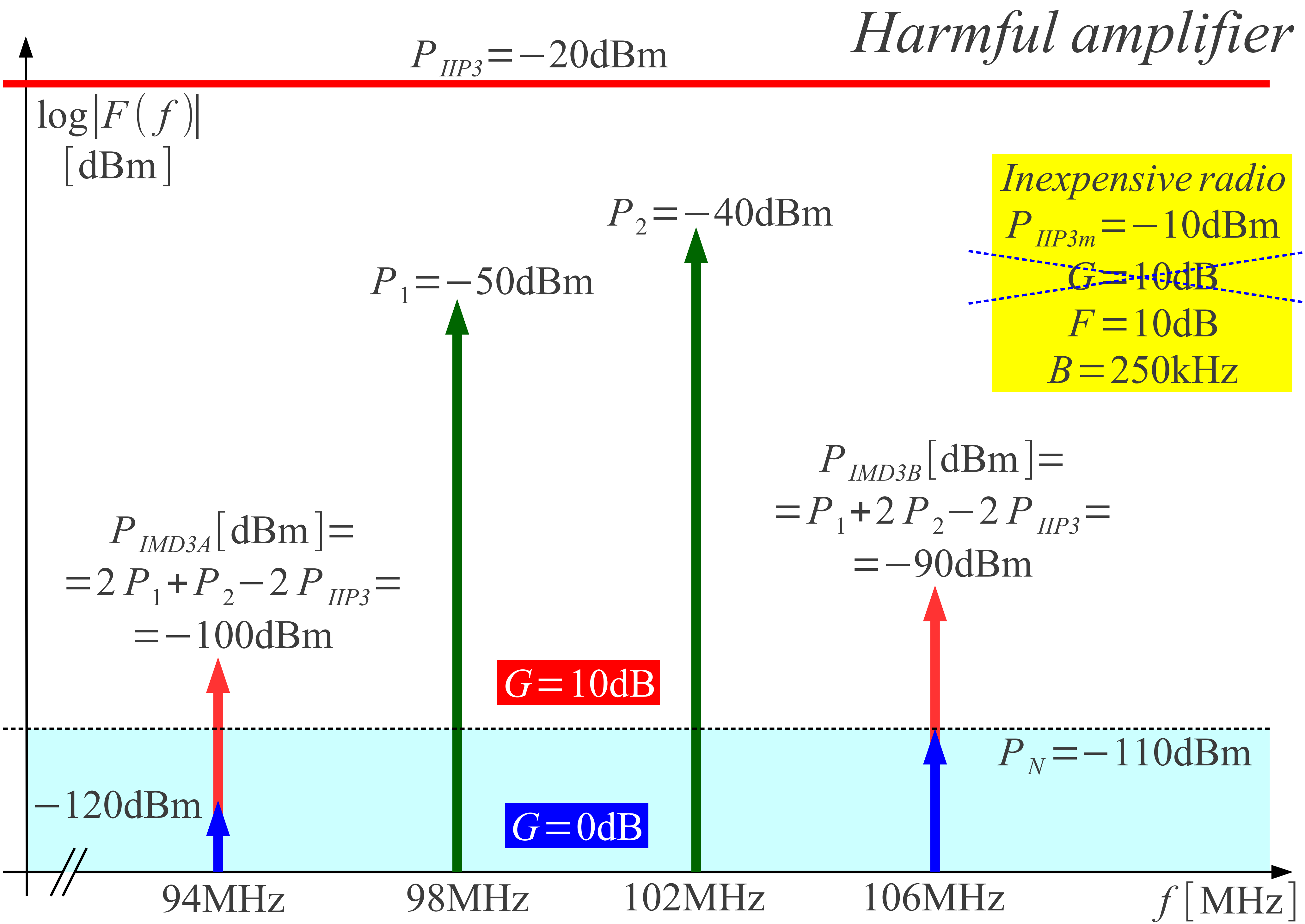
# Receiver Piip3

$$P_{IIP3chain} \approx \frac{P_{IIP3m}}{G_1 \cdot G_2}$$

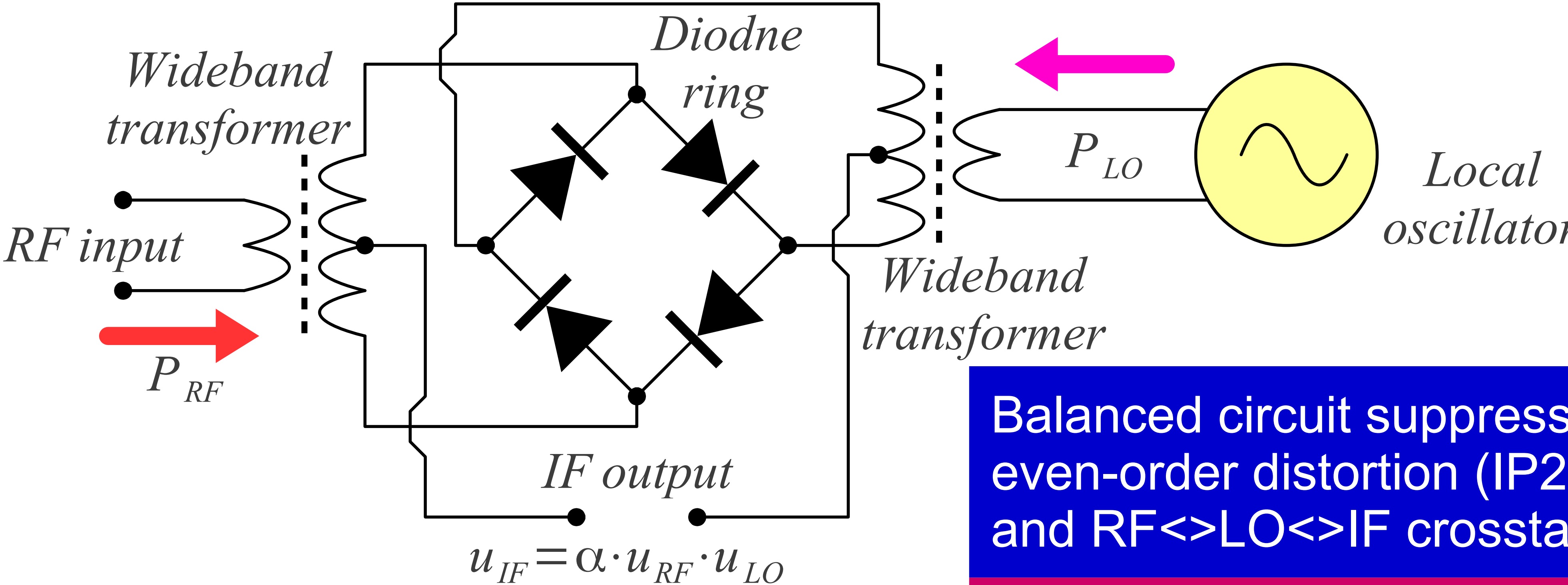
*Opposite requirements for mixer :*

(1) *NONLINEAR for mixing*

(2) *LINEAR for large* $P_{IIP3m}$




$$P_N = 10\log B + 10\log k_B T_0 + F_{dB} \approx 54\text{dB.Hz} - 174\text{dBm/Hz} + 10\text{dB} = -110\text{dBm}$$




Balanced circuit suppresses even-order distortion (IP2) and RF<>LO<>IF crosstalk

Balanced circuit has no effect on odd-order distortion nor $P_{IIP3}$ (IMD3)

$$\log P_{1dBinput} \approx \log P_{LO} - 6dB$$

$$\log P_{IIP3} \approx \log P_{1dBinput} + 15dB$$

| $P_{LO}$ | $P_{1dBinput}$ | $P_{IIP3}$ |
|---|---|---|
| +7dBm | +1dBm | +16dBm |
| +17dBm | +11dBm | +26dBm |
| +25dBm | +19dBm | +34dBm |

# *Doubly − balanced mixer*

# *Receiver dynamic range*

$$D = \frac{P_{MAX}}{P_{MIN}} = \sqrt[3]{\frac{P_{IIP3}^2}{P_N^2}} = \left(\frac{P_{IIP3}}{P_N}\right)^{2/3}$$

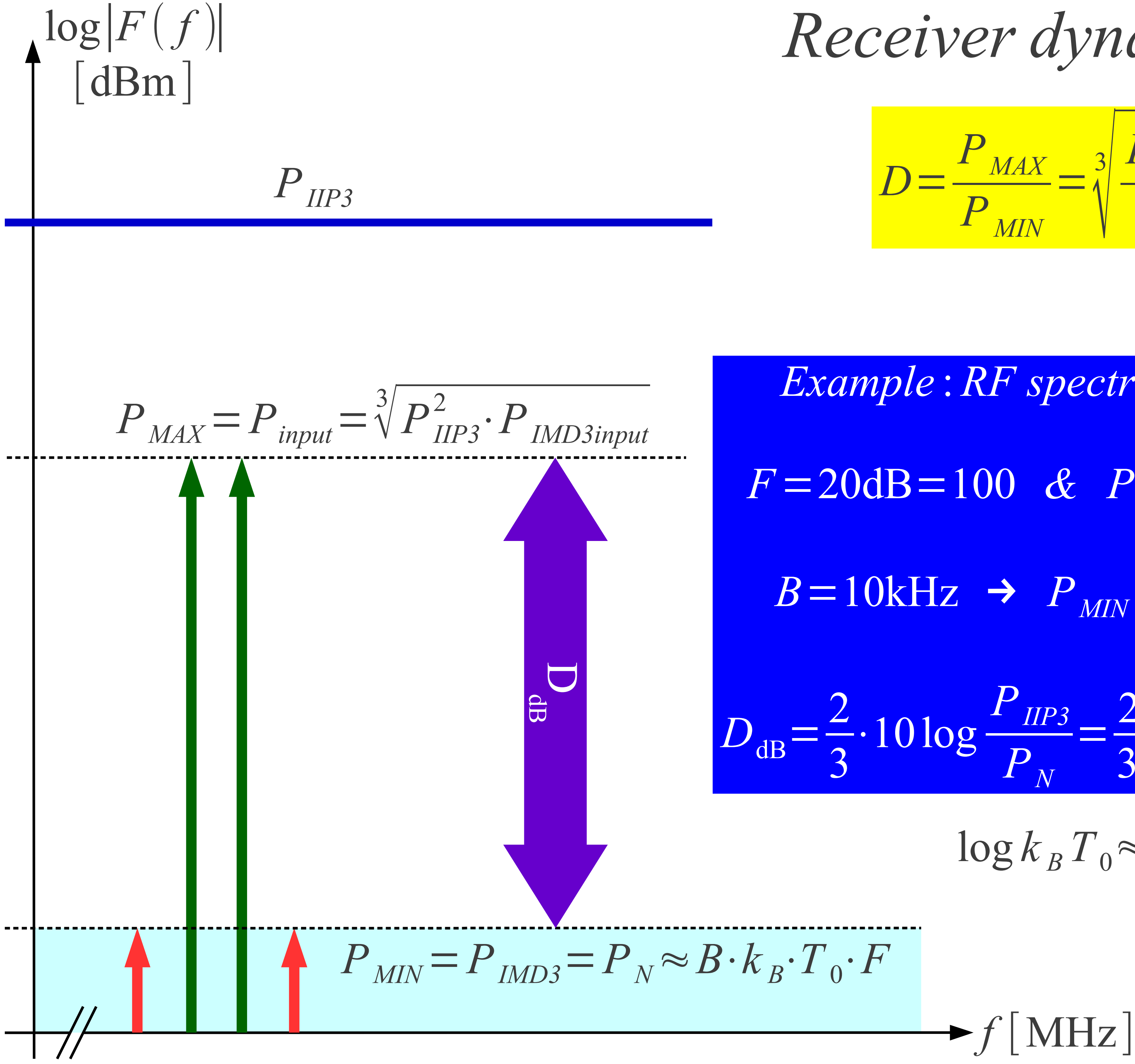
*Example : RF spectrum analyzer*

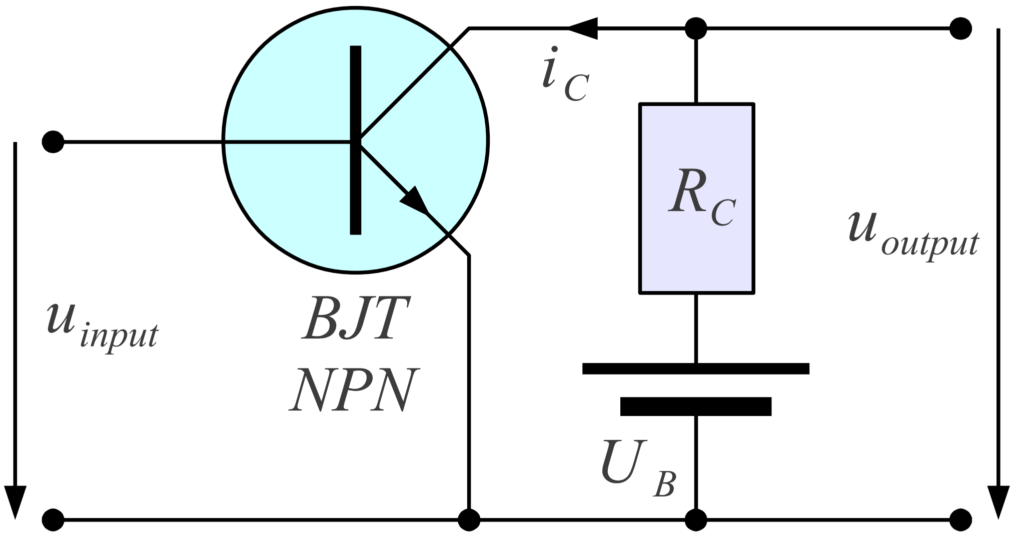
$$F = 20\text{dB} = 100 \quad \& \quad P_{IIP3} = +30\text{dBm}$$

$$B = 10\text{kHz} \;\rightarrow\; P_{MIN} = -114\text{dBm}$$

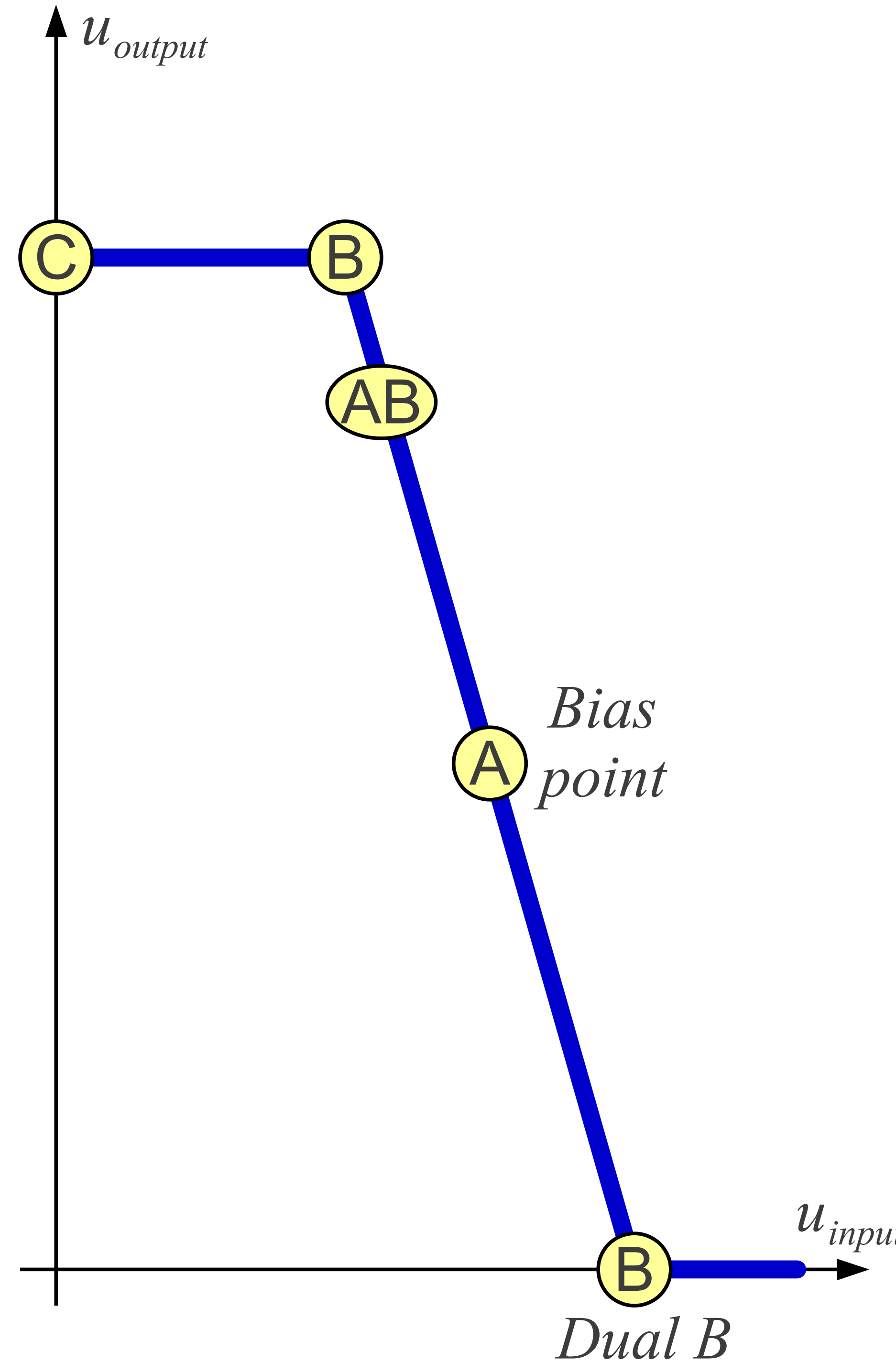
$$D_{\text{dB}} = \frac{2}{3}\cdot 10\log\frac{P_{IIP3}}{P_N} = \frac{2}{3}\cdot 144\text{dB} = 96\text{dB}$$

$$\log k_B T_0 \approx -174\text{dBm/Hz}$$

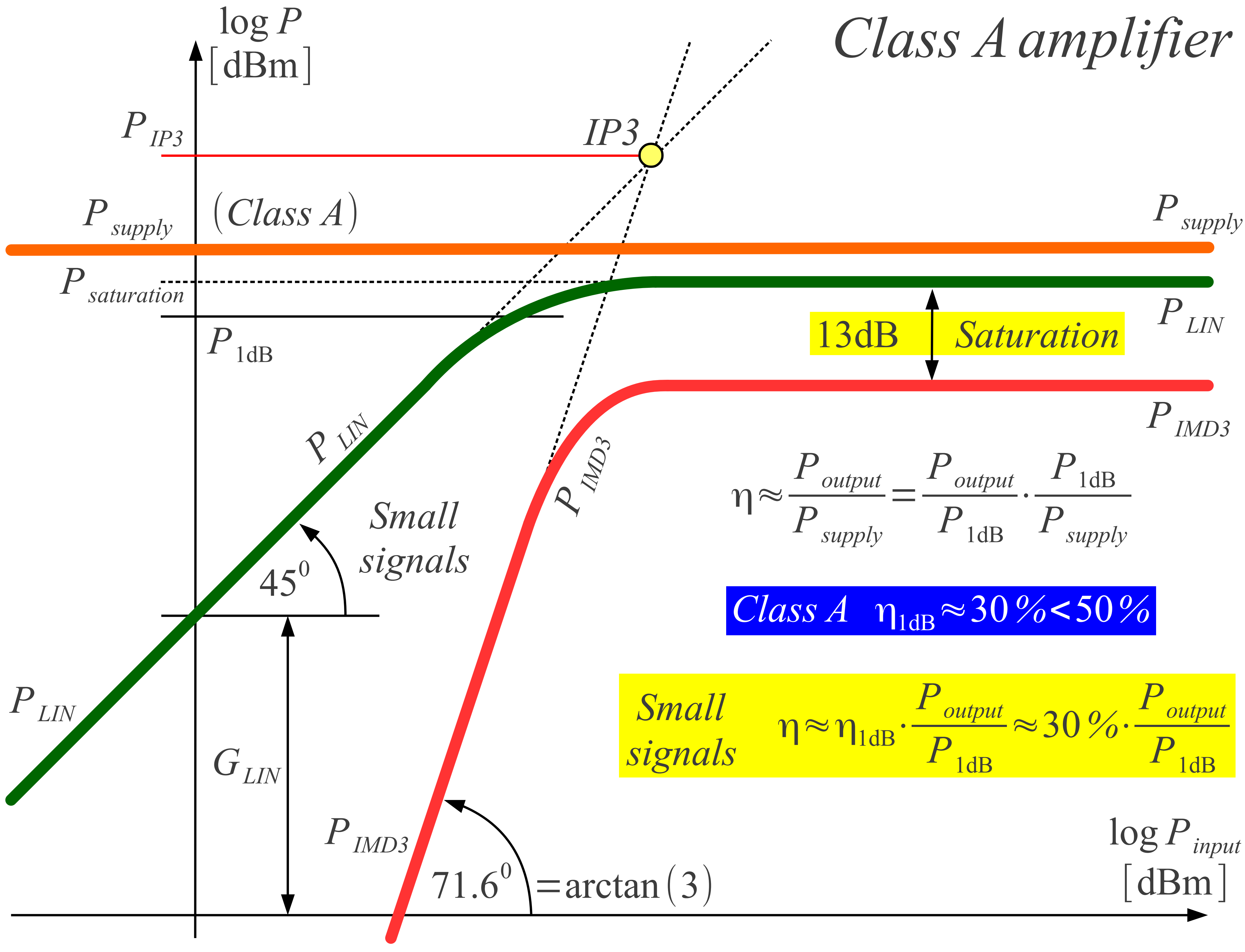




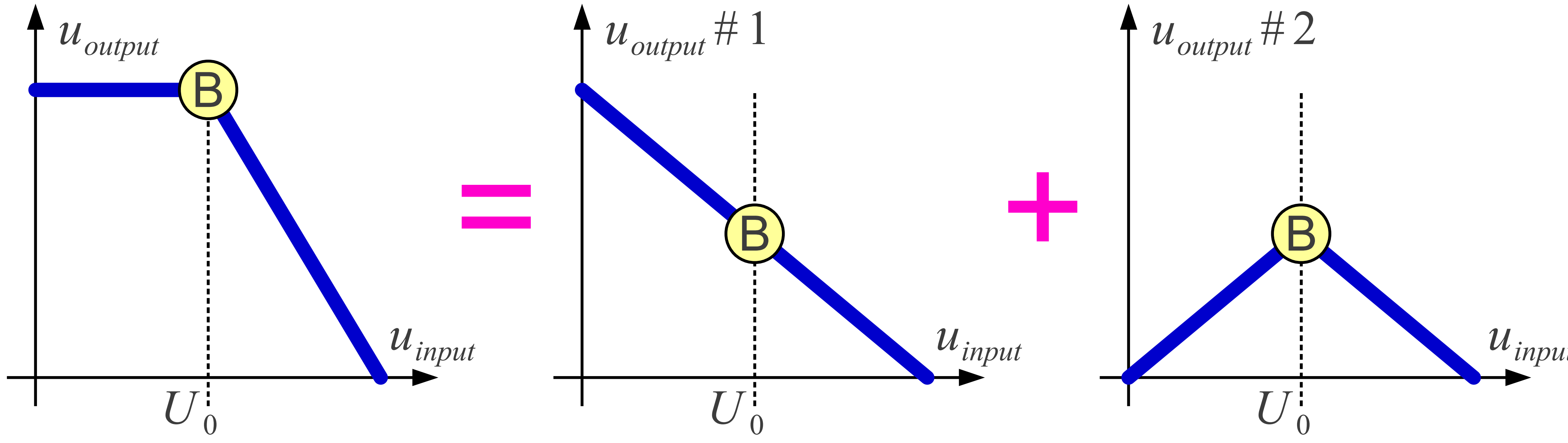

| | | $R_C \rightarrow L_C$ | |
|---|---|---|---|
| Class | Distortion | Theory $\eta$ | Available $\eta_{1dB}$ |
| A | small | 50% | ~30% |
| B | moderate | 78.5% | ~50% |
| C | large | 100% | ~70% |




# *Amplifier class of operation*



Class A amplifier
log P [dBm]
$P_{IP3}$
IP3
$P_{supply}$
(Class A)
$P_{supply}$
$P_{saturation}$
$P_{1dB}$
$P_{LIN}$
13dB
Saturation
$P_{IMD3}$
$P_{LIN}$
$P_{IMD3}$
Small signals
$45^0$
$\eta \approx \frac{P_{output}}{P_{supply}} = \frac{P_{output}}{P_{1dB}} \cdot \frac{P_{1dB}}{P_{supply}}$
Class A $\eta_{1dB} \approx 30\% < 50\%$
$P_{LIN}$
$G_{LIN}$
Small signals
$\eta \approx \eta_{1dB} \cdot \frac{P_{output}}{P_{1dB}} \approx 30\% \cdot \frac{P_{output}}{P_{1dB}}$
$P_{IMD3}$
$71.6^0 = \arctan(3)$
log $P_{input}$ [dBm]




$U_0 \equiv bias\ point$

$$u_{output}\#1 = \alpha_0{}' + \alpha_1 \cdot (u_{input} - U_0) \equiv rectilinear$$

$$u_{output\#2}(u_{input} - U_0) \equiv even\ function$$

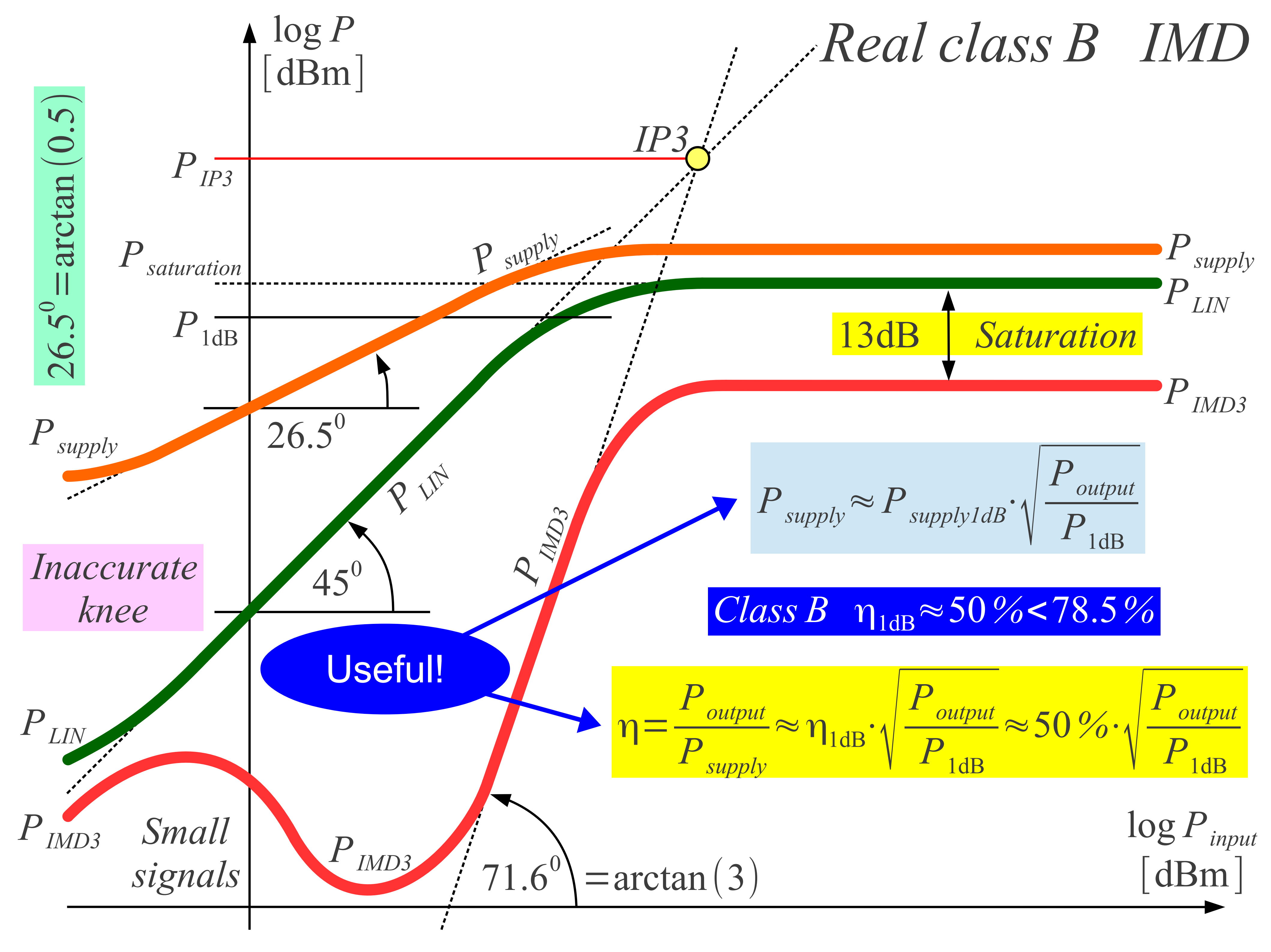
$$u_{output}\#2 = \alpha_0{}'' + \alpha_2 \cdot (u_{input} - U_0)^2 + \alpha_4 \cdot (u_{input} - U_0)^4 + \alpha_6 \cdot (u_{input} - U_0)^6 + \ldots$$

$$u_{output} = u_{output\#1} + u_{output\#2}$$

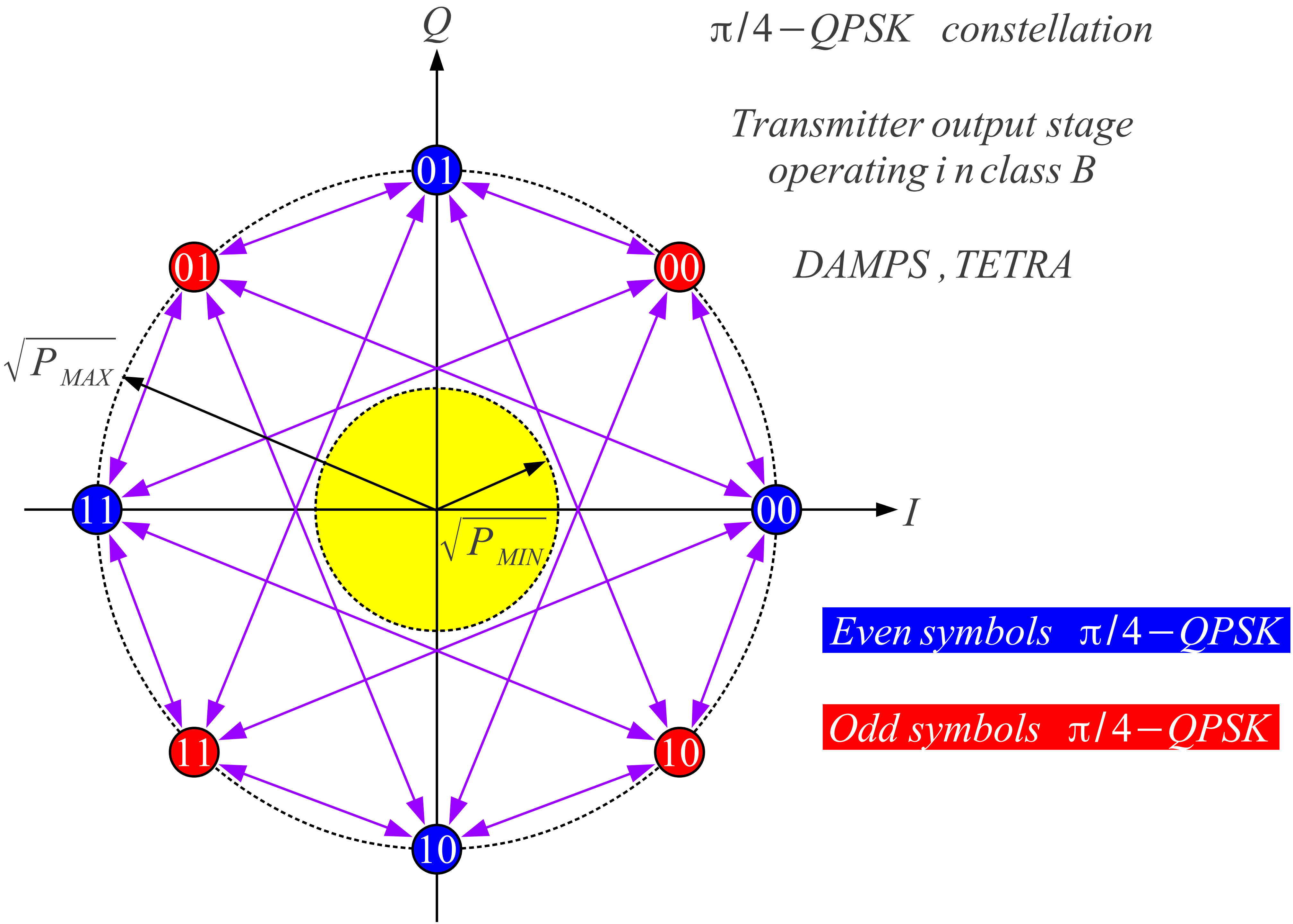
*An ideal class B amplifier has no odd terms* 3, 5, 7, 9 ... ≡ *NOT generating IMD*:

$$u_{output} = \alpha_0 + \alpha_1 \cdot (u_{input} - U_0) + \alpha_2 \cdot (u_{input} - U_0)^2 + \alpha_4 \cdot (u_{input} - U_0)^4 + \alpha_6 \cdot (u_{input} - U_0)^6 + \ldots$$

# *Class B transfer function*

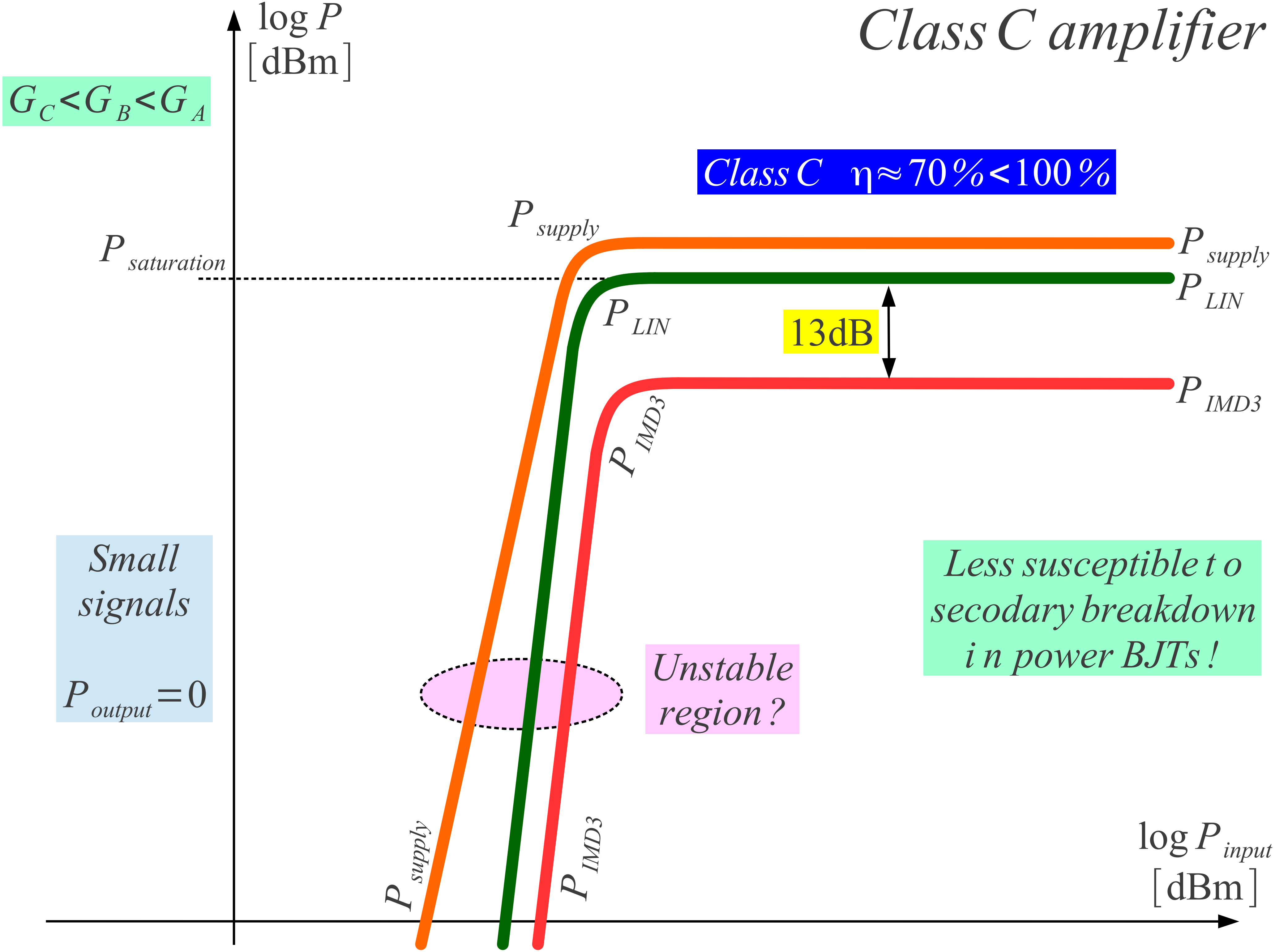


Real class B IMD
log P [dBm]
$26.5^0 = \arctan(0.5)$
$P_{IP3}$
IP3
$P_{saturation}$
$P_{supply}$
$P_{1dB}$
$26.5^0$
$P_{supply}$
$P_{LIN}$
$45^0$
Inaccurate knee
Useful!
$P_{IMD3}$
$P_{LIN}$
$P_{IMD3}$
Small signals
$P_{IMD3}$
$71.6^0 = \arctan(3)$
log $P_{input}$ [dBm]
$P_{supply}$
$P_{LIN}$
13dB Saturation
$P_{IMD3}$
$P_{supply} \approx P_{supply1dB} \cdot \sqrt{\frac{P_{output}}{P_{1dB}}}$
Class B $\eta_{1dB} \approx 50\% < 78.5\%$
$\eta = \frac{P_{output}}{P_{supply}} \approx \eta_{1dB} \cdot \sqrt{\frac{P_{output}}{P_{1dB}}} \approx 50\% \cdot \sqrt{\frac{P_{output}}{P_{1dB}}}$

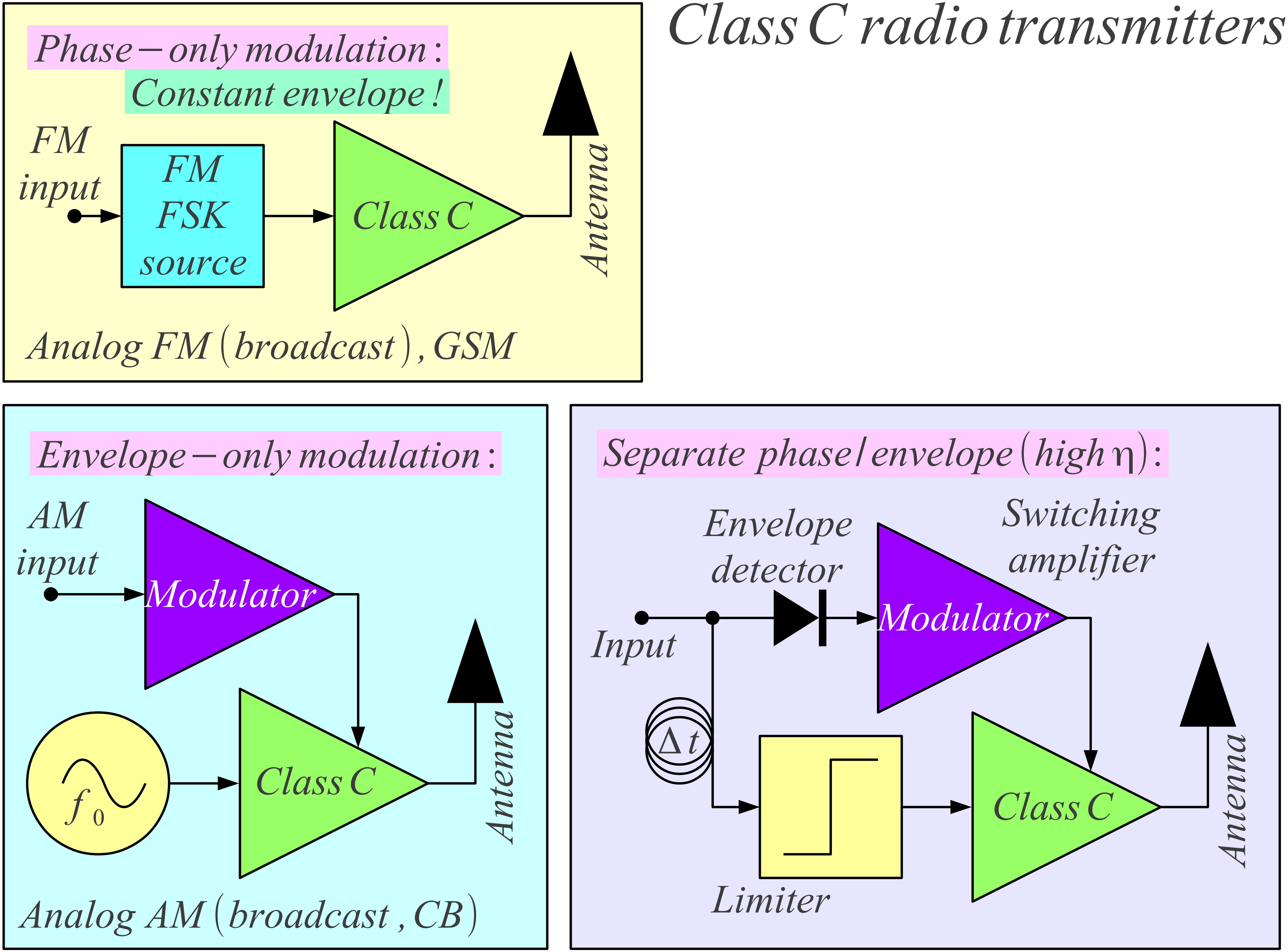


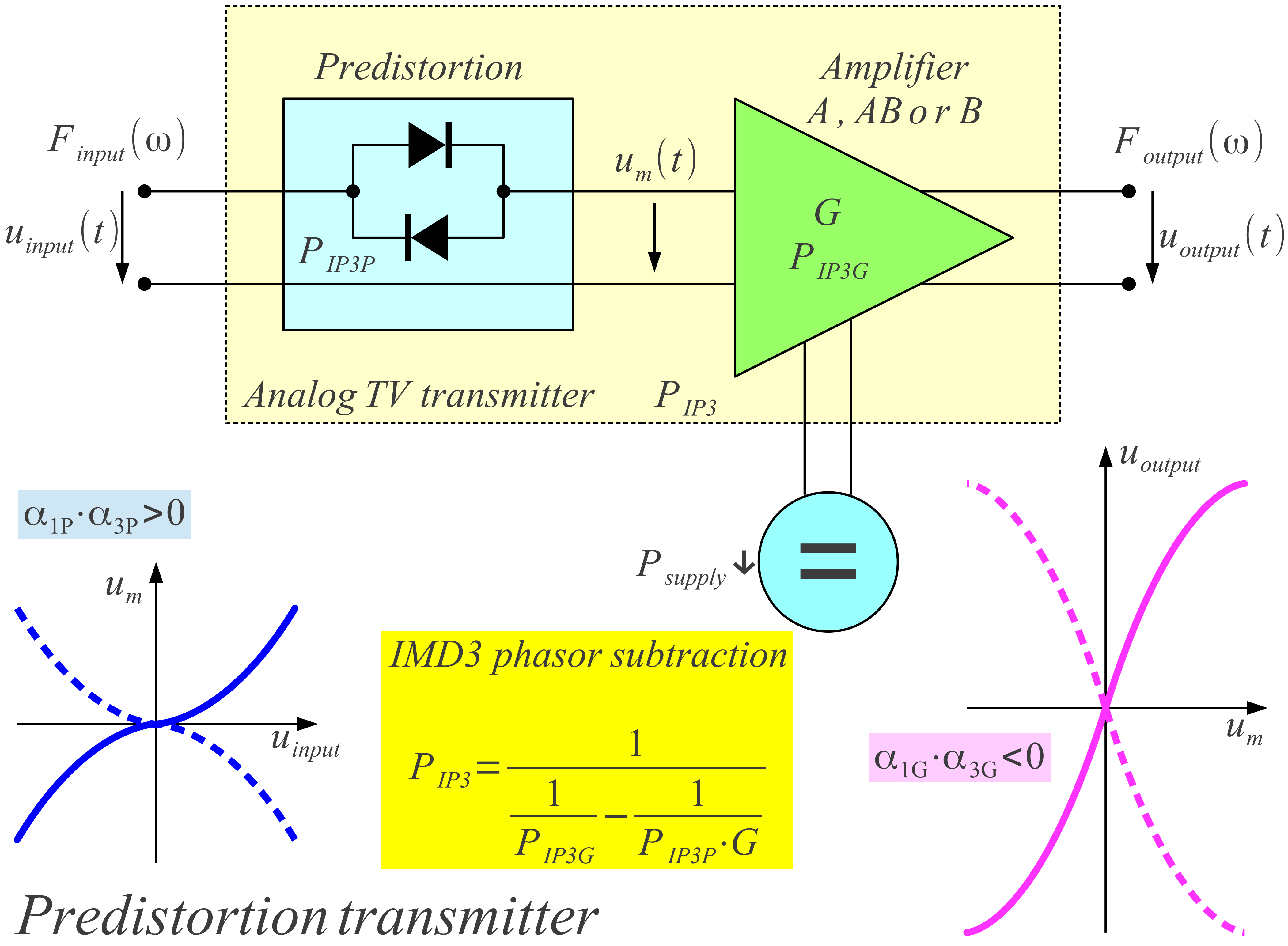

# Class B for π/4−QPSK modulation

# Class C amplifier

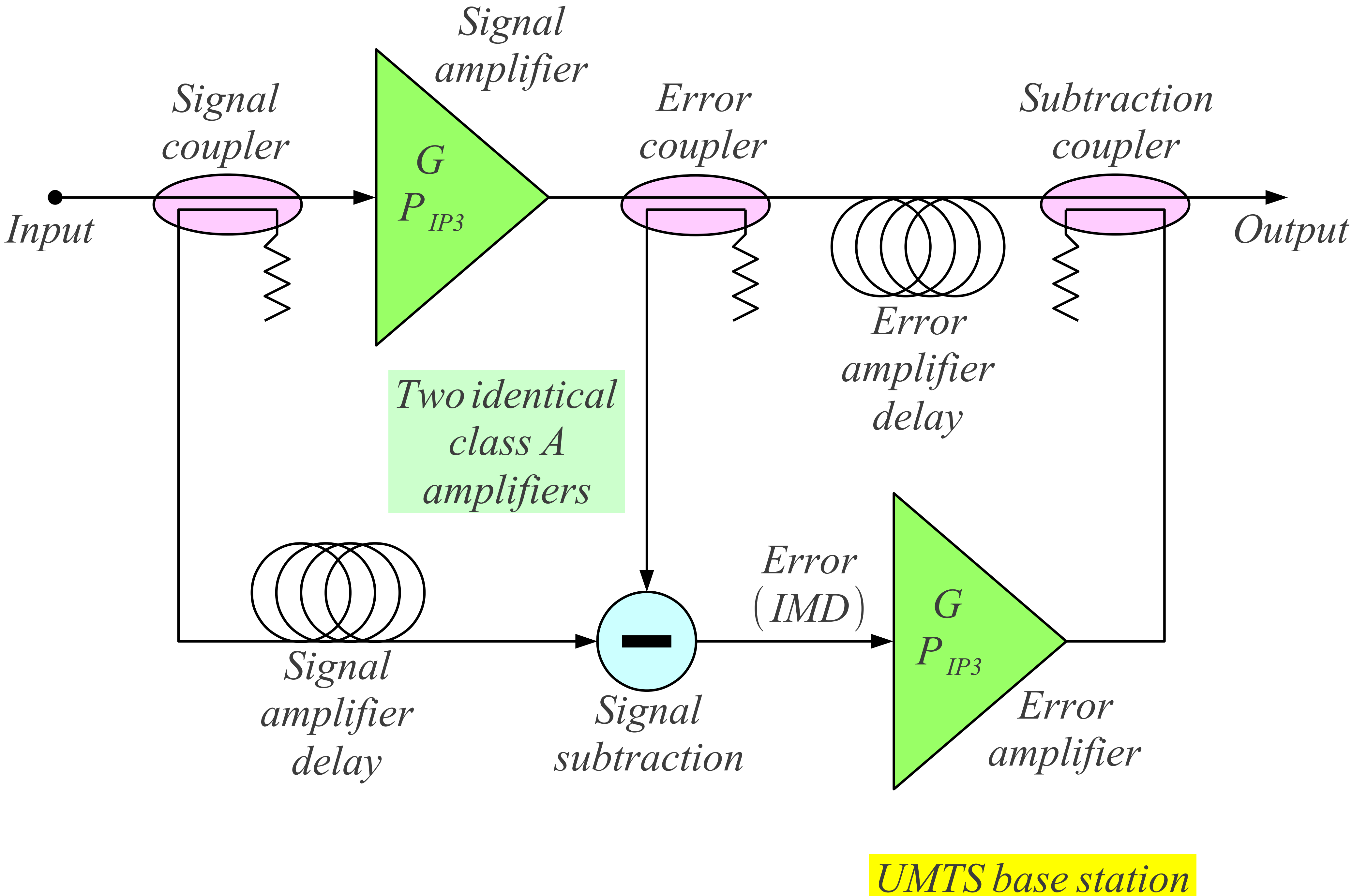


# Class C radio transmitters



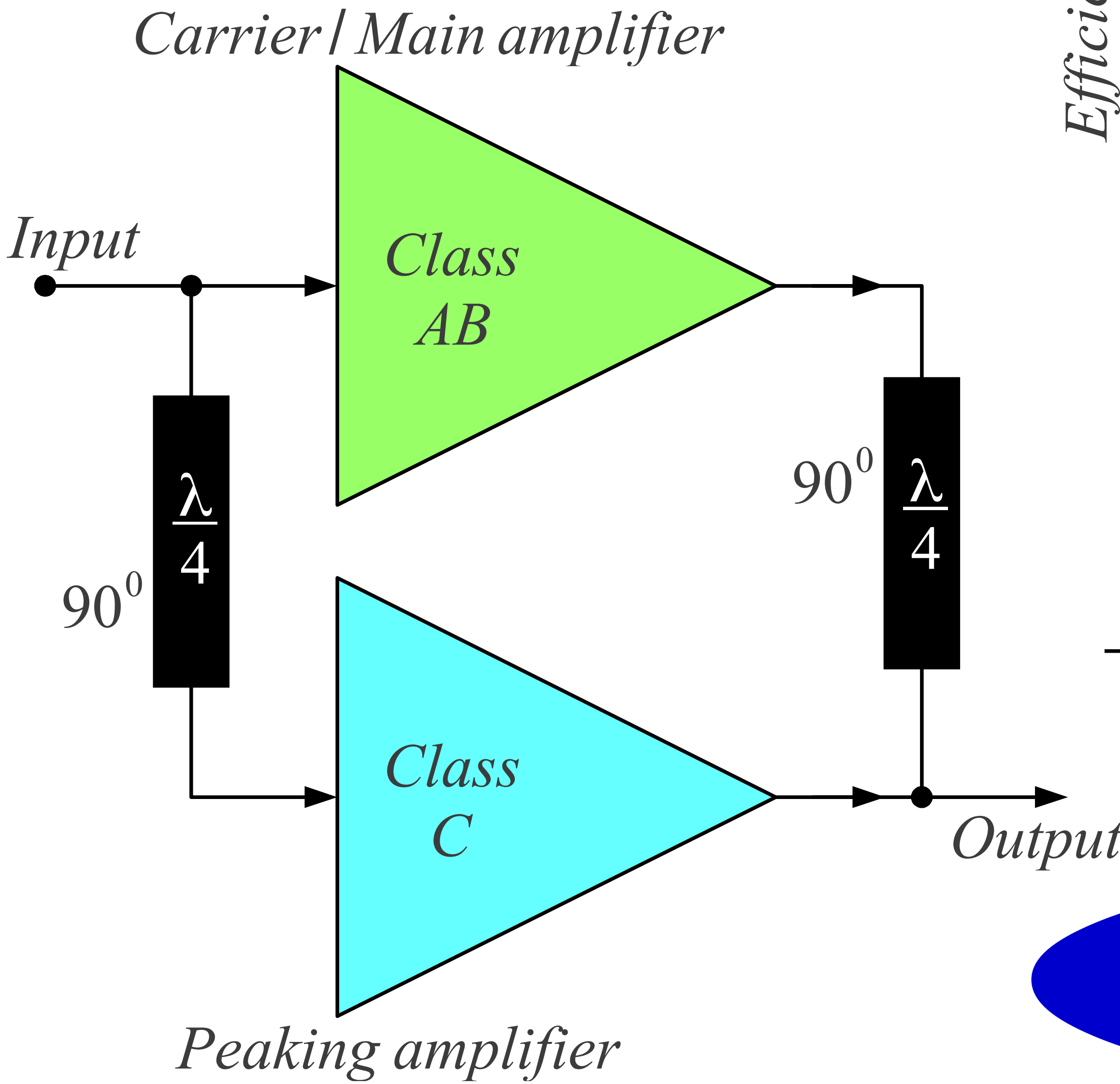


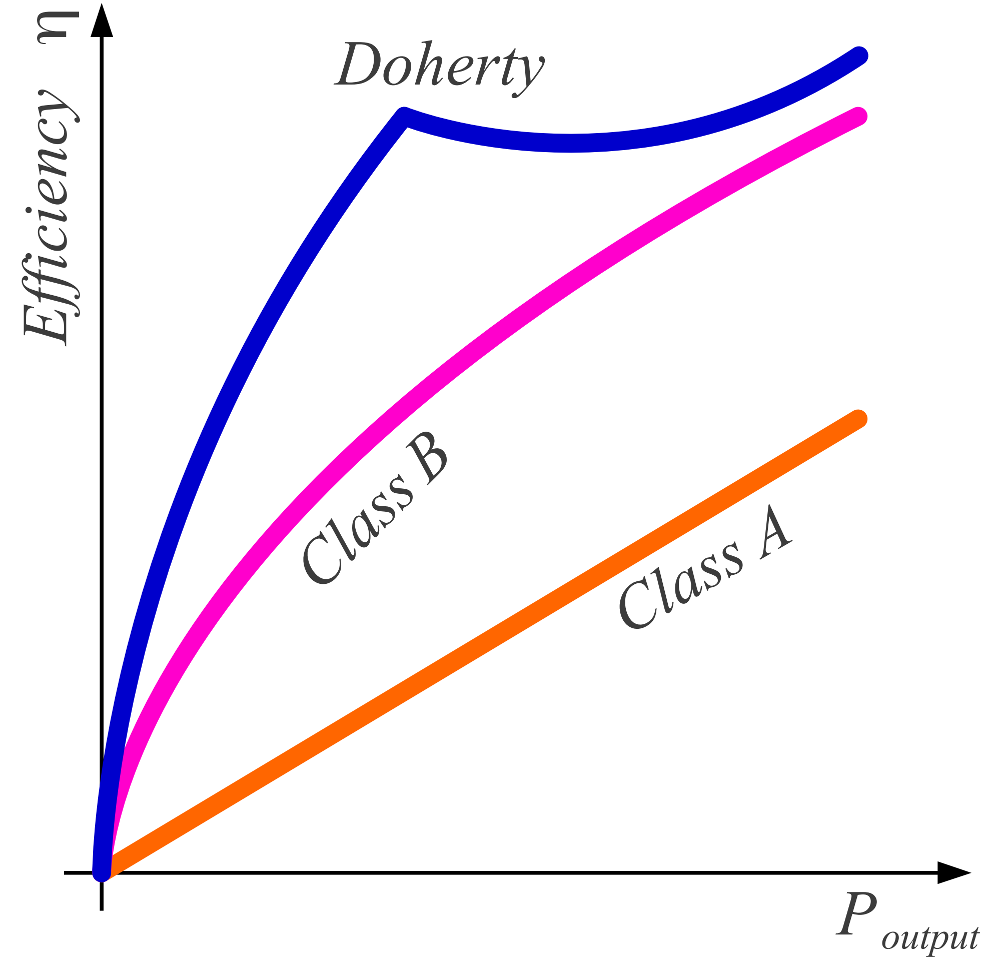
Predistortion
Amplifier
A, AB or B
$F_{input}(\omega)$
$u_{input}(t)$
$u_m(t)$
$F_{output}(\omega)$
$u_{output}(t)$
$P_{IP3P}$
G
$P_{IP3G}$
Analog TV transmitter
$P_{IP3}$
$P_{supply}$
$\alpha_{1P} \cdot \alpha_{3P} > 0$
$u_m$
$u_{input}$
IMD3 phasor subtraction
$P_{IP3} = \frac{1}{\frac{1}{P_{IP3G}} - \frac{1}{P_{IP3P} \cdot G}}$
$u_{output}$
$u_m$
$\alpha_{1G} \cdot \alpha_{3G} < 0$
Predistortion transmitter




*Feed – forward amplifer*

# *Doherty amplifier*



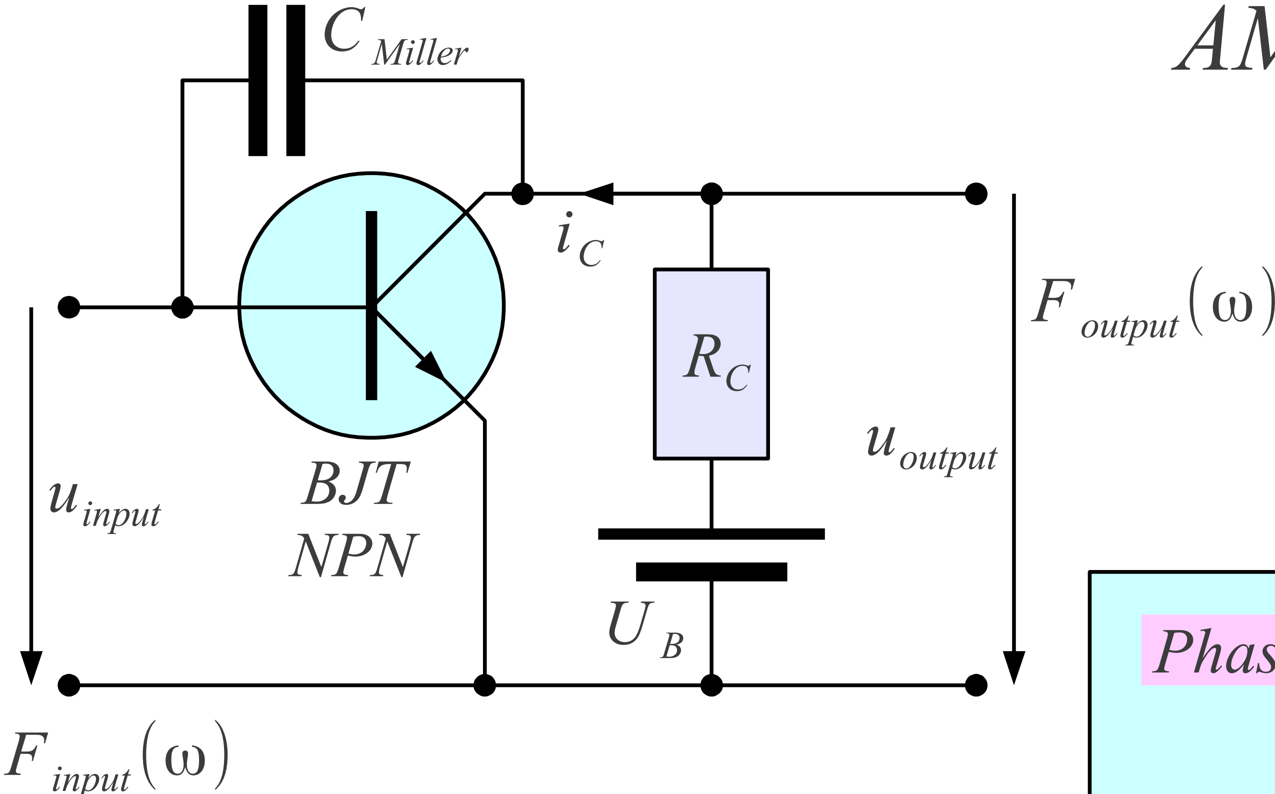

William H. Doherty 1936

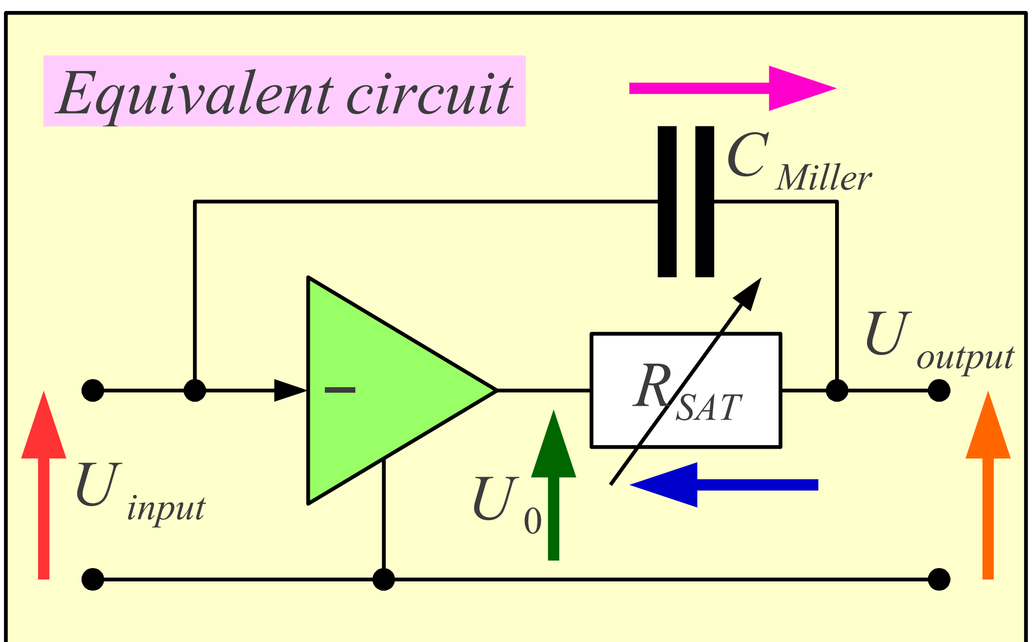
$P_{1dB} \rightarrow P_{3dB}$

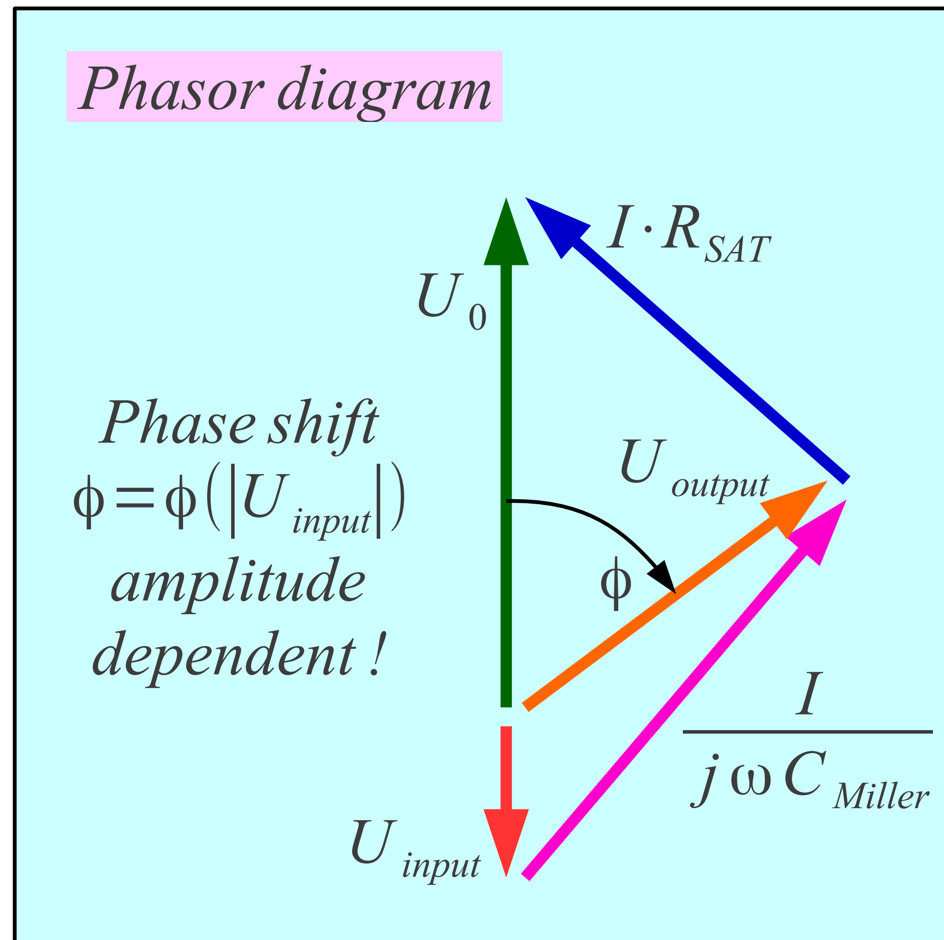
*OFDM transmitters for DVB – T & LTE*
*(+adaptive digital predistortion)*

# $AM \rightarrow \phi M$ conversion

**Four − wave mixing (optical − fiber IMD) caused exclusively by $AM \rightarrow \phi M$ conversion**

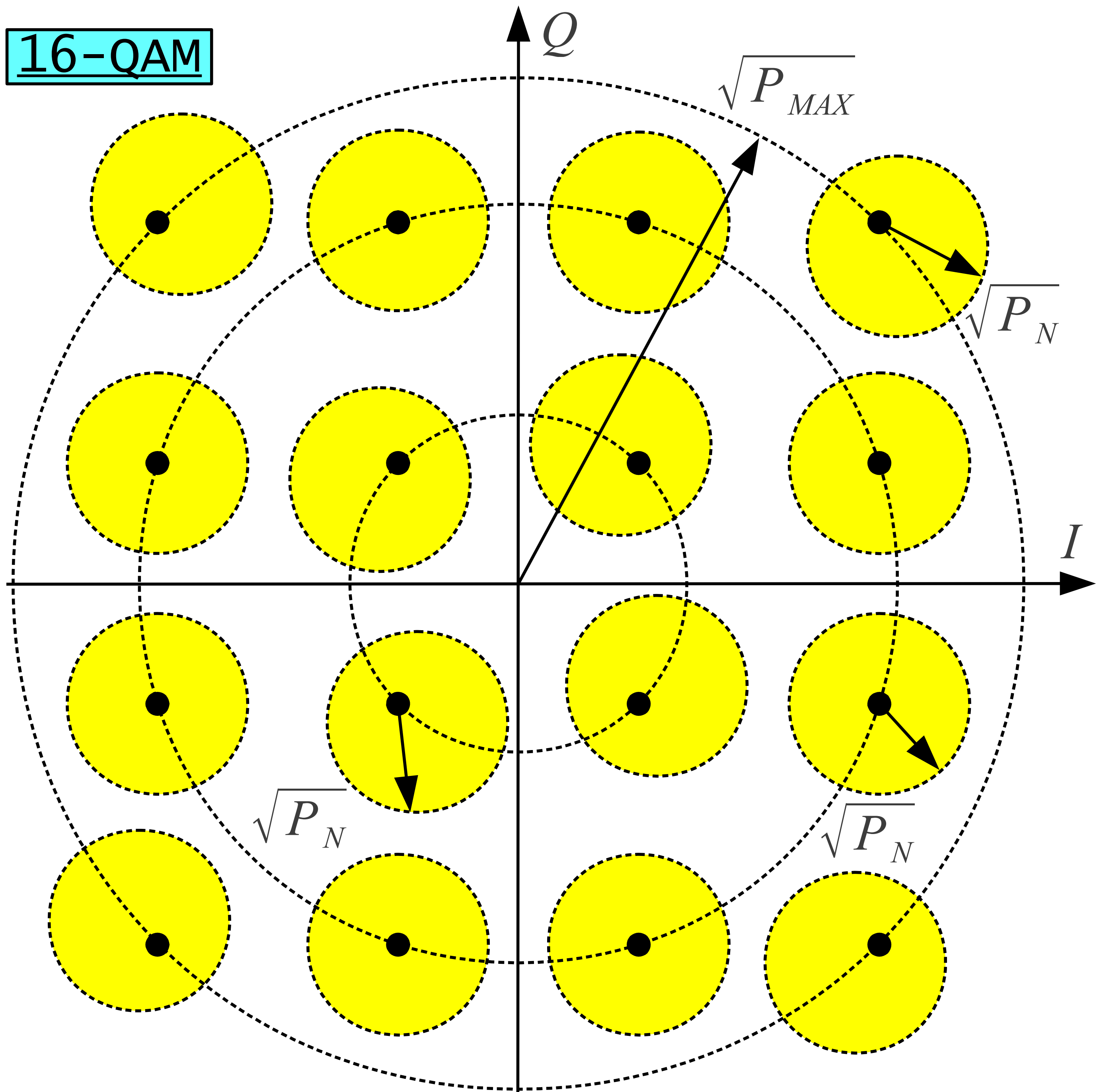


*AM / φ M → mutual rotation inside constellation*

*EVM ≡ Error Vector Magnitude*




$$EVM\,[\%] = \sqrt{\frac{\langle P_N \rangle}{P_{MAX}}}$$

*or*

$$EVM\,[\%] = \sqrt{\frac{\langle P_N \rangle}{\langle P_S \rangle}}$$

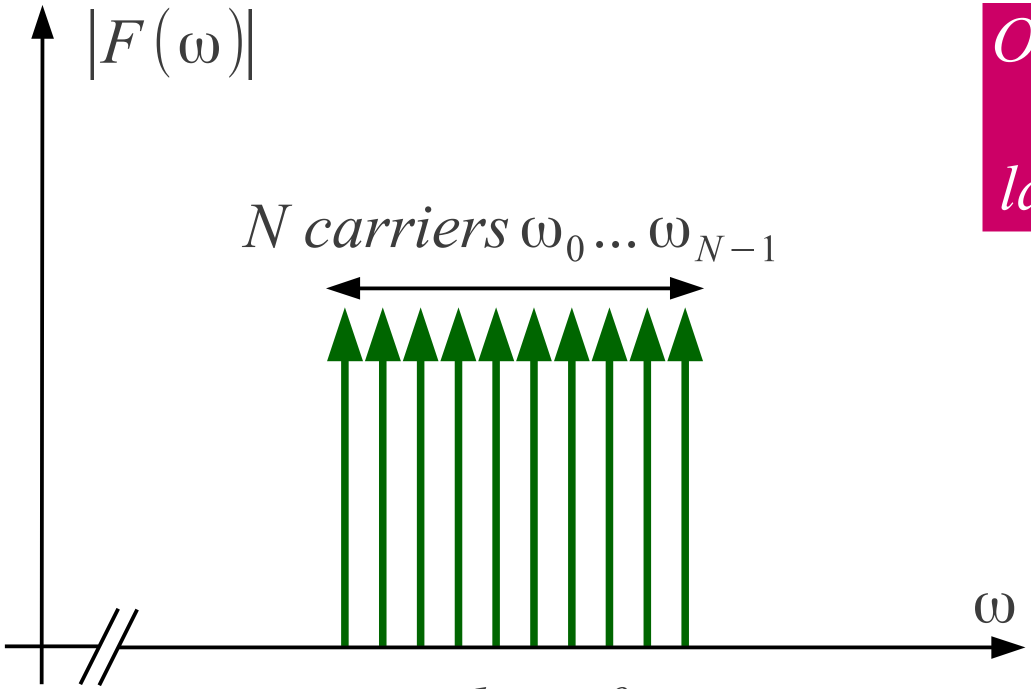
$$EVM_{dB} = 20 \log_{10} EVM$$

*EVM* & *MER* include noise, interference & distortion!

$$MER_{dB} = 10 \log_{10} \left( \frac{\langle P_S \rangle}{\langle P_N \rangle} \right)$$

*MER ≡ Modulation Error Ratio*

# *EVM & MER*




$N \equiv$ number of carriers

$P_0 \equiv$ single − carrier power

$\langle P \rangle = N \cdot P_0 \equiv$ average power

$P_{MAX} = N^2 \cdot P_0 \equiv$ peak power

$$PAPR = \frac{P_{MAX}}{\langle P \rangle} = \frac{N^2 \cdot P_0}{N \cdot P_0} = N$$

OFDM typically $N = 48 \ldots 27265$ carriers

large $PAPR = N$ → poor efficiency $\eta_{TX}$

Large number of independent carriers:

Rayleigh distribution of the power probability density

$$p(P) = \frac{1}{\langle P \rangle} \cdot e^{-\frac{P}{\langle P \rangle}}$$

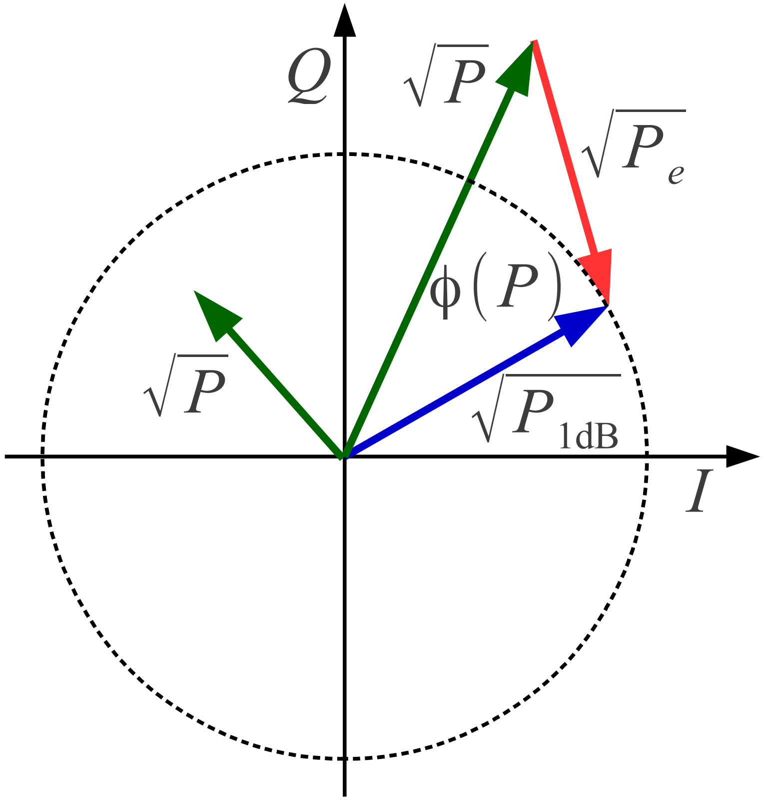
Probability density of large powers

$P \gg \langle P \rangle$

is extremely small !

Sensible choice for $P_{MAX} \approx P_{1dB} = ?$

# Peak − to − Average Power Ratio (PAPR)




$$P_e(P) = P + P_{1dB} - 2\sqrt{P \cdot P_{1dB}}\cos\phi(P)$$

$$P_e(P) \equiv \textit{measurable but demanding}$$

$$\langle P_e \rangle = \int_0^\infty P_e(P) \cdot p(P)\, dP$$

$$\textit{Rough estimate} \begin{cases} P_e(P) = 0 & @ \quad P < P_{1dB} \\ P_e(P) \approx P & @ \quad P > P_{1dB} \end{cases}$$

$$\langle P_e \rangle \approx \int_{P_{1dB}}^\infty \frac{P}{\langle P \rangle} \cdot e^{-\frac{P}{\langle P \rangle}} dP = P_{1dB} \cdot e^{-\frac{P_{1dB}}{\langle P \rangle}} + \langle P \rangle \cdot e^{-\frac{P_{1dB}}{\langle P \rangle}}$$

$$EVM = \sqrt{\frac{\langle P_e \rangle}{\langle P \rangle}} \equiv \textit{Error Vector Magnitude}$$

$$EVM \approx \sqrt{\left(\frac{P_{1dB}}{\langle P \rangle} + 1\right) \cdot e^{-\frac{P_{1dB}}{\langle P \rangle}}}$$

| $10\log_{10}\frac{P_{1dB}}{\langle P \rangle}$ | $EVM$ |
|---|---|
| 0dB | 86% |
| 5dB | 42% |
| 10dB | 2.2% |
| 15dB | 0.000078% |
| 20dB | 1.9E-19% |

# *OFDM backoff*