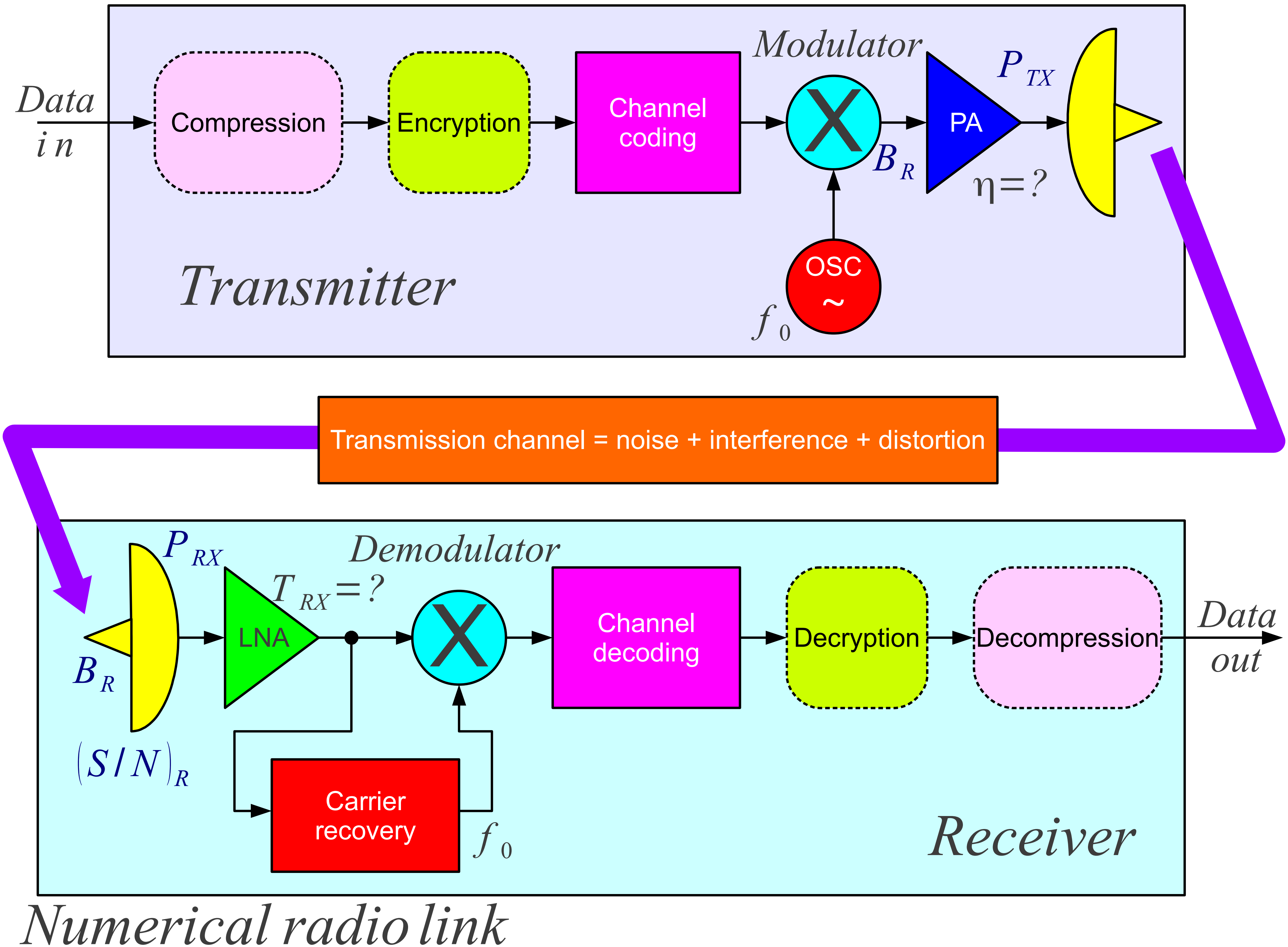
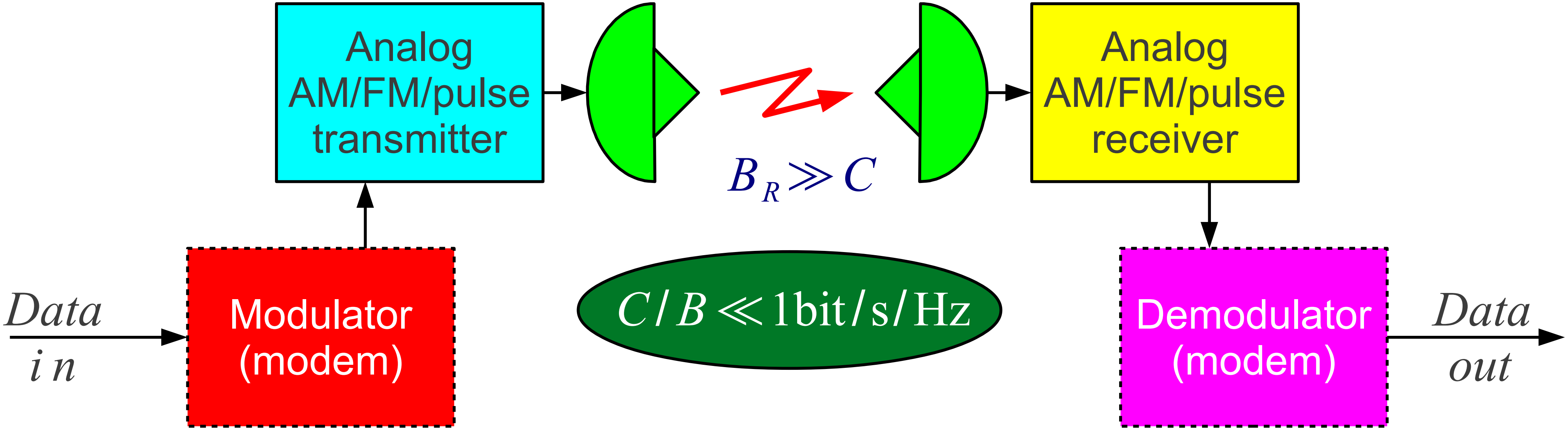
# Communication Electronics

## Lecture 11:

## Numerical modulations



Data in
Compression
Encryption
Channel coding
Modulator
$B_R$
PA
$P_{TX}$
$\eta = ?$
OSC ~
$f_0$
Transmitter
Transmission channel = noise + interference + distortion
$P_{RX}$
Demodulator
$T_{RX} = ?$
LNA
Channel decoding
Decryption
Decompression
Data out
$B_R$
$(S/N)_R$
Carrier recovery
$f_0$
Receiver
Numerical radio link




- ASK ≡ Amplitude Shift Keying
- FSK ≡ Frequency Shift Keying
- AASK ≡ Audio Amplitude Shift Keying
- AFSK ≡ Audio Frequency Shift Keying
- OOK ≡ On-Off Keying
- PAM ≡ Pulse-Amplitude Modulation
- PPM ≡ Pulse-Position Modulation

Large radio bandwidth $B_R$ >>C

Poor spectral efficiency C/B<<1bit

Inefficient non-coherent receiver

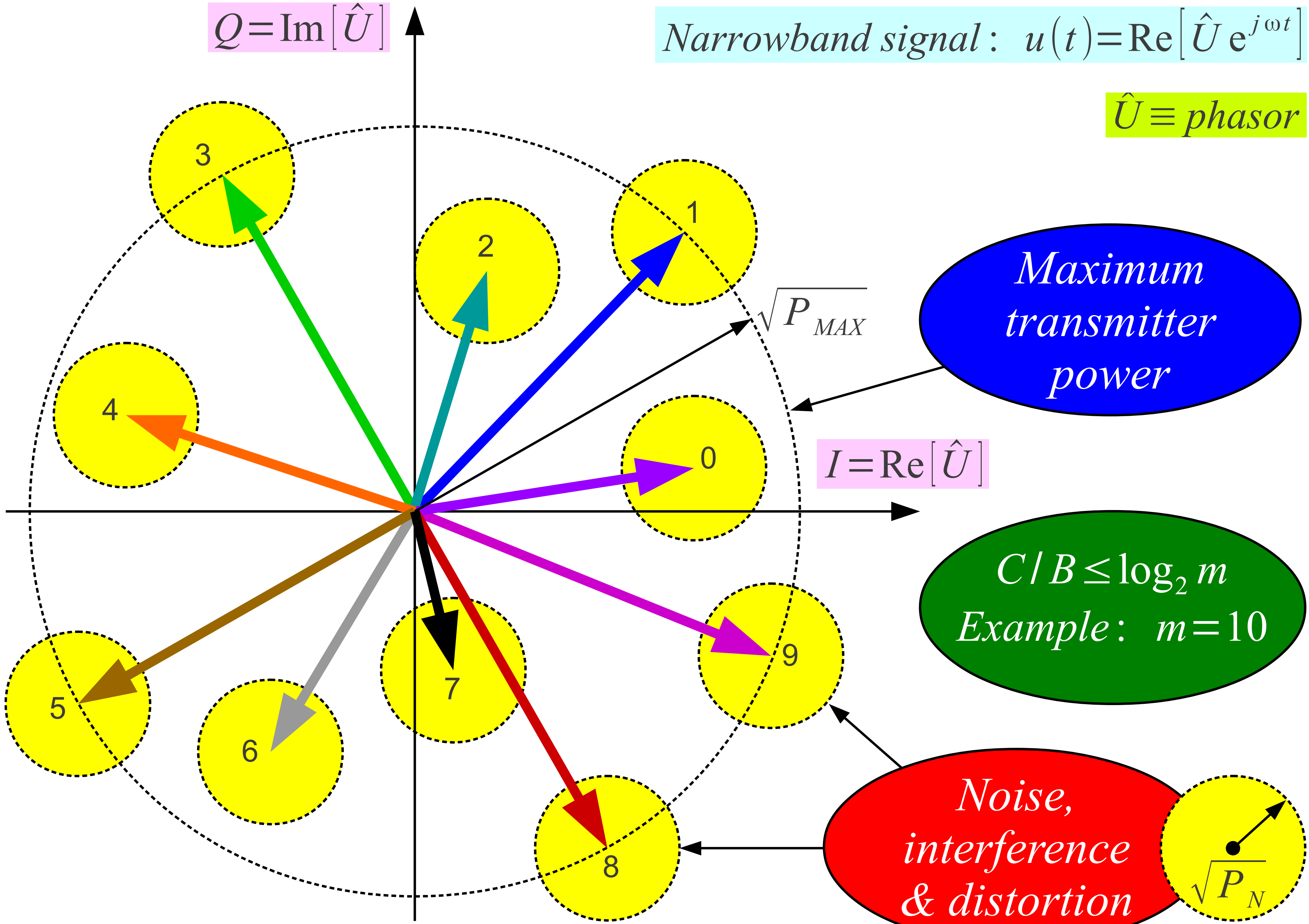
20dB...50dB worse than Shannon

HISTORICAL PATCHING!

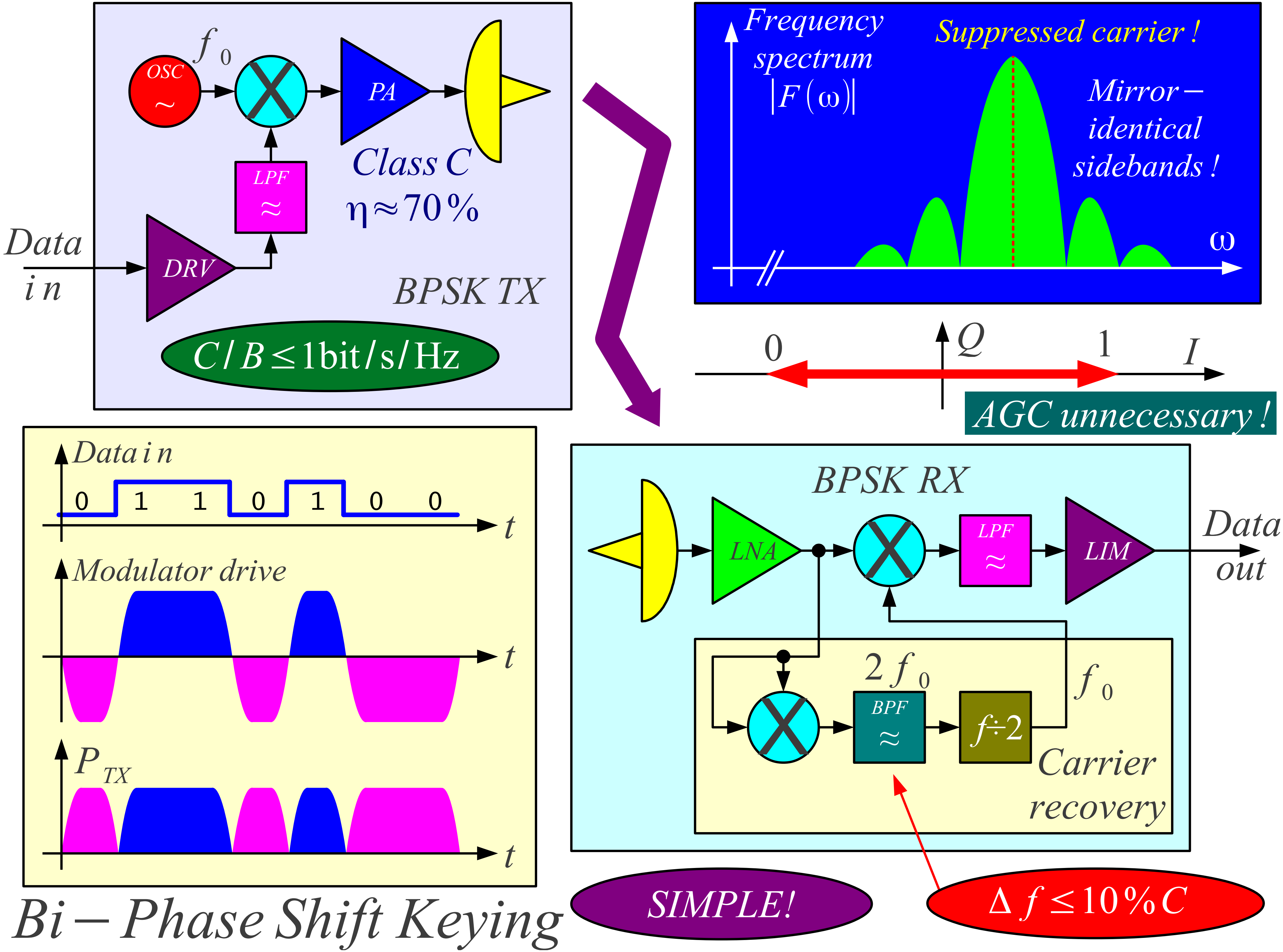
Simple transmitter and receiver

Insensitive to carrier-frequency errors

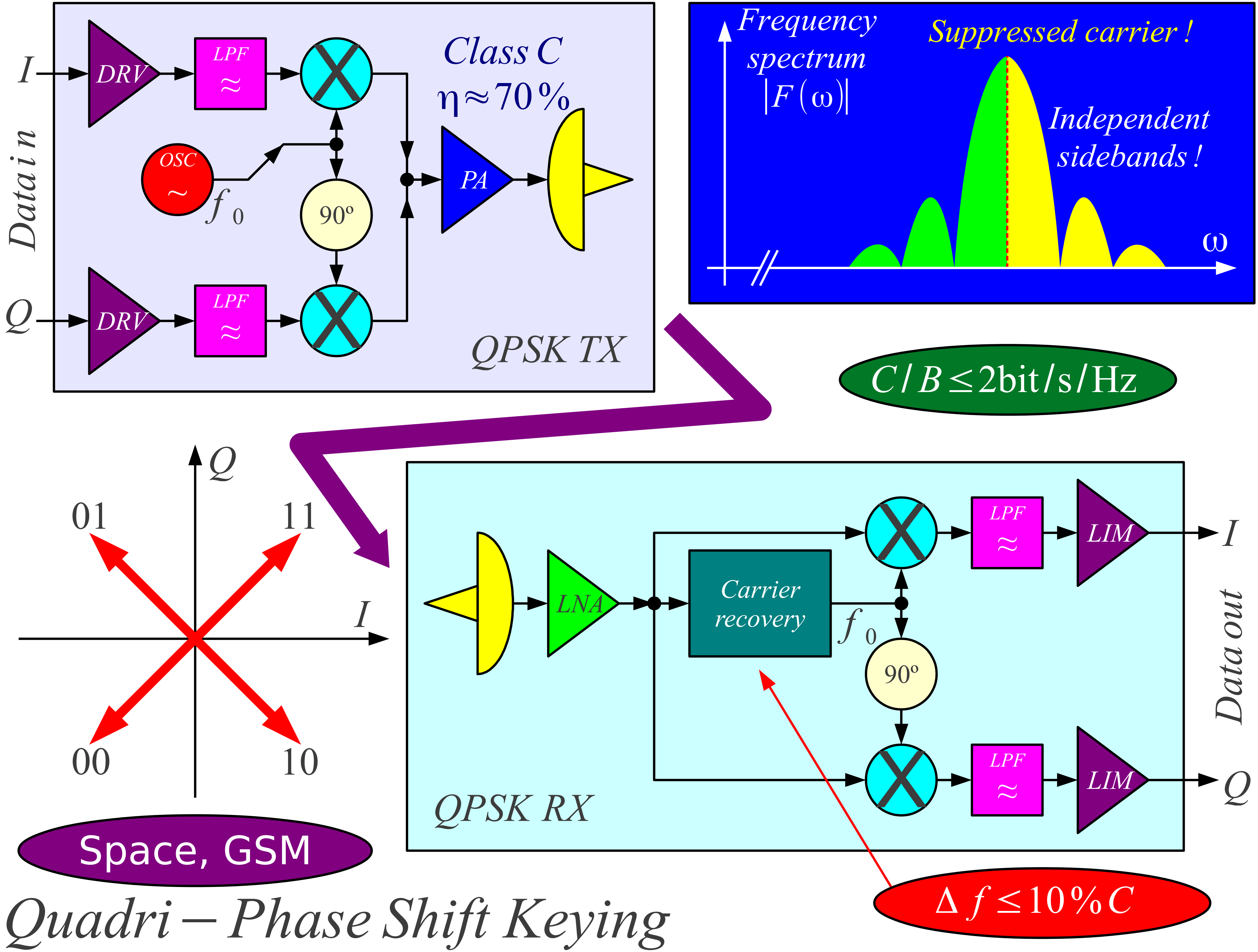
*Modem radio link*



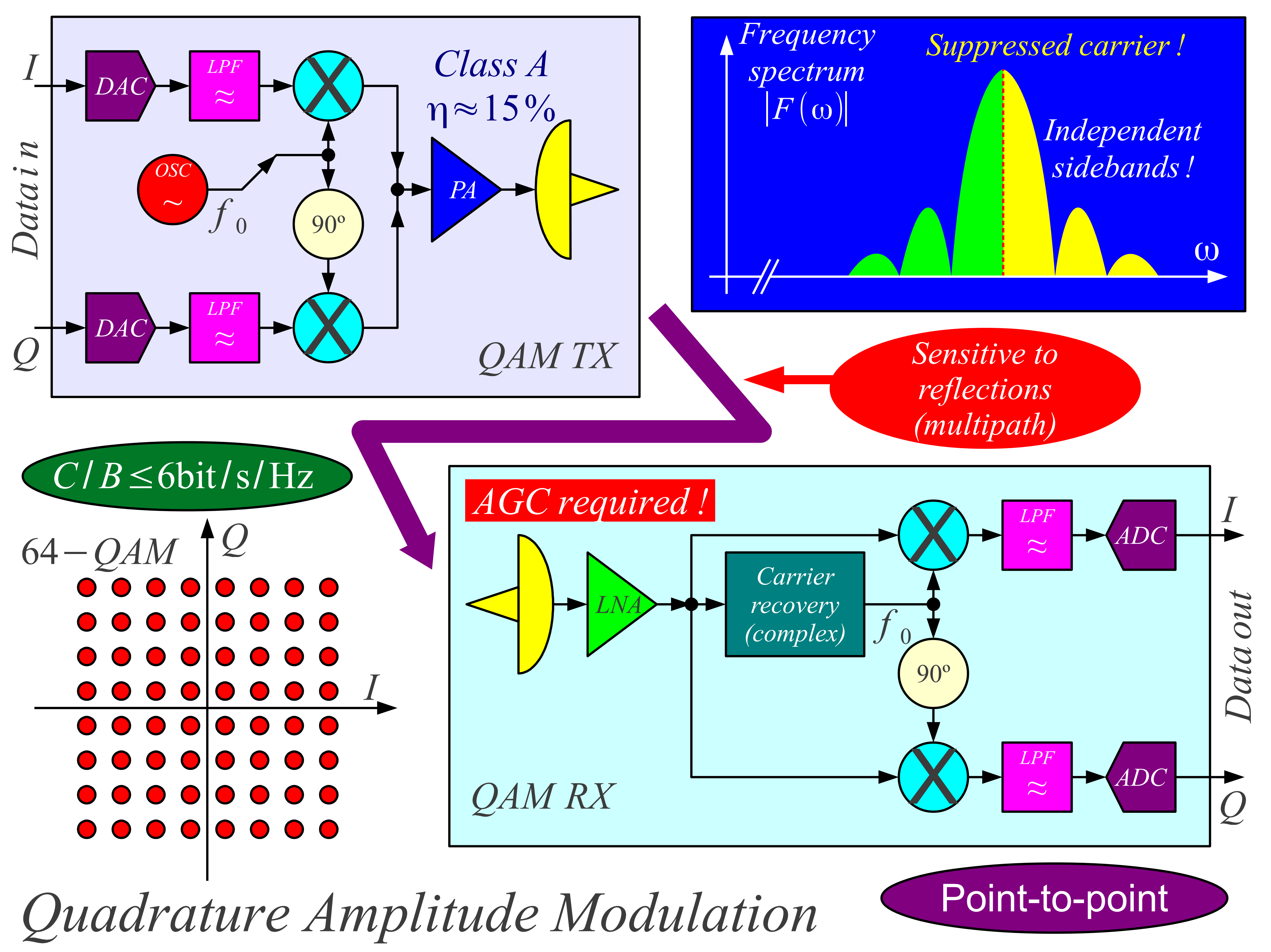
Modulation constellation

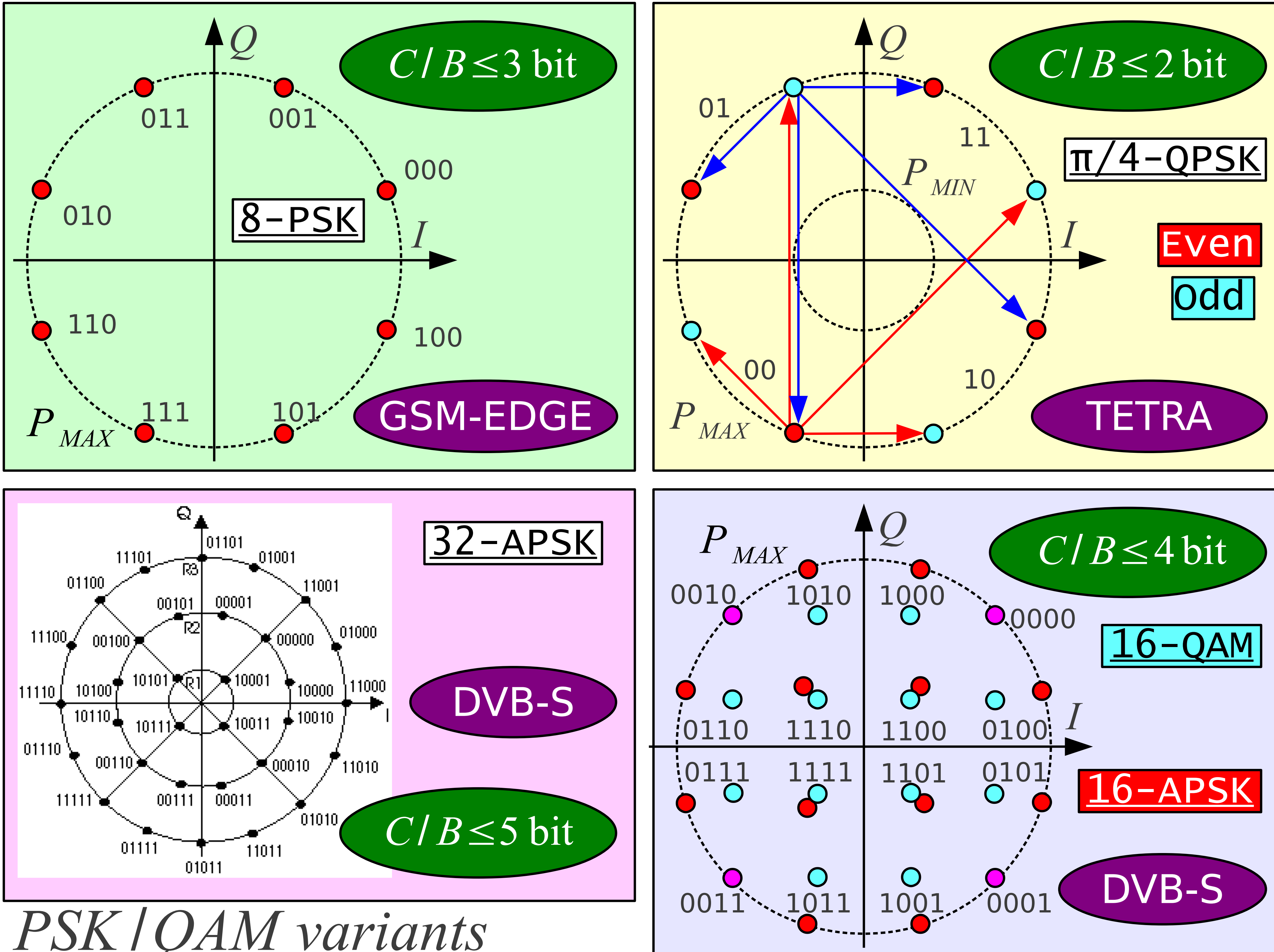


OSC
$f_0$
PA
Class C
η≈70%
LPF
Data in
DRV
BPSK TX
C/B≤1bit/s/Hz
Frequency spectrum |F(ω)|
Suppressed carrier!
Mirror – identical sidebands!
ω
0
Q
1
I
AGC unnecessary!
Data in
0 1 1 0 1 0 0
t
Modulator drive
t
$P_{TX}$
t
BPSK RX
LNA
LPF
LIM
Data out
2 $f_0$
$f_0$
BPF
f÷2
Carrier recovery
SIMPLE!
Δf≤10%C
Bi – Phase Shift Keying

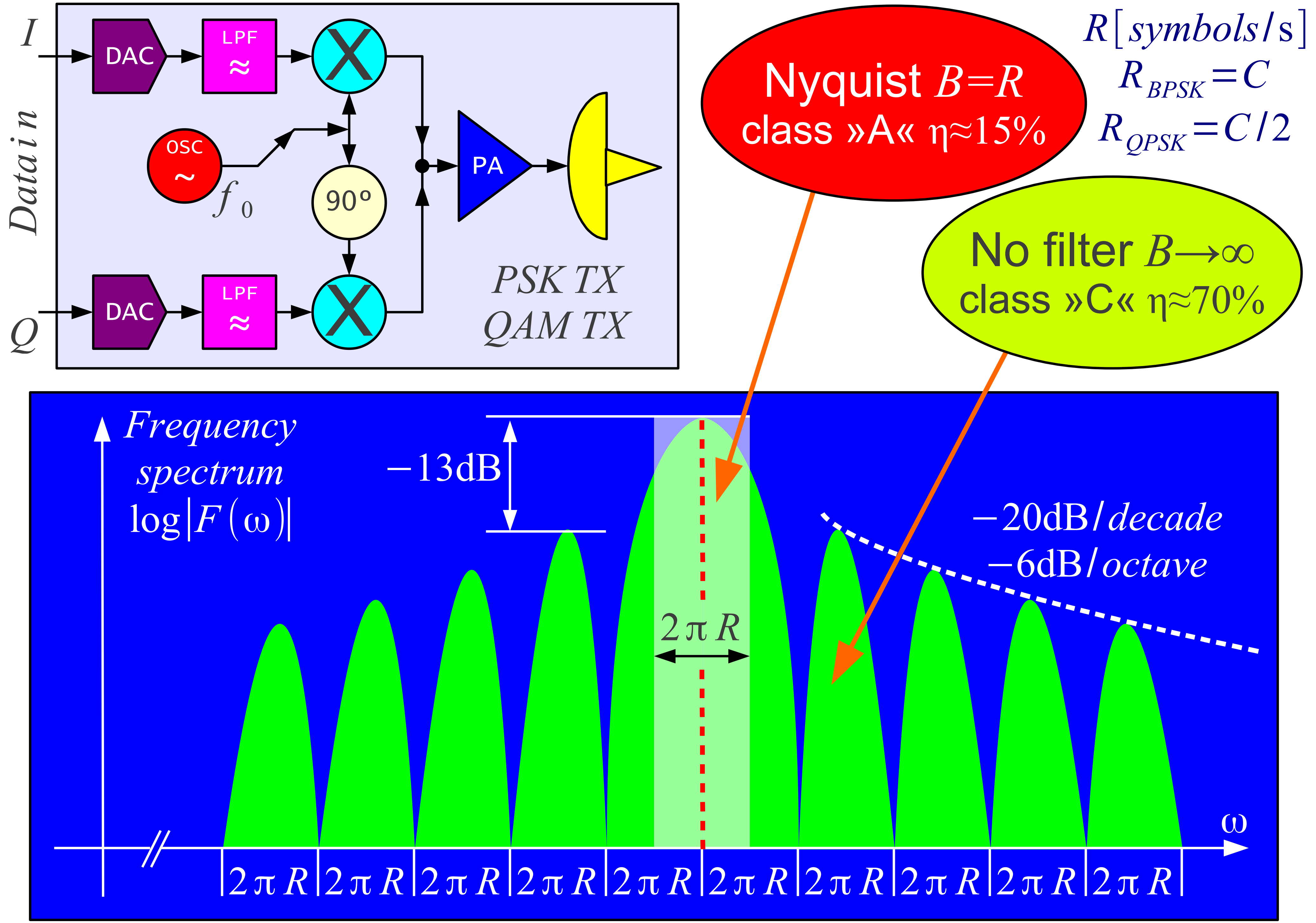


I
Q
Data in
DRV
DRV
LPF ≈
LPF ≈
OSC ~
f 0
90º
Class C
η≈70%
PA
QPSK TX
Frequency spectrum |F(ω)|
Suppressed carrier !
Independent sidebands !
ω
C/B≤2bit/s/Hz
Q
I
01
11
00
10
Space, GSM
LNA
Carrier recovery
f 0
90º
LPF ≈
LPF ≈
LIM
LIM
I
Q
Data out
QPSK RX
Δf≤10%C
Quadri−Phase Shift Keying

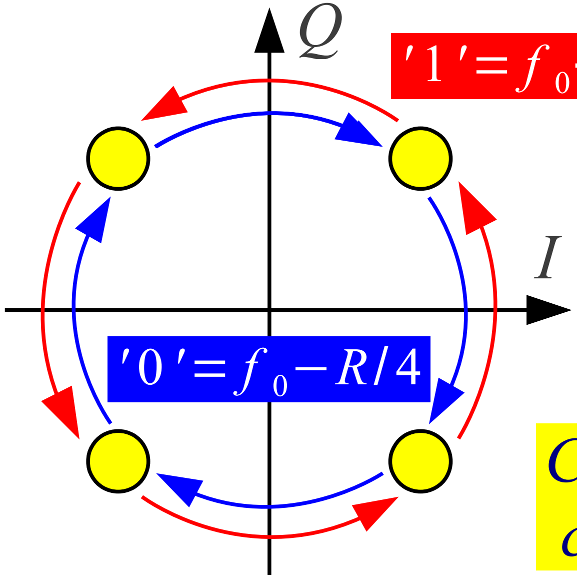

I
Q
Data in
DAC
DAC
LPF ≈
LPF ≈
OSC ~
$f_0$
90º
PA
Class A
η≈15%
QAM TX
Frequency spectrum $|F(\omega)|$
Suppressed carrier !
Independent sidebands !
ω
Sensitive to reflections (multipath)
$C/B \leq 6$bit/s/Hz
64−QAM
Q
I
AGC required !
LNA
Carrier recovery (complex)
$f_0$
90º
LPF ≈
LPF ≈
ADC
ADC
I
Q
Data out
QAM RX
Point-to-point
Quadrature Amplitude Modulation




*PSK/QAM variants*




PSK / QAM spectrum shaping




$R\,[symbols/s]$

$R_{MSK} = C = 2\,R_{QPSK}$

*Constant envelope*

*class C* $\eta \approx 70\%$

*Non-coherent (G)MSK:* $f \approx f_0 \pm R/4$

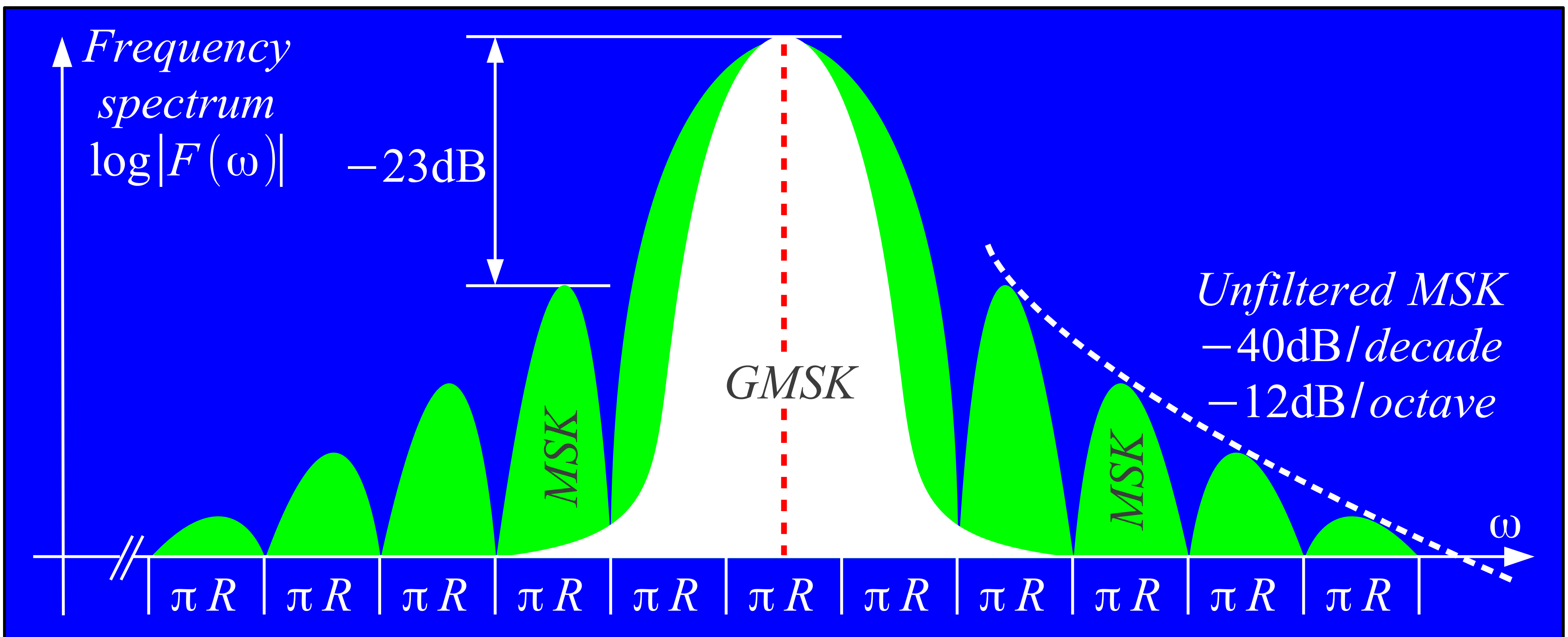
*Directly-modulated VCO*

*Usage: DECT*

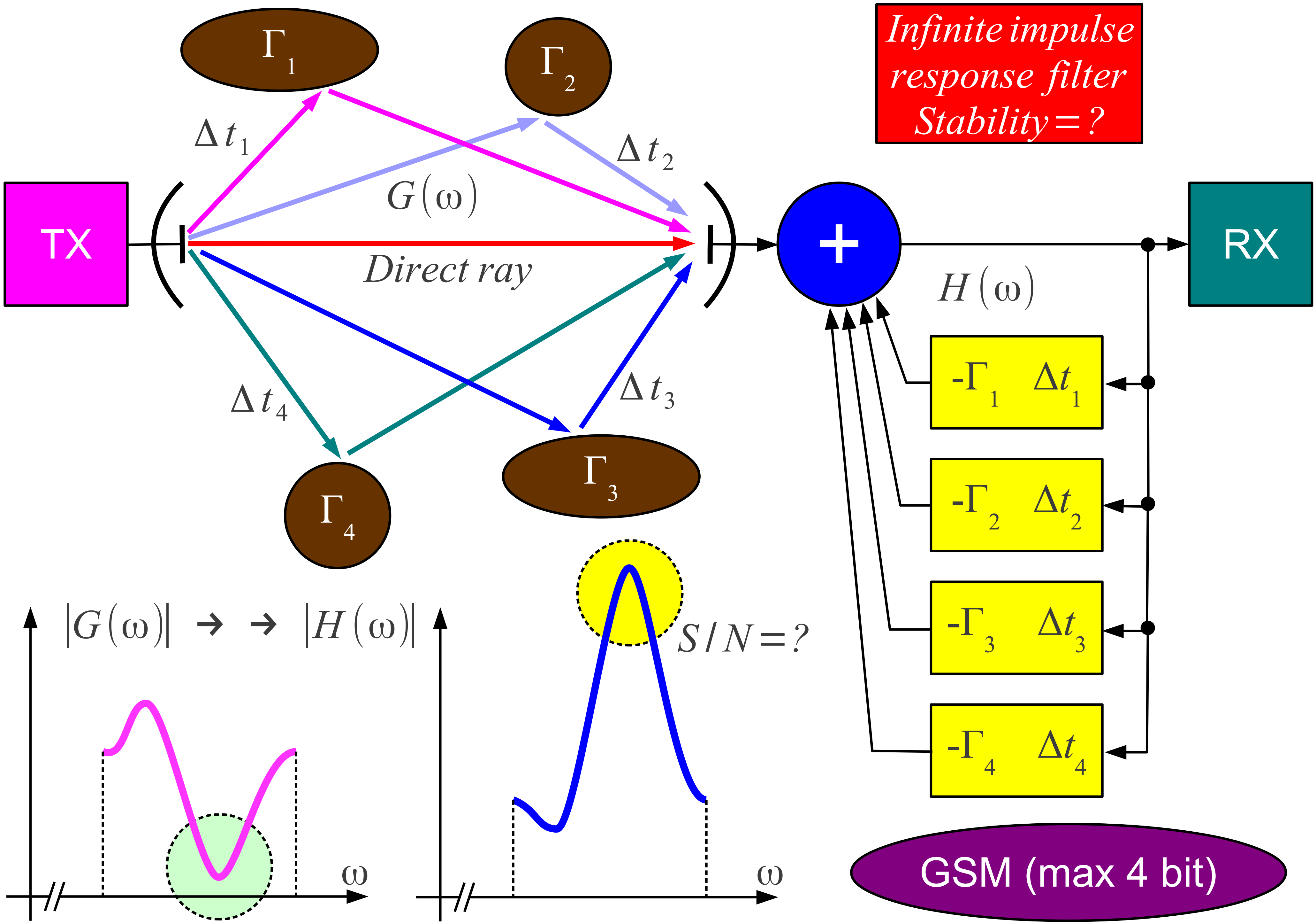
*Coherent (G)MSK:* $f = f_0 \pm R/4$

*Quadrature modulator*

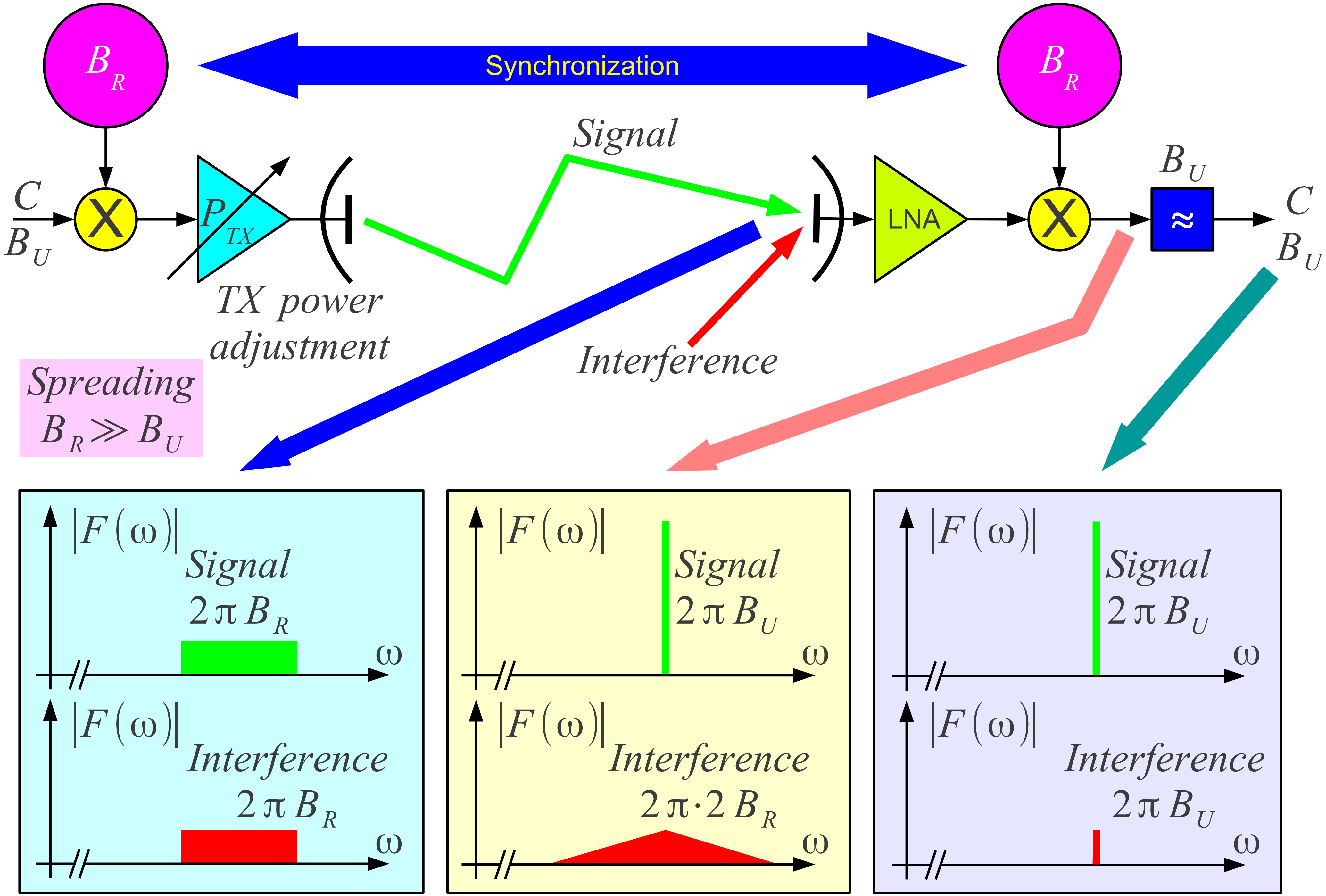
*Usage: GSM*



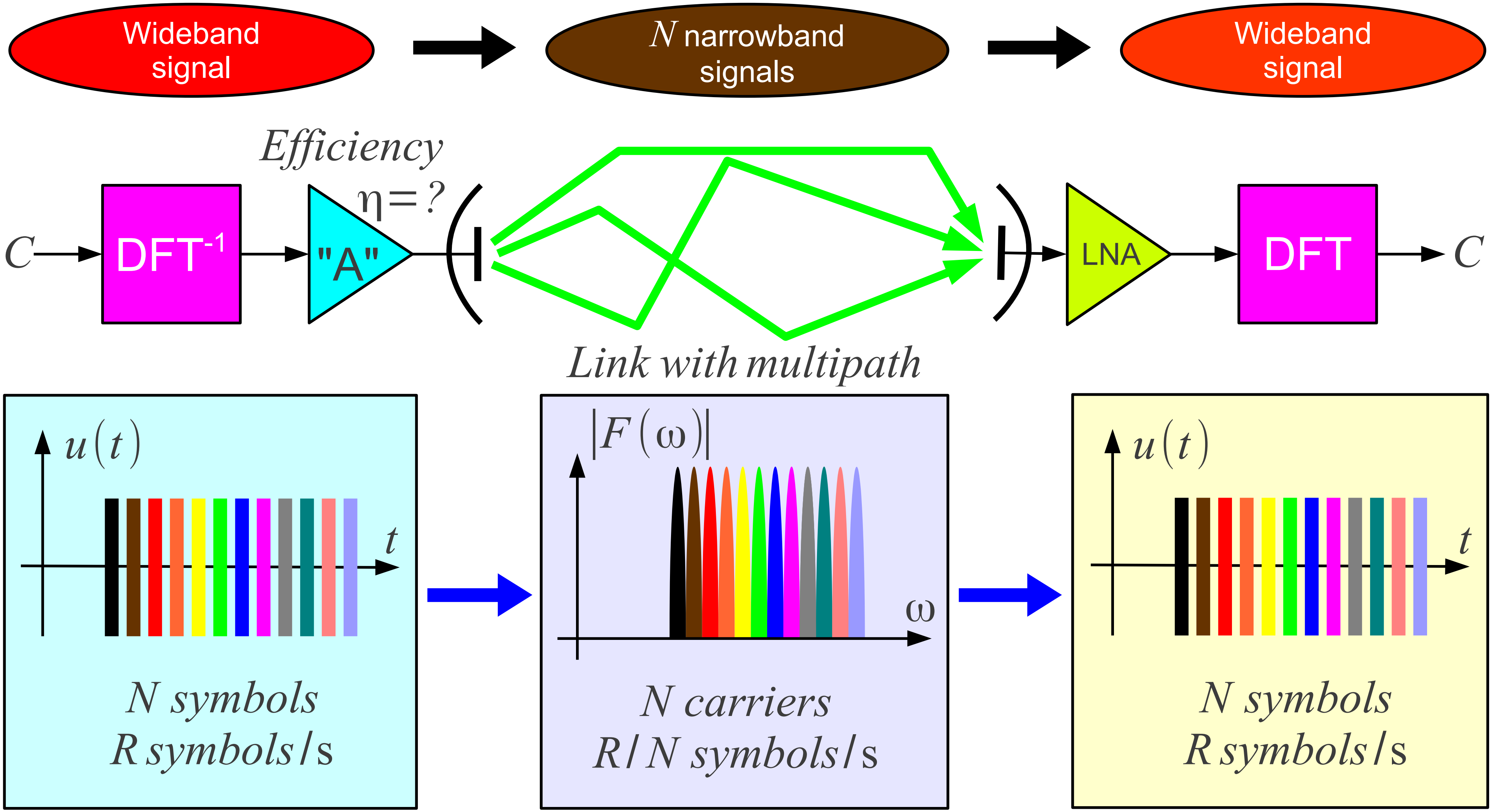

# (Gaussian) Minimum Shift Keying




*Multipath equalizing filter*

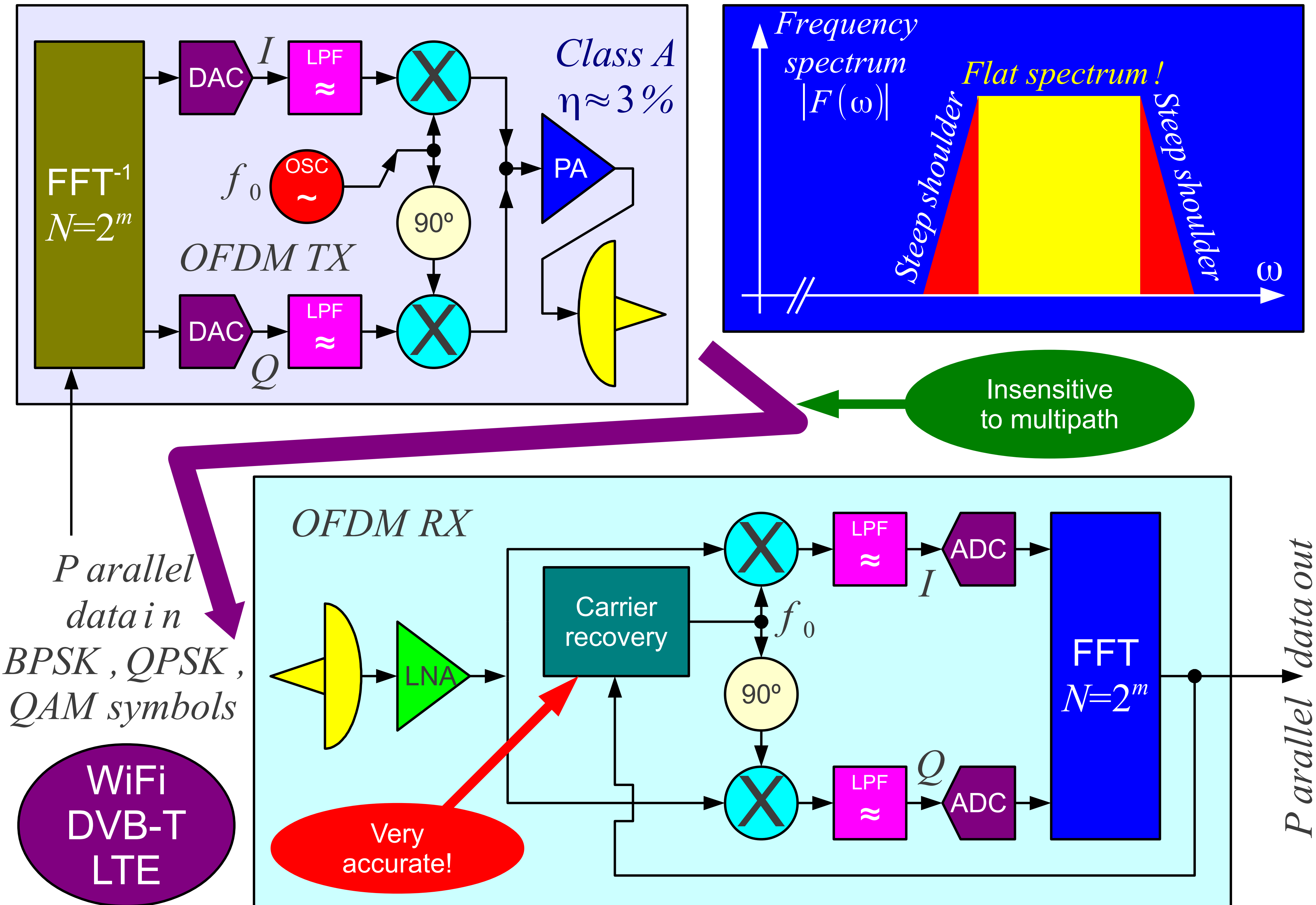



*Spread spectrum* → *CDMA=?*

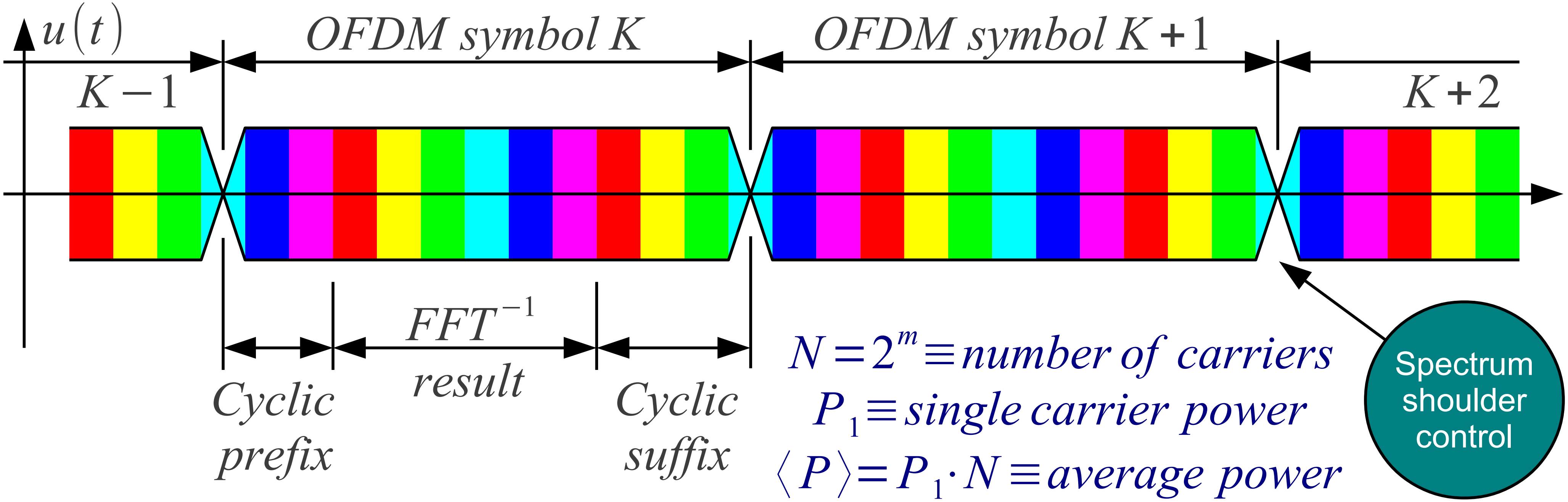



- ~1950 analog multitone modem for ionospheric links
- ~1990 wavelength-division multiplex (WDM) in optical fibers
- ~2000 numerical DFT → OFDM WLAN (WiFi) 802.11a (FFT)

# *Multipath mitigation with multitone transmission*




# Orthogonal Frequency Division Multiplex




$N = 2^m \equiv$ *number of carriers*

$P_1 \equiv$ *single carrier power*

$\langle P \rangle = P_1 \cdot N \equiv$ *average power*

$P_{MAX} = P_1 \cdot N^2 \equiv$ *peak power*

$$t_{prefix} + t_{suffix} \geq \Delta t_{multipath}$$

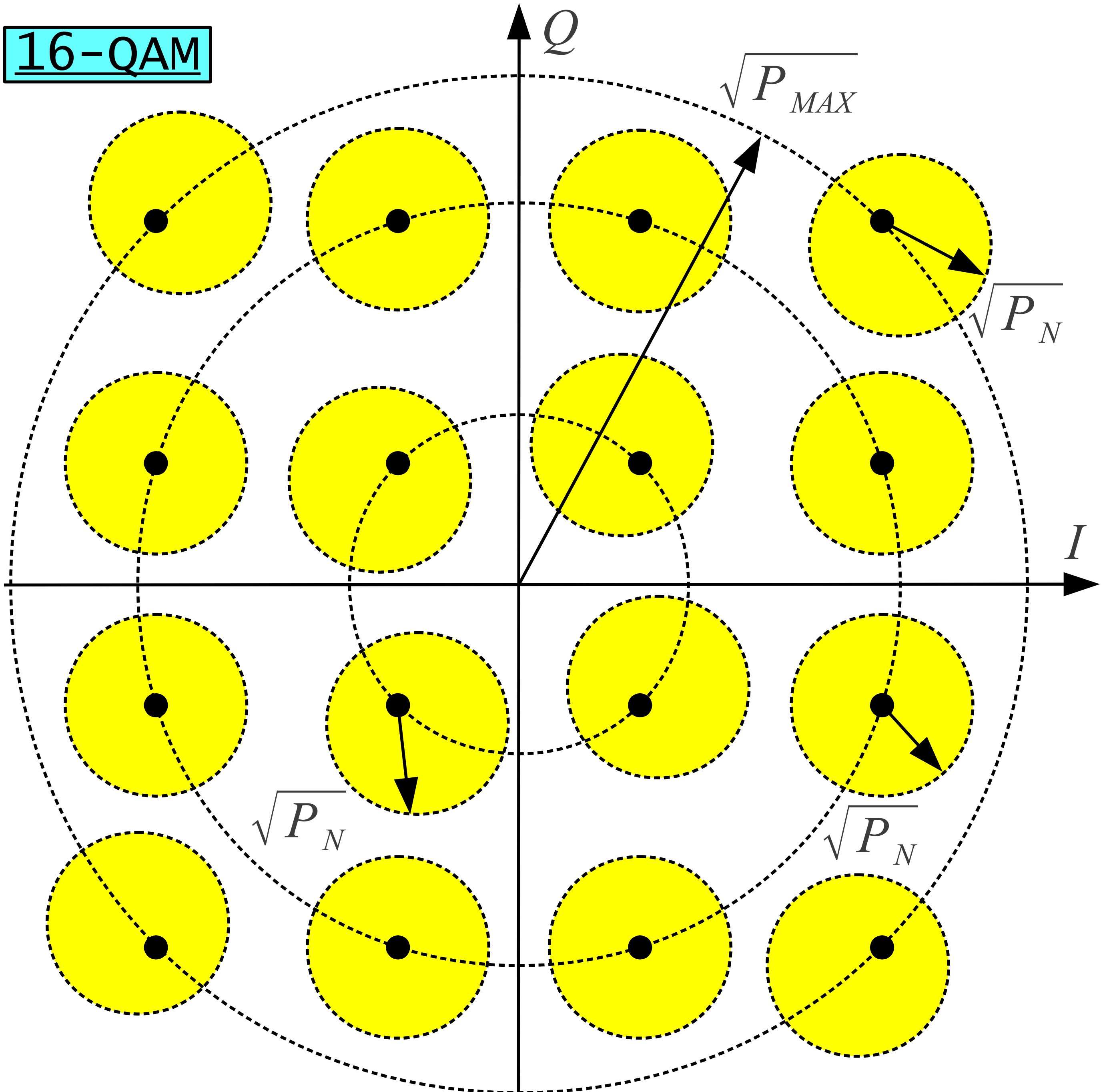
- Adjustable resistance to multipath
- Nearly rectangular frequency spectrum
- Weak FEC sufficient
- Spectral efficiency $C/B$ reaches theory for BPSK, QPSK, QAM
- Allows single-frequency networks (SFN)

- High $PAPR = P_{MAX} / \langle P \rangle = N$ causes poor transmitter efficiency $\eta \approx 3\%$
- FFT requires $N \bullet \log_2 N$ calculations
- Narrowband carriers require high frequency stability $\Delta f \leq 10\%\ R/N$
- Large symbols ~12000 bit ($N \approx 2000$, $C/B \approx 6$ bit) for some protocols
- Narrowband interference kills sync

# *OFDM properties*

*AM / φ M → mutual rotation inside constellation*

*EVM ≡ Error Vector Magnitude*




$$EVM\,[\%] = \sqrt{\frac{\langle P_N \rangle}{P_{MAX}}}$$

or

$$EVM\,[\%] = \sqrt{\frac{\langle P_N \rangle}{\langle P_S \rangle}}$$

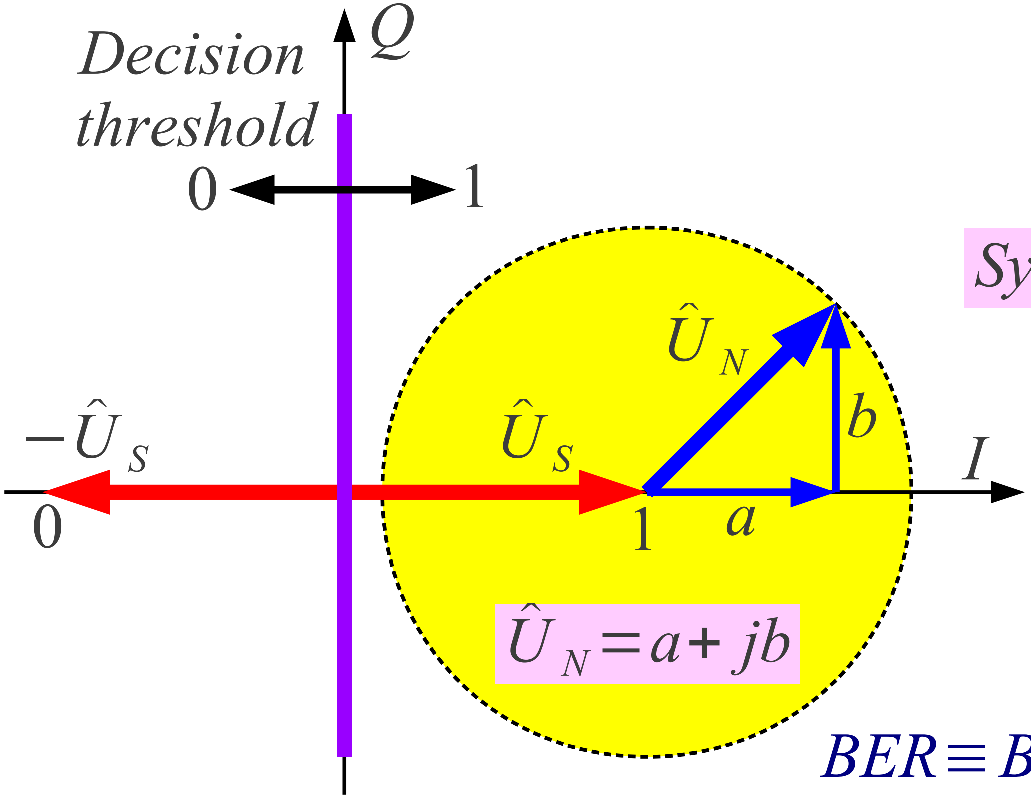
$$EVM_{dB} = 20 \log_{10} EVM$$

*EVM* & *MER* include noise, interference & distortion!

$$MER_{dB} = 10 \log_{10} \left( \frac{\langle P_S \rangle}{\langle P_N \rangle} \right)$$

*MER ≡ Modulation Error Ratio*

# *EVM & MER*




$$\hat{U}_N = a + jb$$

$$P_{1\to 0} = \int_{-\infty}^{-|\hat{U}_S|} p(a)\,da$$

$$P_{0\to 1} = \int_{|\hat{U}_S|}^{\infty} p(a)\,da$$

*Symmetric threshold* : $P_{1\to 0} = P_{0\to 1} = BER$

$$BER = \int_{|\hat{U}_S|}^{\infty} \frac{1}{\sqrt{\pi\langle|\hat{U}_N|^2\rangle}}\, e^{-\frac{a^2}{\langle|\hat{U}_N|^2\rangle}}\, da$$

*BER ≡ Bit − Error Rate*

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2}\, du$$

*Gaussian distribution of the probability density of inphase a & quadrature jb noise components*

$$p(a) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{a^2}{2\sigma^2}}$$

$$p(b) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{b^2}{2\sigma^2}}$$

$$BER = \frac{1}{2}\operatorname{erfc}\left(\frac{|\hat{U}_S|}{\sqrt{\langle|\hat{U}_N|^2\rangle}}\right)$$

$$P_S = \alpha|\hat{U}_S|^2 \qquad P_N = \alpha\langle|\hat{U}_N|^2\rangle$$

$$\langle|\hat{U}_N|^2\rangle = \langle a^2\rangle + \langle b^2\rangle = 2\sigma^2$$

$$BER = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{P_S}{P_N}}$$

# *BER calculation for BPSK*

$$BPSK: \quad BER = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{P_S}{P_N}} = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{W_{bit}}{N_0}}$$

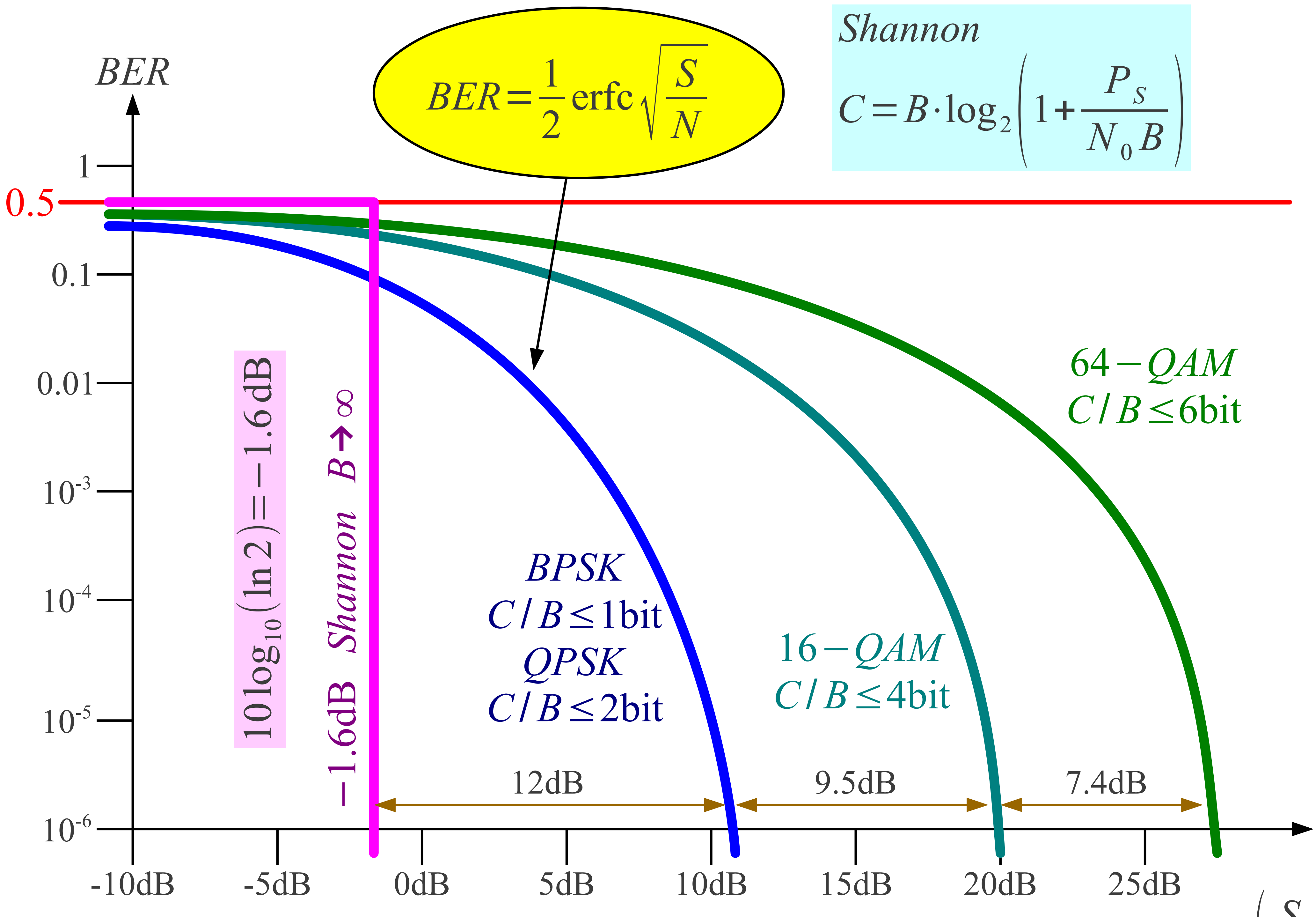
| S/N | BER |
|---|---|
| −5dB | 23.6% |
| −4dB | 18.6% |
| −3dB | 15.9% |
| −2dB | 13.1% |
| −1dB | 10.4% |
| 0dB | 7.9% |
| 1dB | 5.7% |
| 2dB | 3.8% |
| 3dB | 2.3% |
| 4dB | 1.3% |
| 5dB | 0.6% |
| 6dB | 0.24% |
| 7dB | $7.7\cdot10^{-4}$ |
| S/N | BER |

| S/N | BER |
|---|---|
| 8dB | $1.9\cdot10^{-4}$ |
| 9dB | $3.4\cdot10^{-5}$ |
| 10dB | $3.9\cdot10^{-6}$ |
| 11dB | $2.6\cdot10^{-7}$ |
| 12dB | $9\cdot10^{-9}$ |
| 13dB | $1.3\cdot10^{-10}$ |
| 14dB | $6.8\cdot10^{-13}$ |
| 15dB | $9.2\cdot10^{-16}$ |
| 16dB | $2.3\cdot10^{-19}$ |
| 17dB | $6.8\cdot10^{-24}$ |
| 18dB | $1.4\cdot10^{-29}$ |
| 19dB | $10^{-36}$ |
| 20dB | $10^{-45}$ |
| S/N | BER |

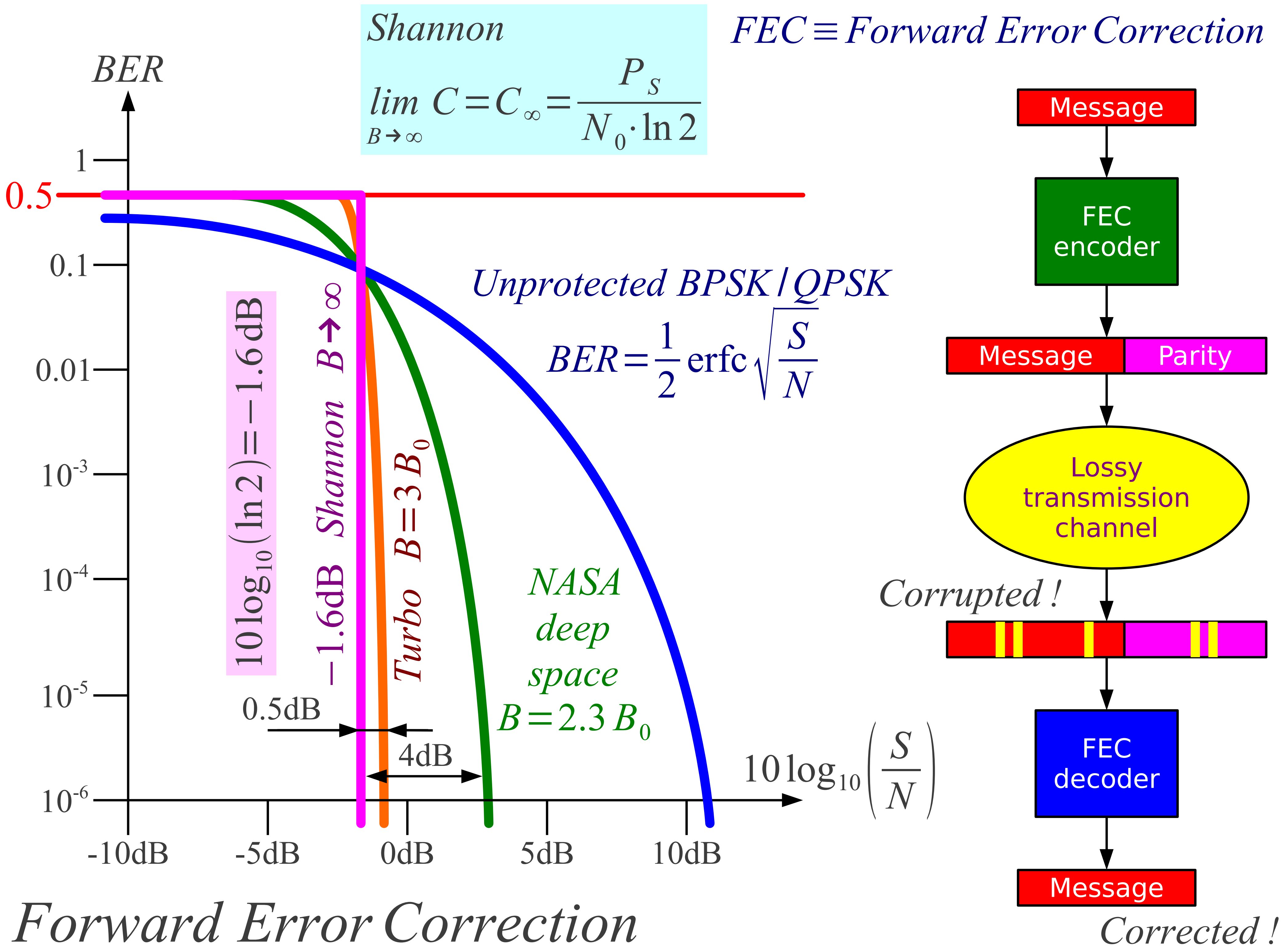
| BER | S/N |
|---|---|
| 30% | −8.6dB |
| 10% | −0.8dB |
| 3% | 2.5dB |
| 1% | 4.3dB |
| 0.3% | 5.8dB |
| $10^{-3}$ | 6.8dB |
| $3\cdot10^{-4}$ | 7.7dB |
| $10^{-4}$ | 8.4dB |
| $3\cdot10^{-5}$ | 9.1dB |
| $10^{-5}$ | 9.6dB |
| $3\cdot10^{-6}$ | 10.1dB |
| $10^{-6}$ | 10.4dB |
| $3\cdot10^{-7}$ | 11dB |
| BER | S/N |

| BER | S/N |
|---|---|
| $10^{-7}$ | 11.3dB |
| $3\cdot10^{-8}$ | 11.7dB |
| $10^{-8}$ | 12dB |
| $3\cdot10^{-9}$ | 12.3dB |
| $10^{-9}$ | 12.6dB |
| $10^{-10}$ | 13.1dB |
| $10^{-11}$ | 13.5dB |
| $10^{-12}$ | 13.9dB |
| $10^{-13}$ | 14.3dB |
| $10^{-14}$ | 14.7dB |
| $10^{-15}$ | 15dB |
| $10^{-16}$ | 15.3dB |
| $10^{-17}$ | 15.6dB |
| BER | S/N |

*BER table for BPSK*

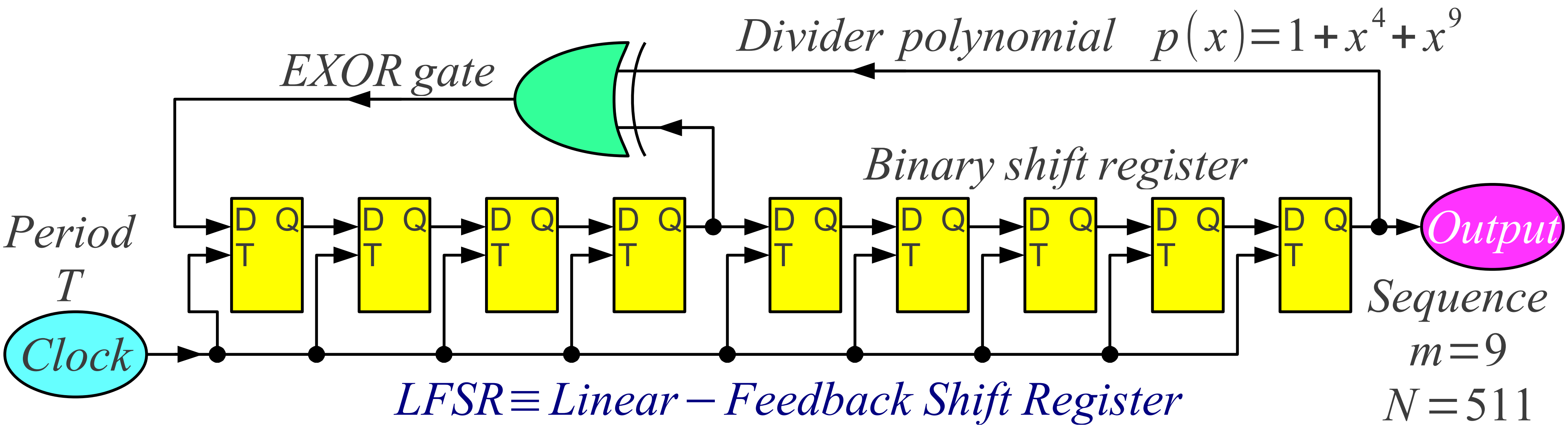



*BER plots for different modulations*



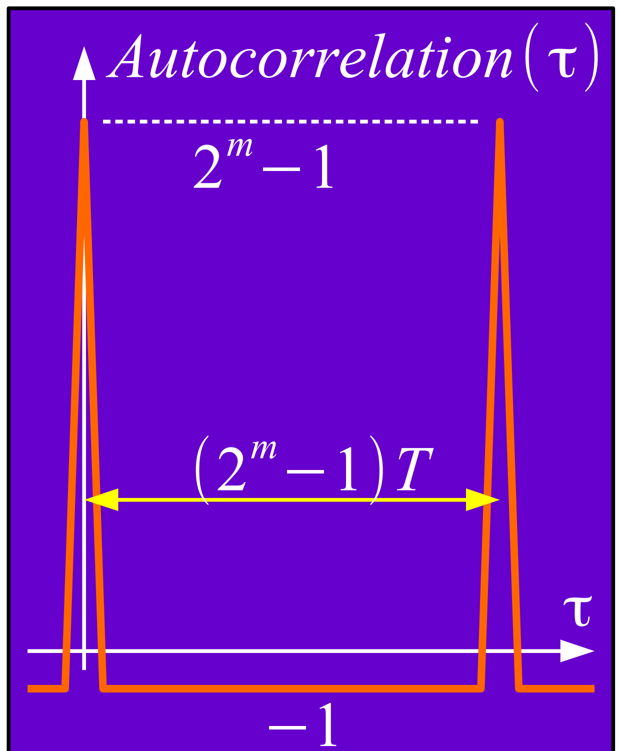

# Forward Error Correction

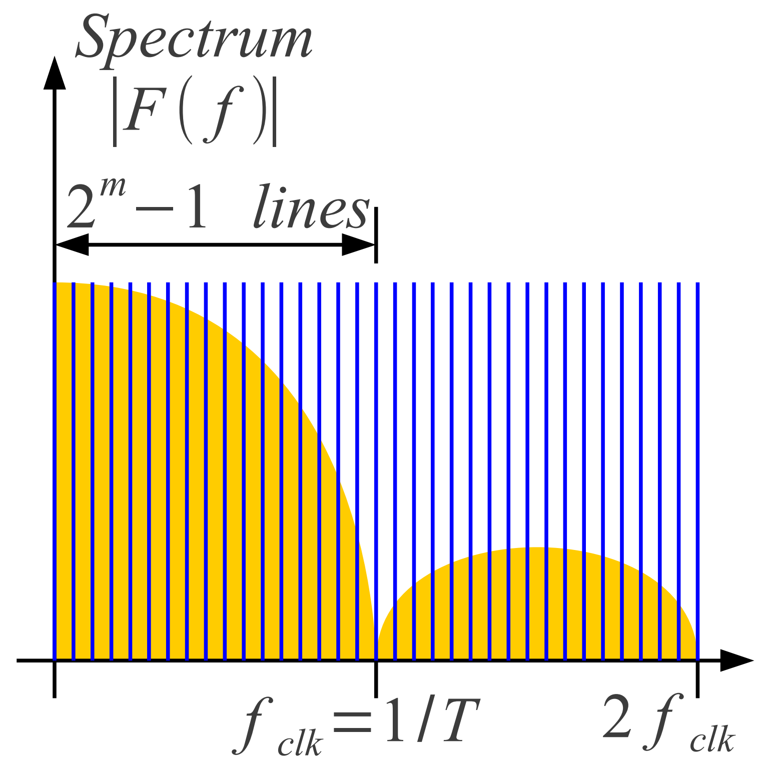
# LFSR pseudorandom sequences



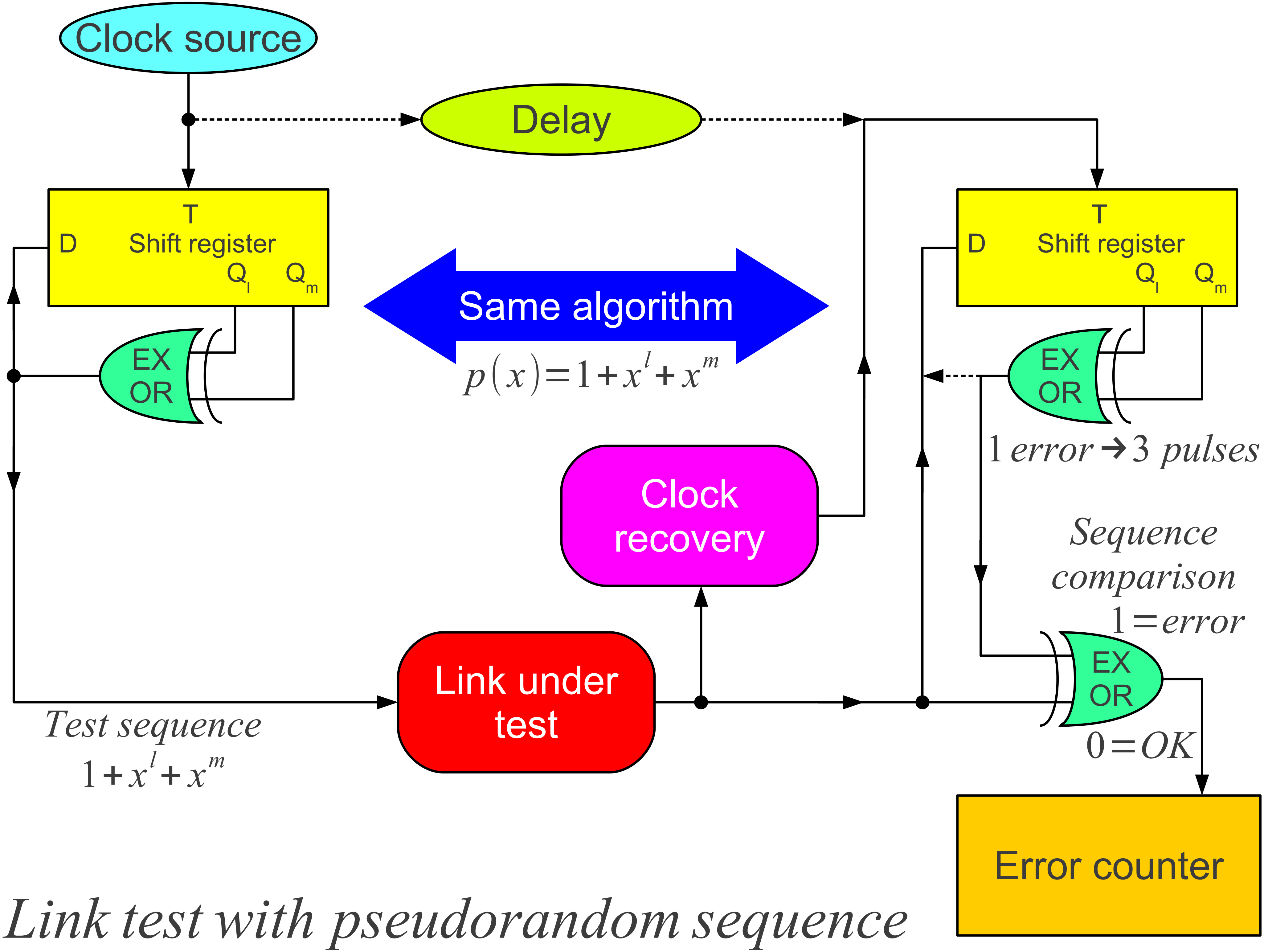

*Primitive polynomial* $p(x) = 1 + x^l + x^m \rightarrow$ *max-length sequence* $N = 2^m - 1$

$2^{m-1}$ *ones* & $2^{m-1}$-1 *zeros*
*arranged in groups of*
1X m *ones*, m-1 *zeros*
1X m-2 *ones & zeros*
2X m-3 *ones & zeros*
4X m-4 *ones & zeros*
.................
$2^{m-5}$ *triplets* 111 & 000
$2^{m-4}$ *pairs* 11 & 00
$2^{m-3}$ *individual* 1 & 0

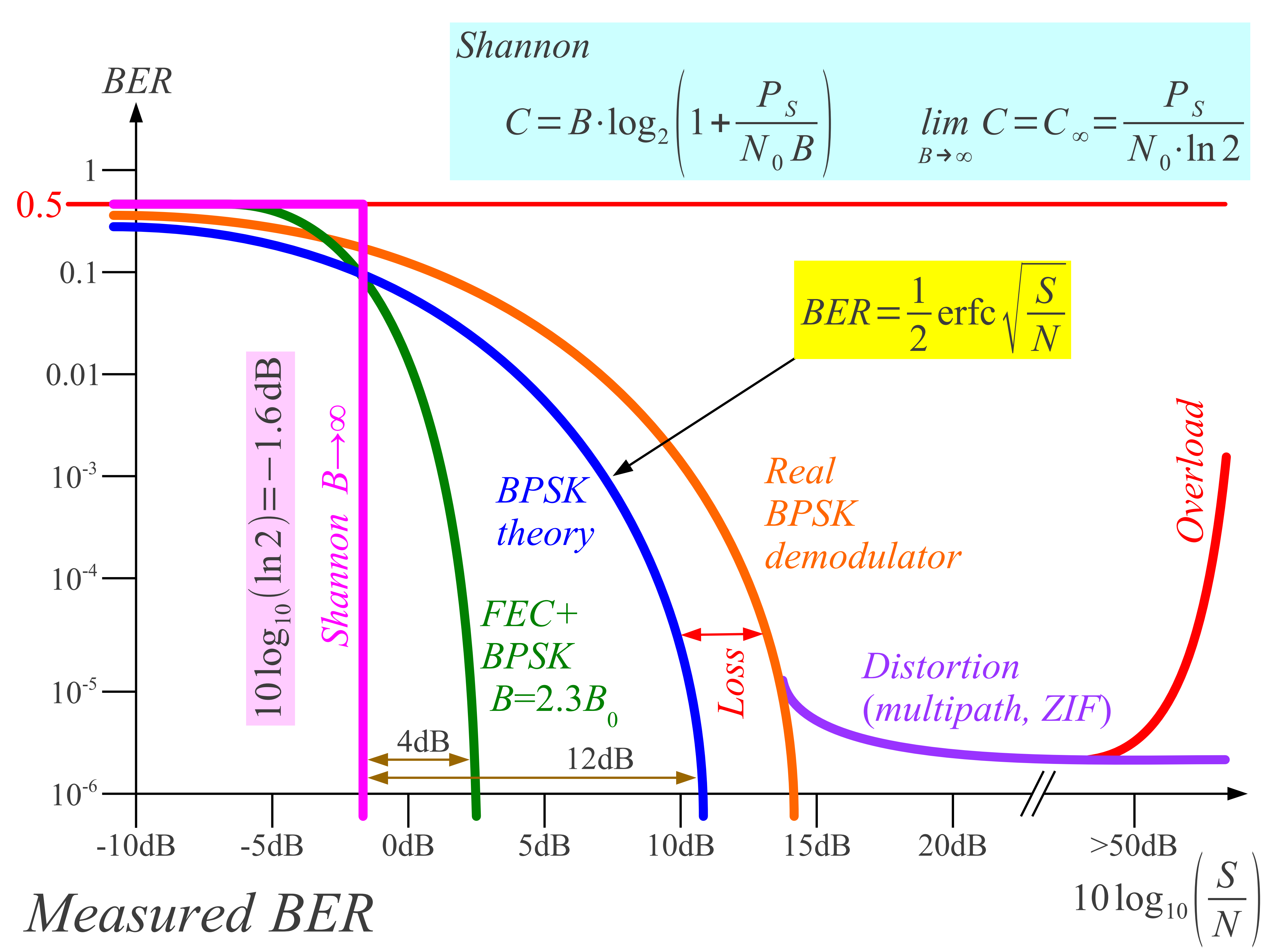



*Sounds as white noise!*




*Link test with pseudorandom sequence*



Shannon
$C = B \cdot \log_2\left(1 + \frac{P_S}{N_0 B}\right)$
$\lim_{B \to \infty} C = C_\infty = \frac{P_S}{N_0 \cdot \ln 2}$
BER
1
0.5
0.1
0.01
$10^{-3}$
$10^{-4}$
$10^{-5}$
$10^{-6}$
$BER = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{S}{N}}$
$10 \log_{10}(\ln 2) = -1.6\,\text{dB}$
Shannon $B \to \infty$
BPSK theory
FEC+ BPSK $B=2.3B_0$
Real BPSK demodulator
Loss
Distortion (multipath, ZIF)
Overload
4dB
12dB
-10dB
-5dB
0dB
5dB
10dB
15dB
20dB
>50dB
$10 \log_{10}\left(\frac{S}{N}\right)$
Measured BER