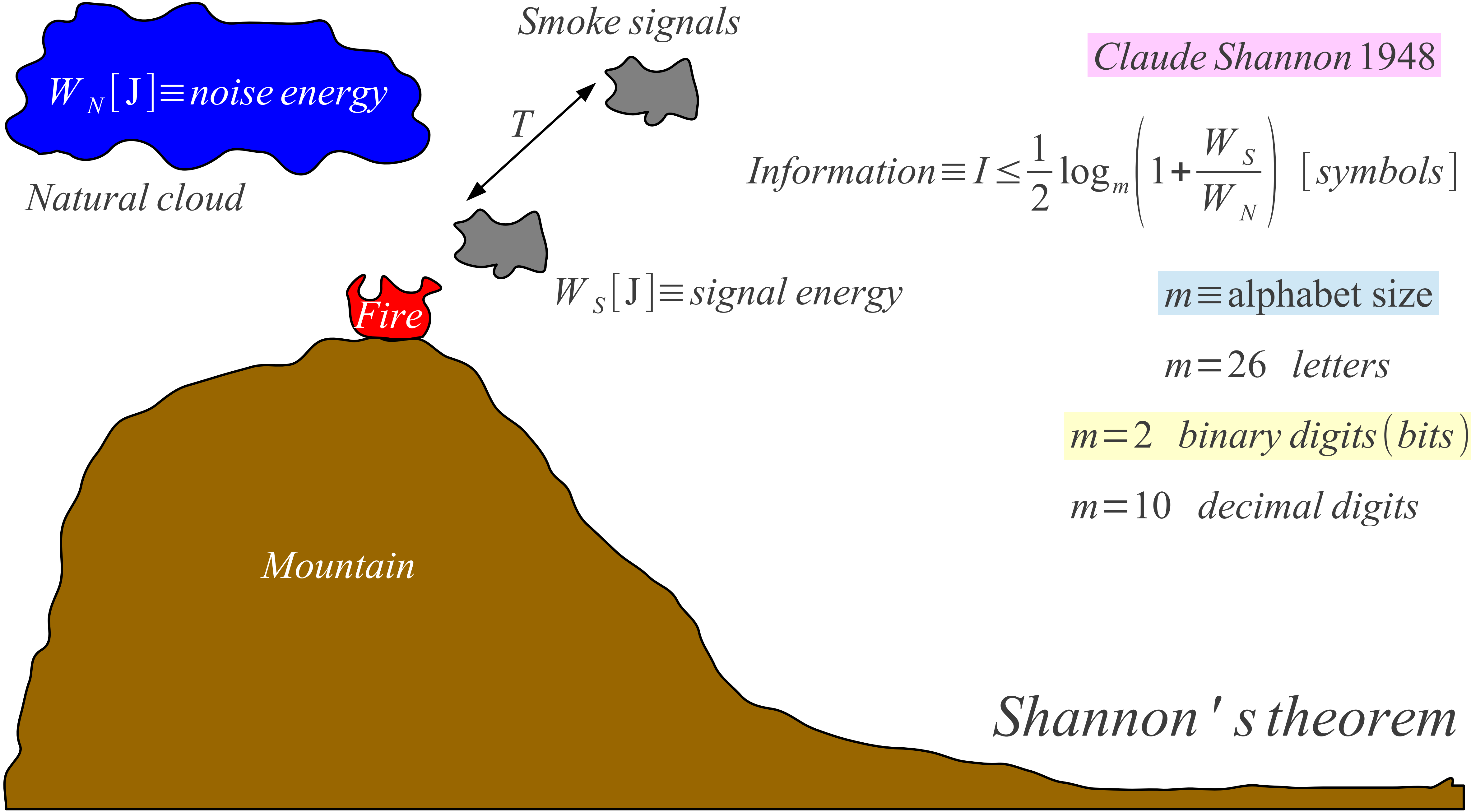
Communication Electronics

Lecture 1:

Range and capacity of wired and wireless links

Shannon's theorem



Claude Shannon 1948

$$Information \equiv I \le \frac{1}{2}\log_m\left(1 + \frac{W_S}{W_N}\right) \quad [symbols]$$

$m \equiv$ alphabet size

$m = 26$ *letters*

$m = 2$ *binary digits (bits)*

$m = 10$ *decimal digits*

$$Capacity \equiv C = \frac{dI}{dt} = \frac{1}{2T}\log_m\left(1+\frac{W_S}{W_N}\right) \quad [symbols/\mathrm{s}]$$

$$Signal\ period \equiv T\,[\mathrm{s}]$$

$$Bandwidth \equiv B = \frac{1}{2T} \quad [\mathrm{Hz}]$$

$$C = B\cdot\log_m\left(1+\frac{W_S}{W_N}\right) \quad [symbols/\mathrm{s}]$$

$$Power \equiv P = \frac{dW}{dt} \quad [\mathrm{W}]$$

$$C = B\cdot\log_m\left(1+\frac{P_S}{P_N}\right) \quad [symbols/\mathrm{s}]$$

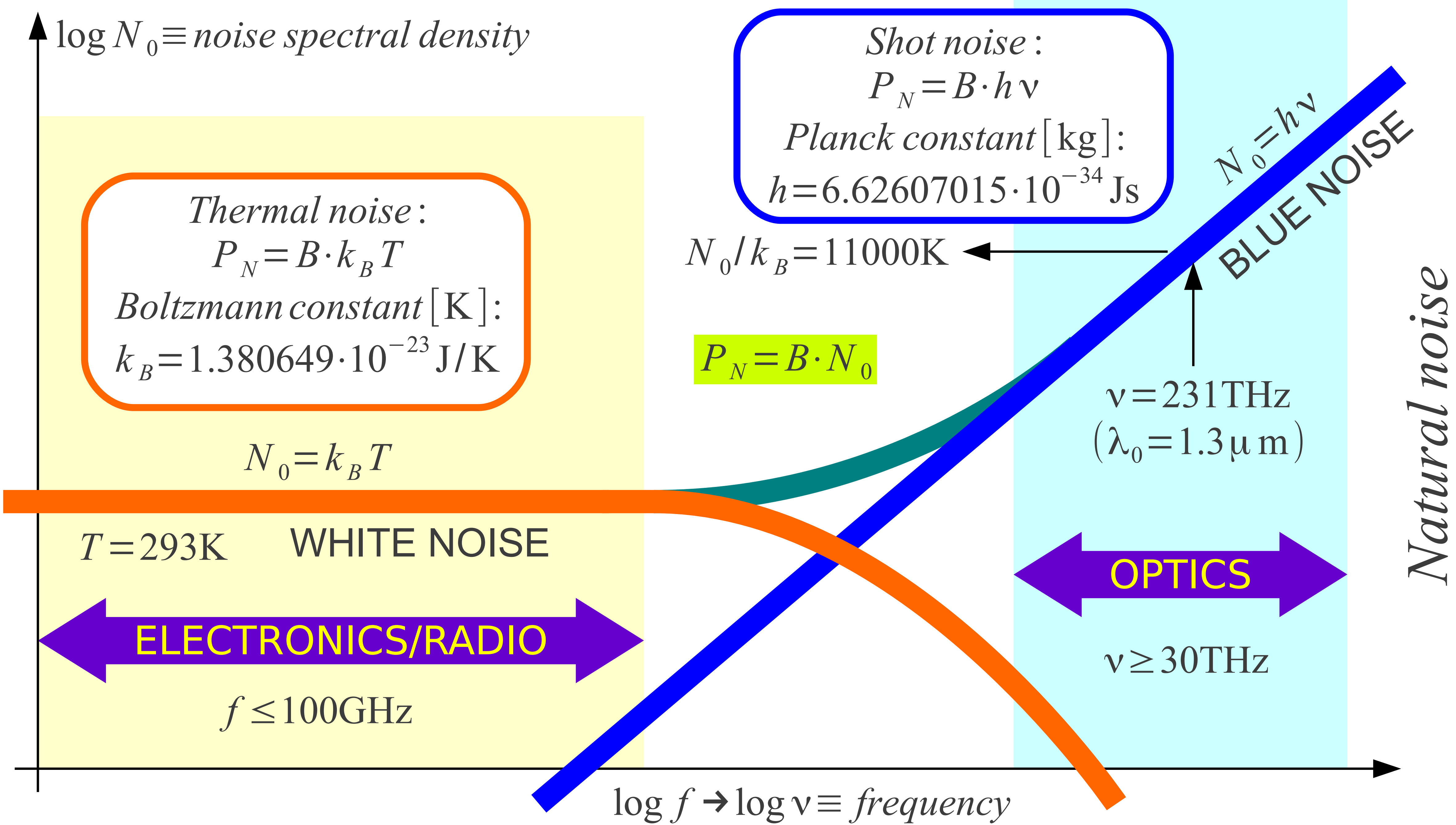
$$C = B\cdot\log_2\left(1+\frac{P_S}{P_N}\right) \quad [\mathrm{bit/s}]$$

$$Noise\ power \equiv P_N = B\cdot N_0$$

$$Noise\ spectral\ density \equiv N_0 \quad \left[\frac{\mathrm{W}}{\mathrm{Hz}} = \mathrm{J}\right]$$

$$C = B\cdot\log_2\left(1+\frac{P_S}{B\cdot N_0}\right) \quad [\mathrm{bit/s}]$$

Link capacity

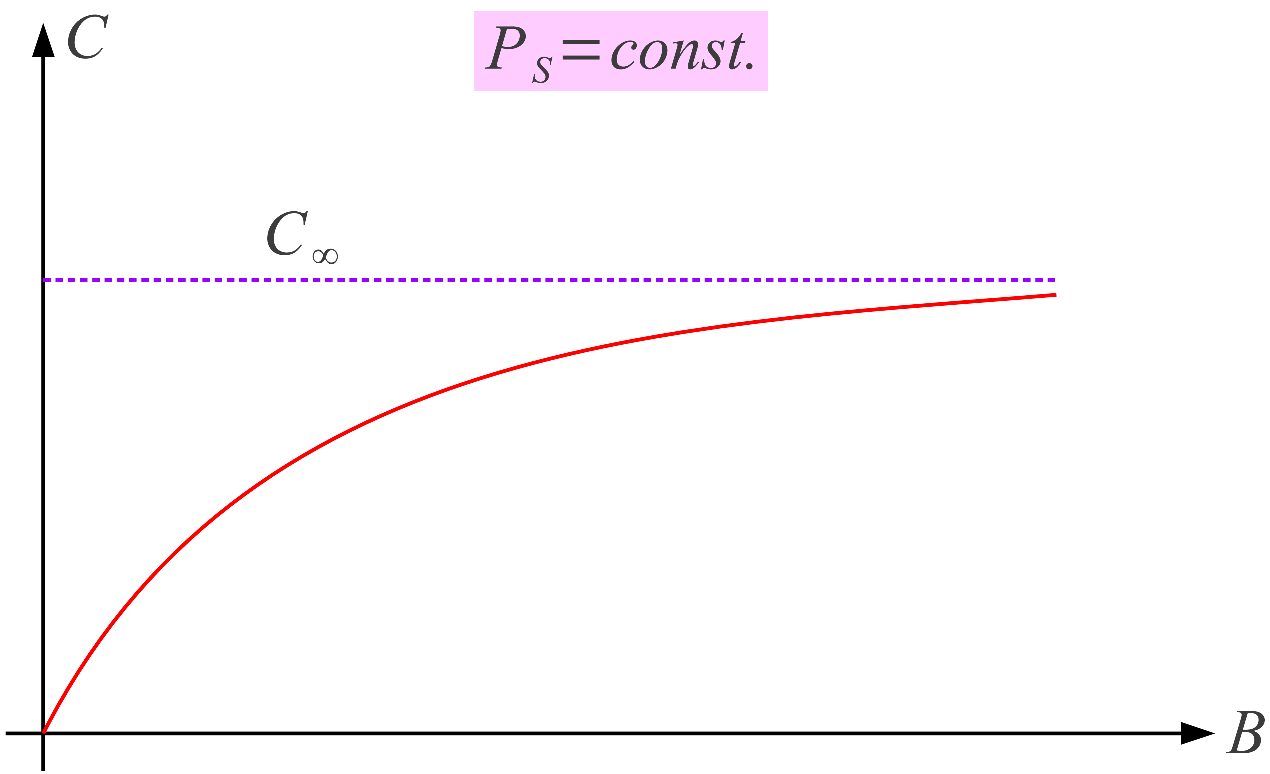


Natural noise

Thermal noise:
$P_N = B \cdot k_B T$
Boltzmann constant $[\mathrm{K}]$:
$k_B = 1.380649 \cdot 10^{-23}\ \mathrm{J/K}$

Shot noise:
$P_N = B \cdot h\nu$
Planck constant $[\mathrm{kg}]$:
$h = 6.62607015 \cdot 10^{-34}\ \mathrm{Js}$

$P_N = B \cdot N_0$



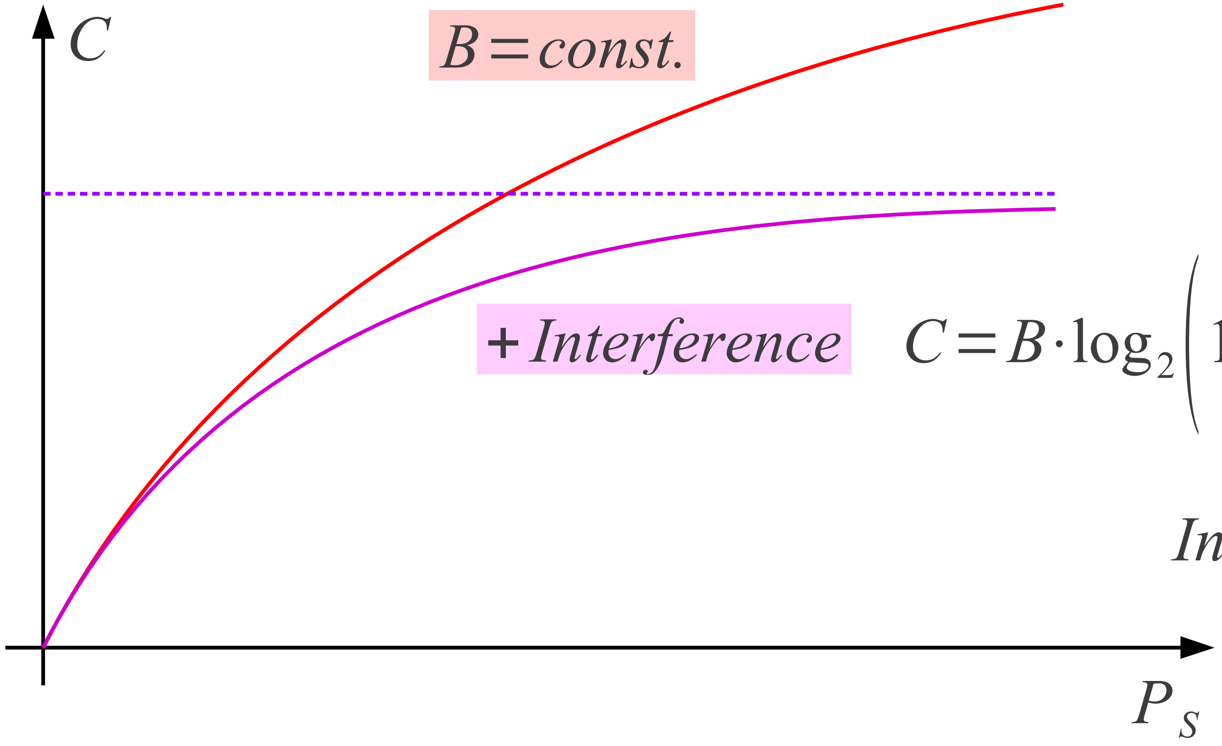
$$C = B \cdot \log_2\left(1 + \frac{P_S}{B \cdot N_0}\right) \quad [\mathrm{bit/s}]$$

$$\boxed{C \le \lim_{B \to \infty} C = C_\infty = \frac{P_S}{N_0 \ln 2}}$$

Strict theoretical limit !

Power – limited link

$$C = B \cdot \log_2\left(1 + \frac{P_S}{B \cdot N_0}\right) \quad [\mathrm{bit/s}]$$



$$\mathit{Spectral\ efficiency} \equiv \frac{C}{B} \quad \left[\frac{\mathrm{bit/s}}{\mathrm{Hz}} = \mathrm{bit}\right]$$

$$C = B \cdot \log_2\left(1 + \frac{P_S}{B \cdot N_0 + P_I}\right)$$

$$\mathit{Interference} \equiv P_I = P_{multipath} + P_{nonlinear} + P_{crosstalk}$$

Practical interference limit :

$$\frac{C}{B} \leq 10\ \mathrm{bit}$$

Bandwidth − limited link

$$\textit{Speech frequency band}\quad 300\text{Hz}\ldots 3400\text{Hz} \;\rightarrow\; B = 3100\text{Hz}$$

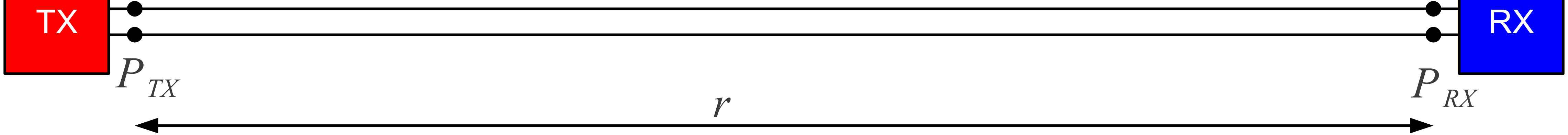
$$\textit{Guaranteed signal/noise ratio}\quad S/N \approx 35\text{dB} = 10^{\frac{35}{10}} \approx 3162$$

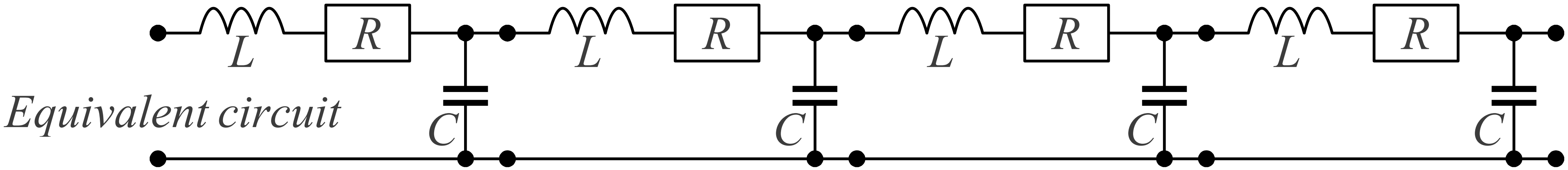
$$C = B \cdot \log_2\left(1 + \frac{P_S}{P_N}\right) = B \cdot \frac{\ln\left(1 + \frac{P_S}{P_N}\right)}{\ln 2} \approx 3100\text{Hz} \cdot \frac{\ln(1 + 3162)}{\ln 2} \approx 3100\text{Hz} \cdot \frac{8.059}{0.693} \approx 36\text{kbit/s}$$

How to make a 56kbit/s *modem?*

Example: analog-telephone modem

Coaxial − cable link





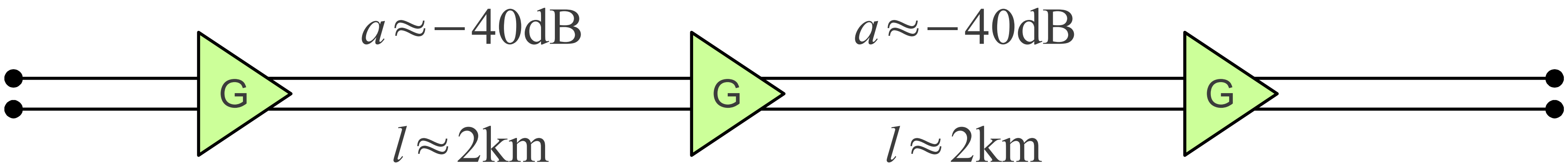
$$\text{Characteristic impedance} \equiv Z_K \approx \sqrt{\frac{L/l}{C/l}} \quad [\Omega]$$

$$\text{Attenuation coefficient} \equiv \alpha \approx \frac{R/l}{2Z_K} \quad \left[\frac{\mathrm{Np}}{\mathrm{m}}\right]$$

$$U_{RX} = U_{TX} \cdot e^{-\alpha \cdot r} \qquad P_{RX} = P_{TX} \cdot e^{-2\alpha \cdot r}$$

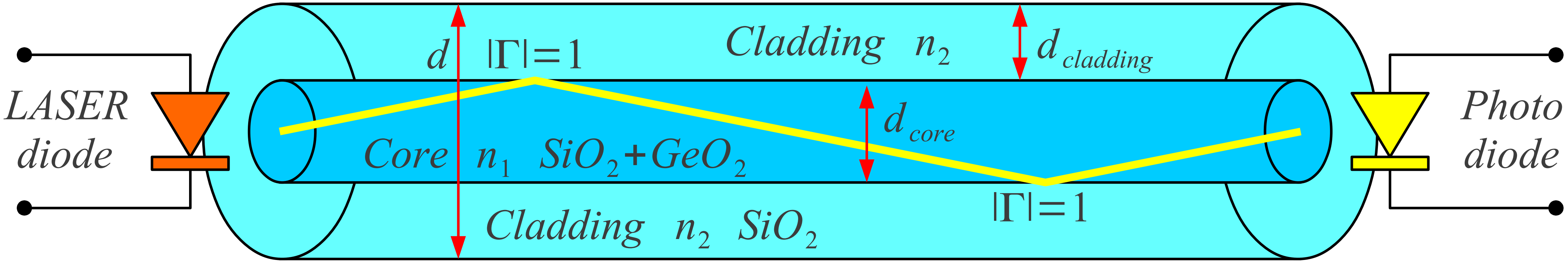
$$\text{Attenuation/length} \equiv a/l = \frac{-20}{\ln 10} \cdot \alpha \quad \left[\frac{\mathrm{dB}}{\mathrm{m}}\right]$$

Coaxial cable attenuation $a/l \approx -20\,\mathrm{dB/km}\ @\ B \approx 100\,\mathrm{MHz}$



Optical − fiber link

$d_{cladding} > 30\,\mu m \gg \lambda$



$d_{core} \approx 3\,\mu m \dots 60\,\mu m$

Total reflection → $n_1 > n_2$

1960 $a/l \approx -1000\,dB/km$ → useless for long − distance communications

1970 $a/l \approx -17\,dB/km$ Corning → competititve to coaxial cable !

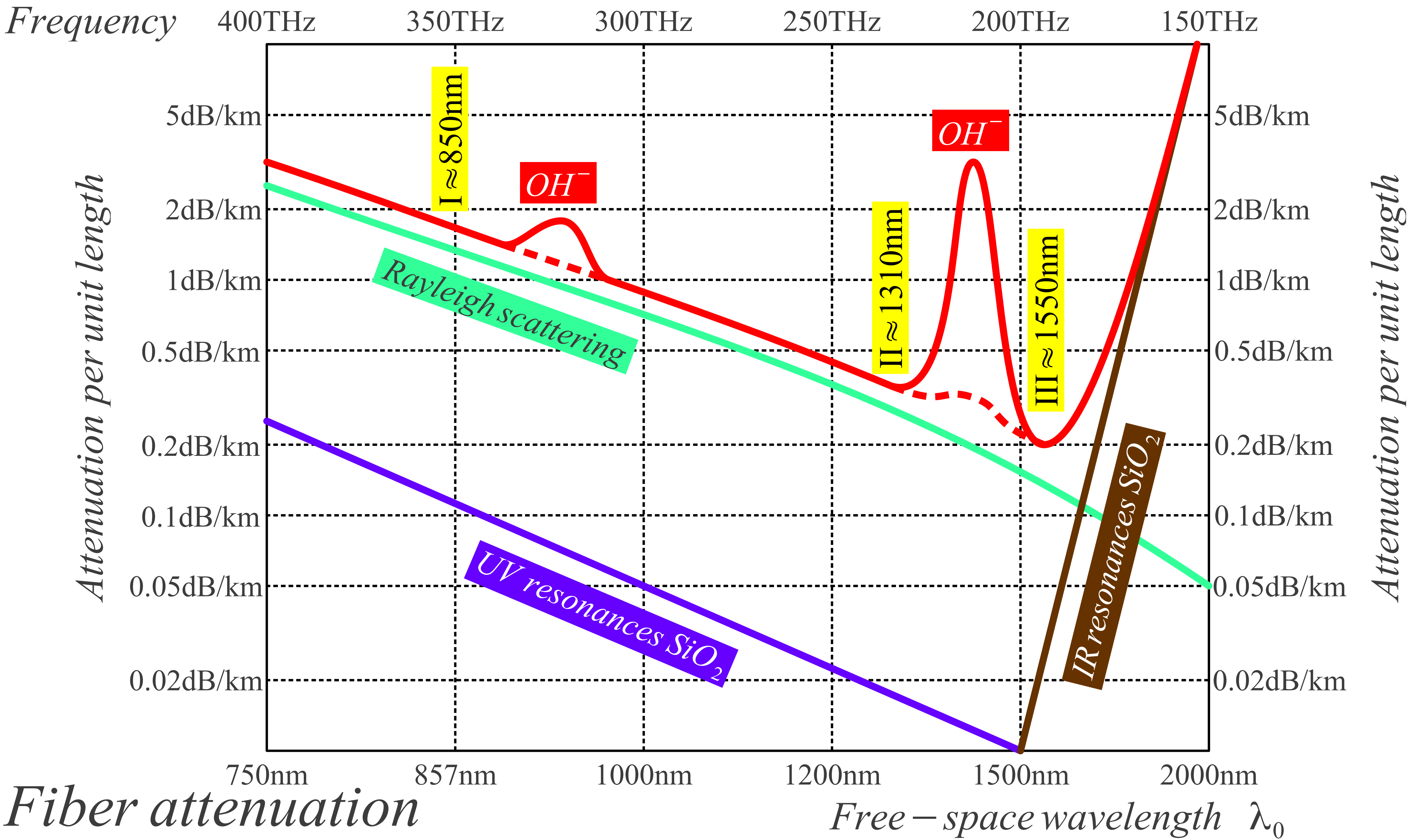
1977 $a/l \approx -0.2\,dB/km$ almost theoretical limit for SiO_2 glass @ $\lambda_0 \approx 1.55\,\mu m$

$TlCl_4$ theoretical $a/l \approx -0.001\,dB/km$ → practical $-1\,dB/m$ @ $\lambda_0 \approx 3\,\mu m$

Glass $d = 250\,\mu m \dots 80\,\mu m$ → $125\,\mu m$

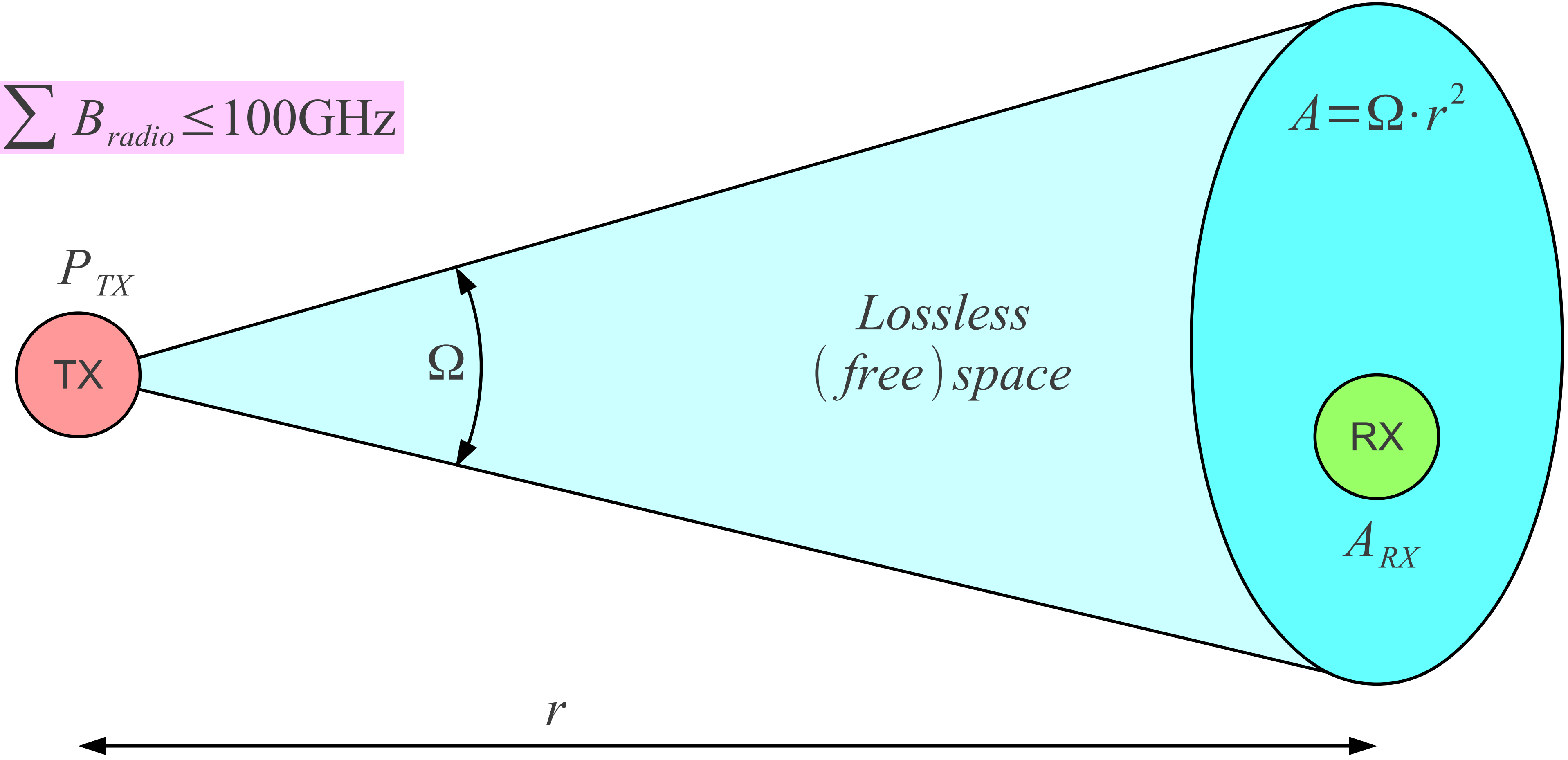
Plastic $a/l \approx -150\,dB/km$

Fiber attenuation



Wireless link → wave propagation

$$\sum B_{radio} \leq 100\text{GHz}$$



$$S=\frac{P_{TX}}{A}=\frac{P_{TX}}{\Omega\cdot r^2}$$

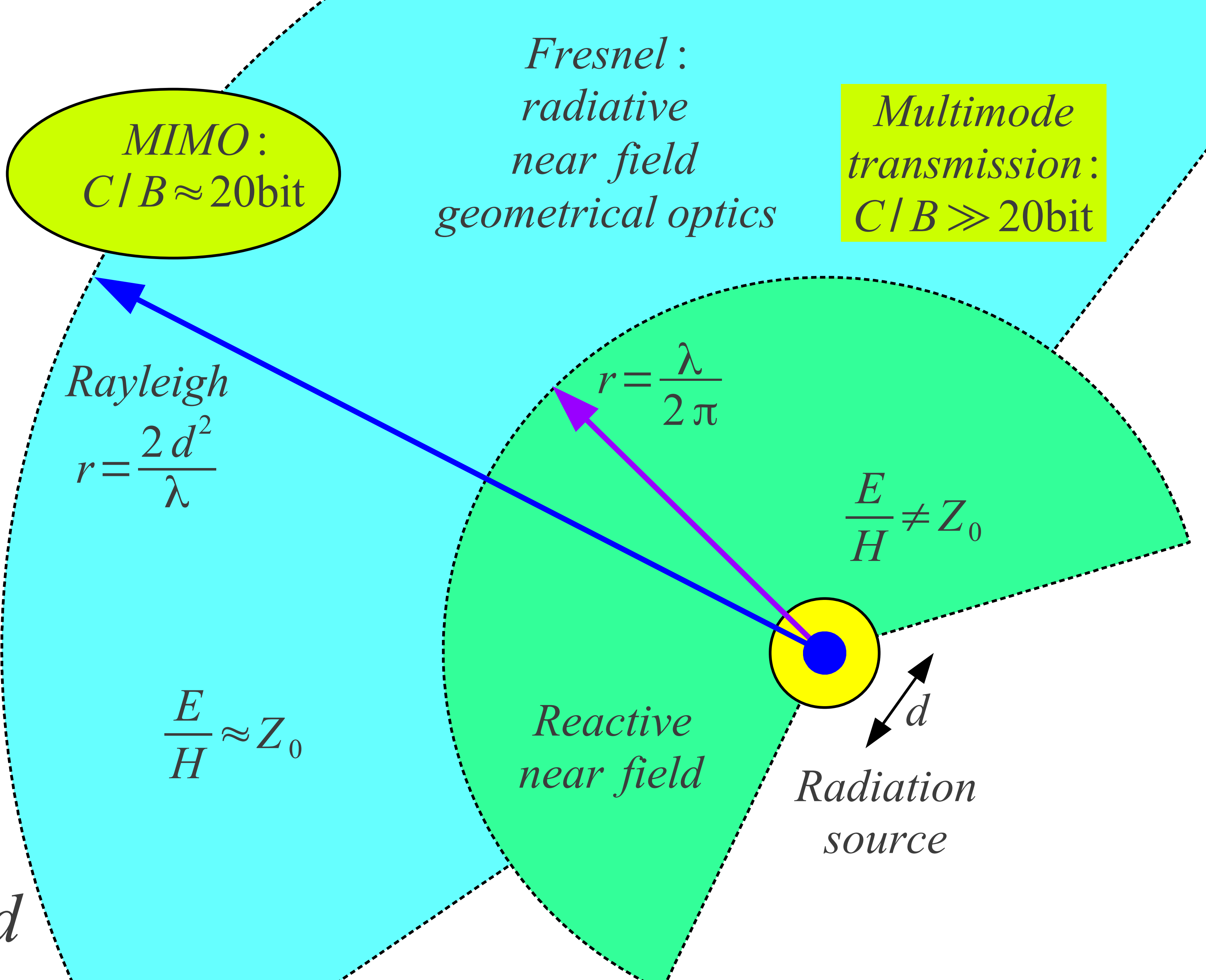
$$P_{RX}=S\cdot A_{RX}=P_{TX}\frac{A_{RX}}{\Omega\cdot r^2}=\alpha\cdot r^{-2}$$

Obstacle loss
(diffraction)

$$P_{RX}=\alpha\cdot r^{-n}$$

$$n\approx 3\ldots 5$$

Near and far field



Fraunhofer:
far field
wave optics

Two polarizations
(two modes):
$C/B \le 20\text{bit}$

$$\frac{E}{H} = Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\,\Omega$$

Only here exist:
$D, G, F(\Theta, \Phi)$,
Friis equation

MIMO:
$C/B \approx 20\text{bit}$

Fresnel:
radiative
near field
geometrical optics

Multimode
transmission:
$C/B \gg 20\text{bit}$

Rayleigh

$$r = \frac{2d^2}{\lambda}$$

$$\frac{E}{H} \approx Z_0$$

$$r = \frac{\lambda}{2\pi}$$

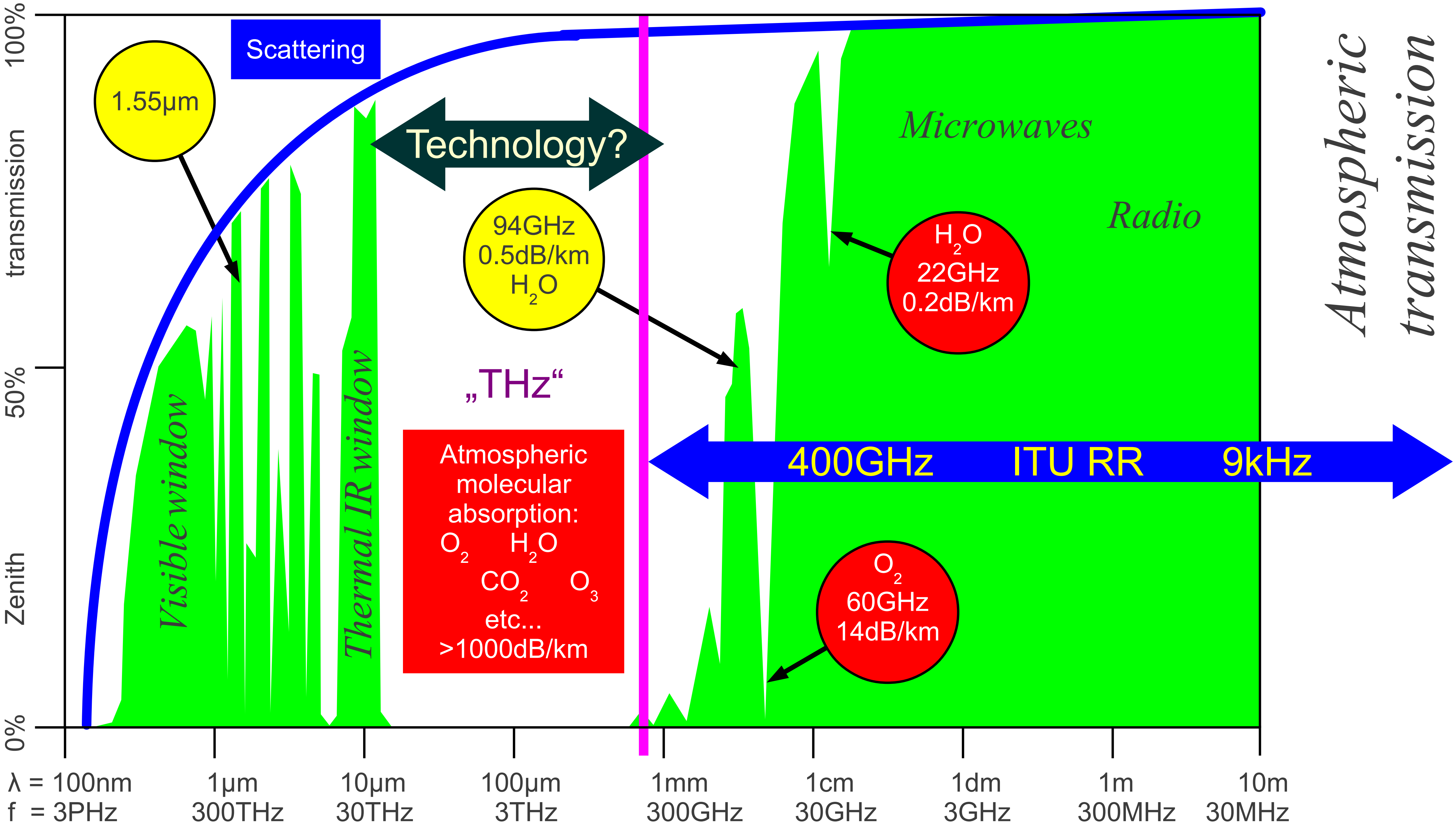
$$\frac{E}{H} \ne Z_0$$

Reactive
near field

d

Radiation
source

Atmospheric transmission



Link comparison