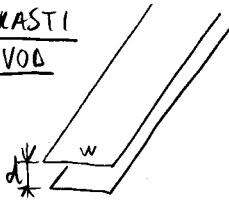


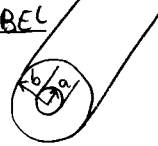
TRAKASTI DVOVOD



$$C/l = \epsilon_0 \frac{w}{d}$$

$$L/l = \mu_0 \frac{d}{w}$$

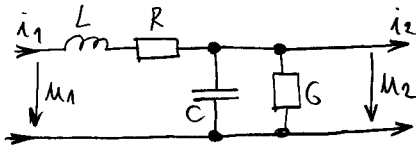
KOAKSIALNI KABEL



$$C/l = \frac{2\pi \epsilon_0}{\ln(b/a)}$$

$$L/l = \frac{\mu_0}{2\pi} \ln(b/a)$$

NADOMESTNO VEZJE



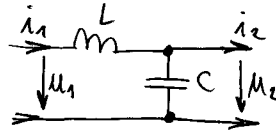
$$\Delta u = u_2 - u_1 = -L \frac{di_1}{dt} - Ri_1$$

$$\Delta i = i_2 - i_1 = -C \frac{du_2}{dt} - Gu_2$$

$$\frac{\partial u(z,t)}{\partial z} = -L/l \frac{\partial i(z,t)}{\partial t} - R/l i(z,t)$$

$$\frac{\partial i(z,t)}{\partial z} = -C/l \frac{\partial u(z,t)}{\partial t} - G/l u(z,t)$$

BREZ IZGUB



$$\Delta u = u_2 - u_1 = -L \frac{di_1}{dt}$$

$$\Delta i = i_2 - i_1 = -C \frac{du_2}{dt}$$

$$\frac{\partial u(z,t)}{\partial z} = -L/l \frac{\partial i(z,t)}{\partial t} \quad / \frac{\partial}{\partial z}$$

$$\frac{\partial i(z,t)}{\partial z} = -C/l \frac{\partial u(z,t)}{\partial t} \quad / \frac{\partial}{\partial t}$$

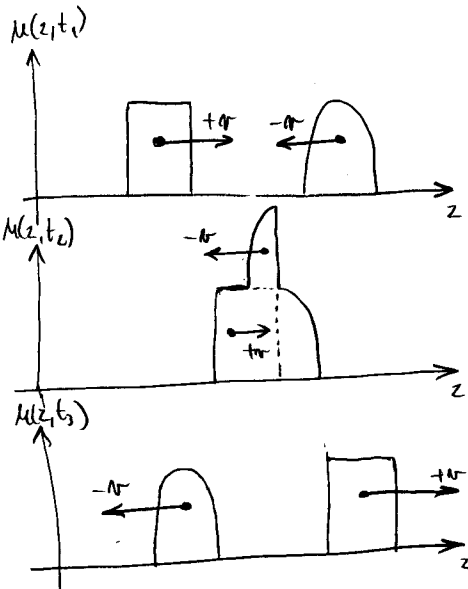
$$\frac{\partial^2 u(z,t)}{\partial z^2} = L/l \cdot C/l \frac{\partial^2 u(z,t)}{\partial t^2}$$

$$u(z,t) = f(x); \quad x = t \pm \frac{z}{v}$$

$$\frac{\partial^2 u(z,t)}{\partial z^2} = f''(x) \frac{1}{v^2}$$

$$\frac{\partial^2 u(z,t)}{\partial t^2} = f''(x)$$

$$\rightarrow v = 1/\sqrt{L/l \cdot C/l}$$



$$u(z,t) = C_1 f_1\left(t - \frac{z}{v}\right) + C_2 f_2\left(t + \frac{z}{v}\right)$$

NAPREBUJOČI VAL      ODBITI VAL

TRAKASTI DVOVOD

$$v = 1/\sqrt{\mu_0 \frac{d}{w} \epsilon_0 \frac{w}{d}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c_0 \approx 3 \cdot 10^8 \text{ m/s}$$

KOAKSIALNI KABEL

$$v = 1/\sqrt{\frac{\mu_0}{2\pi} \ln(b/a) \frac{2\pi \epsilon_0}{\ln(b/a)}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c_0 \approx 3 \cdot 10^8 \text{ m/s}$$

TRAKASTI DVOVOD       $Z_k = \sqrt{\frac{\mu_0 \frac{d}{w}}{\epsilon_0 \frac{w}{d}}} = \frac{d}{w} \sqrt{\frac{\mu_0}{\epsilon_0}}$

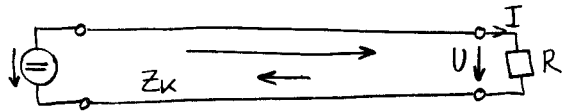
KOAKSIALNI KABEL       $Z_k = \sqrt{\frac{\frac{\mu_0}{2\pi} \ln(b/a)}{2\pi \epsilon_0 / \ln(b/a)}} = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \ln(b/a)$

$$\frac{\partial}{\partial z} u\left(t \pm \frac{z}{v}\right) = -L/l \frac{\partial}{\partial t} i\left(t \pm \frac{z}{v}\right)$$

$$\pm \frac{1}{v} u'\left(t \pm \frac{z}{v}\right) = -L/l i'\left(t \pm \frac{z}{v}\right)$$

$$\frac{u'}{i'} = \frac{u}{i} = \mp v L/l = \mp \sqrt{\frac{L/l}{C/l}} = \mp Z_k$$

PROSTOR:  $\frac{|E|}{|H|} = Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \Omega = 377 \Omega$



$$U = U_N + U_0$$

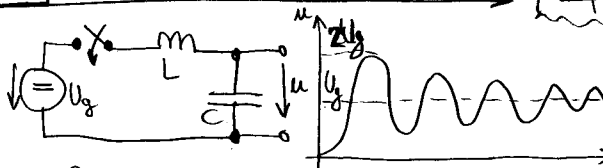
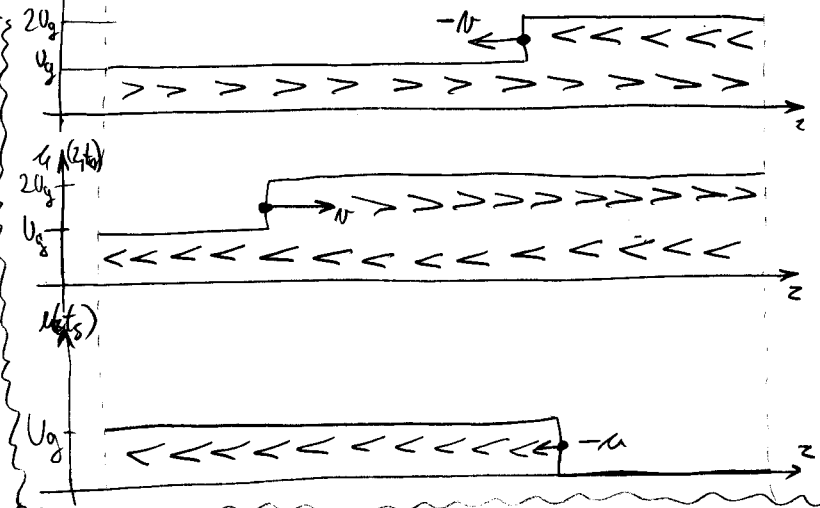
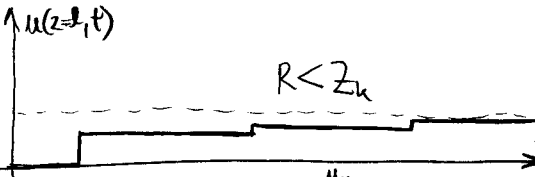
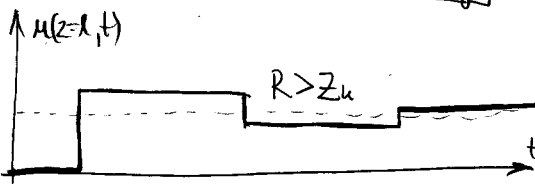
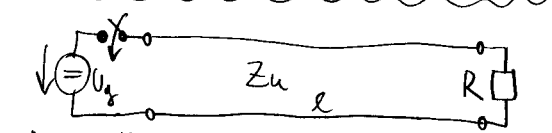
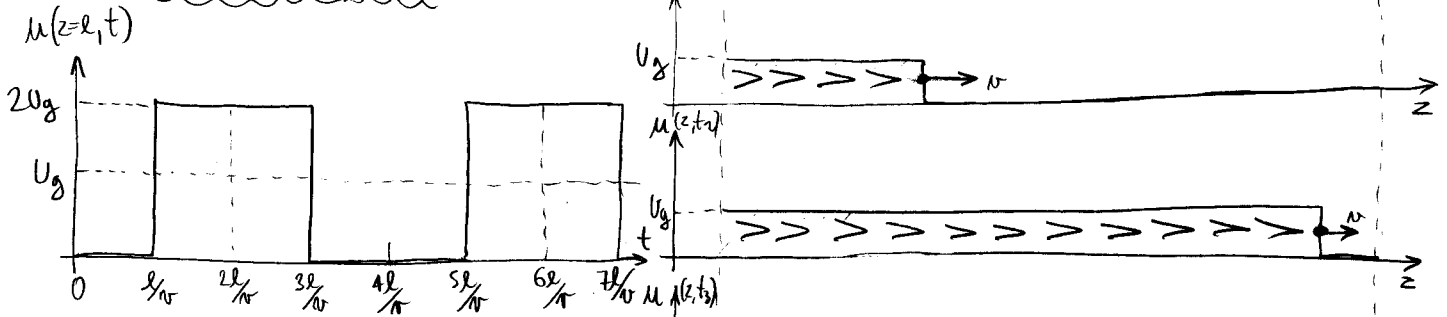
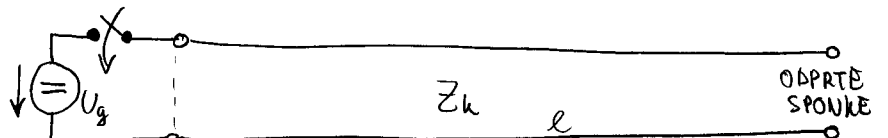
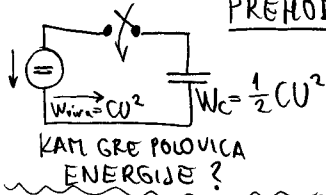
$$I = I_N + I_0 = \frac{U_N}{Z_k} - \frac{U_0}{Z_k} = \frac{U}{R} = \frac{U_N + U_0}{R} \cdot \frac{1}{U_N}$$

ODBOJNOST  $\Gamma = \frac{U_0}{U_N} \rightarrow \frac{1}{Z_k} - \frac{\Gamma}{Z_k} = \frac{1+\Gamma}{R} \rightarrow \boxed{\Gamma = \frac{R - Z_k}{R + Z_k}}$

PASIVNO BREME

OS:  $\Gamma = +1$ , KS:  $\Gamma = -1$ ;  $Z_k = R \rightarrow \Gamma = 0$ ;  $Z_k > R \rightarrow \Gamma < 0$ ;  $Z_k < R \rightarrow \Gamma > 0$ ;  $|\Gamma| \leq 1$

PREHODNI POJAV

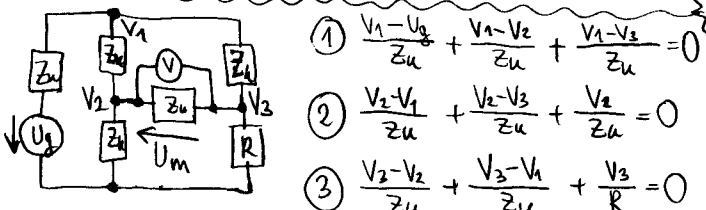


FREKVENČNI PROSTOR  $\frac{\omega}{v} = \beta = \text{fazna konstanta}$

$$u(t,z) = \text{Re} [ U_0 e^{j\omega(t \pm \frac{z}{v})} ] = \text{Re} [ U_0 e^{j(\omega t \pm \beta z)} ]$$

$$\frac{\partial}{\partial t} = j\omega; \quad \frac{\partial}{\partial z} = \pm j\beta \quad \beta = \omega \sqrt{\mu_0 \epsilon_0}$$

$$\beta \lambda = 2\pi \rightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega \sqrt{\mu_0 \epsilon_0}} = \frac{c_0}{f}$$



$$\textcircled{1} \frac{V_1 - U_g}{Z_1} + \frac{V_1 - V_2}{Z_2} + \frac{V_1 - V_3}{Z_4} = 0$$

$$\textcircled{2} \frac{V_2 - V_1}{Z_1} + \frac{V_2 - V_3}{Z_3} + \frac{V_2}{Z_2} = 0$$

$$\textcircled{3} \frac{V_3 - V_2}{Z_3} + \frac{V_3 - V_1}{Z_4} + \frac{V_3}{R} = 0$$

$$\textcircled{1} 3V_1 = U_g + V_2 + V_3 = 9V_2 - 3V_3 \rightarrow 8V_2 = U_g + 4V_3$$

$$\textcircled{2} 3V_2 = V_1 + V_3 \rightarrow V_1 = 3V_2 - V_3$$

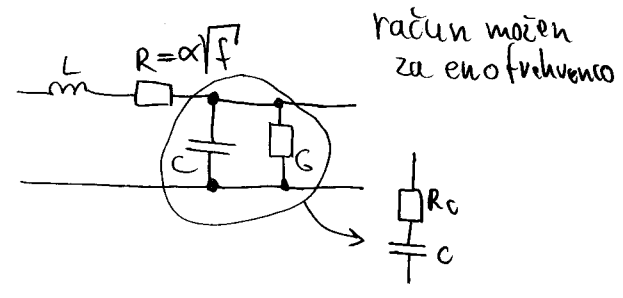
$$\textcircled{3} (2 + \frac{Z_4}{R})V_3 = V_1 + V_2 = 4V_2 - V_3 \rightarrow (3 + \frac{Z_4}{R})V_3 = 4V_2$$

$$(6 + 2\frac{Z_4}{R})V_3 = U_g + 4V_3 \rightarrow V_3 = \frac{U_g}{2(1 + \frac{Z_4}{R})}$$

$$V_2 = \frac{U_g}{8} + \frac{V_3}{2} = \frac{U_g}{8} \frac{3 + \frac{Z_4}{R}}{(1 + \frac{Z_4}{R})}$$

$$U_m = V_3 - V_2 = \frac{U_g}{8} \frac{1 - \frac{Z_4}{R}}{1 + \frac{Z_4}{R}} = \frac{U_g}{8} \Gamma$$

TEŽAVE Z IZGUBAMI



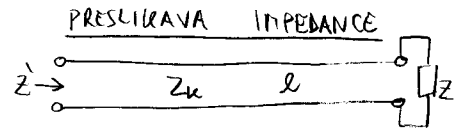
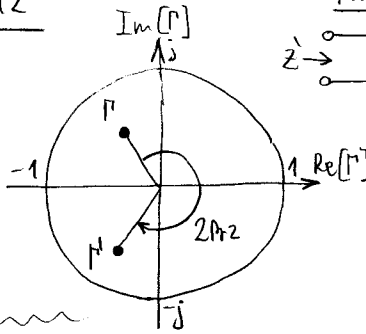
račun močen za eno frekvenco

$$u(z,t) = \text{Re} [U_N e^{j(\omega t - \beta z)} + U_0 e^{j(\omega t + \beta z)}]$$

$$\Gamma = \frac{Z - Z_k}{Z + Z_k} = \frac{Y_k - Y}{Y_k + Y}$$

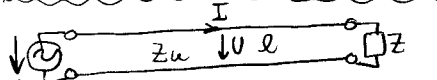
$$|\Gamma| \leq 1 \text{ pasivno breme}$$

BREZIZGUBNI  
VOD



- ①  $\Gamma = \frac{Z - Z_k}{Z + Z_k}$
- ②  $\Gamma' = \Gamma e^{-j2\beta l}$
- ③  $Z' = Z_k \frac{1 + \Gamma'}{1 - \Gamma'}$

$$Z' = Z_k \frac{1 + \Gamma'}{1 - \Gamma'} = Z_k \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} = Z_k \frac{Z + Z_k + (Z - Z_k)e^{-j2\beta l}}{Z + Z_k - (Z - Z_k)e^{-j2\beta l}} = Z_k \frac{Z(e^{j\beta l} + e^{-j\beta l}) + Z_k(e^{j\beta l} - e^{-j\beta l})}{Z(e^{j\beta l} - e^{-j\beta l}) + Z_k(e^{j\beta l} + e^{-j\beta l})} = Z_k \frac{Z \cos \beta l + j Z_k \sin \beta l}{j Z \sin \beta l + Z_k \cos \beta l}$$



$$P = \frac{1}{2} U I^* = \frac{1}{2} (U_N e^{-j\beta z} + U_0 e^{j\beta z}) \left( \frac{U_N^*}{Z_k} e^{j\beta z} - \frac{U_0^*}{Z_k} e^{-j\beta z} \right)$$

$U = U_N e^{-j\beta z} + U_0 e^{j\beta z}$  MOČ  
 $I = \frac{U_N}{Z_k} e^{-j\beta z} - \frac{U_0}{Z_k} e^{j\beta z}$

$$P = \frac{|U_N|^2}{2Z_k} - \frac{|U_0|^2}{2Z_k} + \frac{U_0 U_N^*}{Z_k} e^{j2\beta z} - \frac{U_N U_0^*}{Z_k} e^{-j2\beta z}$$

$$U_0 U_N^* = |U_0 U_N^*| e^{j\varphi}$$

$$U_N U_0^* = |U_0 U_N^*| e^{-j\varphi}$$

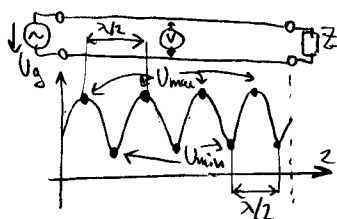
$$P = \frac{|U_N|^2}{2Z_k} - \frac{|U_0|^2}{2Z_k} + j \frac{|U_0 U_N^*|}{Z_k} \sin(2\beta z + \varphi)$$

NAPREJNOČA MOČ      ODEBTA MOČ  
JAKOVA MOČ → ENERGIJA STOJNEGA VALA

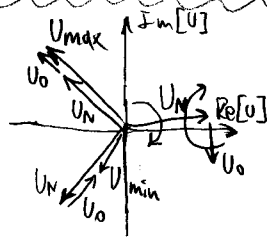
$$\text{Re}[P] = P_N - P_0 \quad P_0 = |\Gamma|^2 P_N$$

$$\text{Re}[P] = P_N (1 - |\Gamma|^2)$$

STOJNI VAL  
(BREZIZGUBNI)



$$\rho = \frac{U_{\max}}{U_{\min}}$$



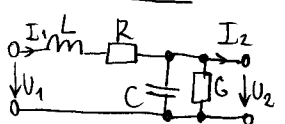
$$U_{\max} = |U_N| + |U_0| = |U_N|(1 + |\Gamma|)$$

$$U_{\min} = |U_N| - |U_0| = |U_N|(1 - |\Gamma|)$$

$$\rho = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

RAZMERJE STOJNEGA VALA  
STANDING-WAVE RATIO  
VALOVITOST

IZGUBNI VOD



$$U_1 - U_2 = (j\omega L + R) I_1 \quad \frac{\partial U}{\partial z} = -(j\omega L + R) I / \frac{\partial z}$$

$$I_1 - I_2 = (j\omega C + G) U_2 \quad \frac{\partial I}{\partial z} = -(j\omega C + G) U$$

$$U = A e^{\pm jkz} \rightarrow -k^2 = (j\omega L + R)(j\omega C + G)$$

$$k = \sqrt{-(j\omega L + R)(j\omega C + G)} = \alpha - j\beta$$

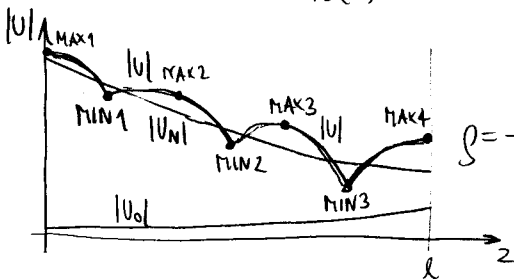
$$Z_k = \frac{U_N}{I_N}; U_N = A_N e^{-jkz}; I_N = \frac{-jk}{-(j\omega L + R)} U_N \rightarrow Z_k = \frac{j\omega L + R}{jk} = \sqrt{\frac{j\omega L + R}{j\omega C + G}} \approx \sqrt{\frac{L/\alpha}{C/\alpha}}$$

ZA NAJMNE IZGUBE!

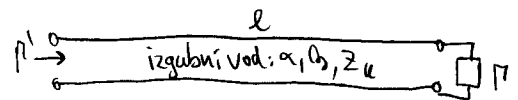
$$|U_N| = |A_N| e^{-\alpha z} \rightarrow P_N(z) = P_N(0) e^{-2\alpha z}$$

$$|U_0| = |A_0| e^{+\alpha z} \rightarrow P_0(z) = P_0(0) e^{+2\alpha z}$$

$$\Gamma(z) = \frac{U_0}{U_N} = \frac{e^{+\alpha z} e^{j\beta z}}{e^{-\alpha z} e^{-j\beta z}} \quad \Gamma(0) = e^{2\alpha z} e^{j2\beta z} \Gamma(0)$$



$$\rho = \frac{U_{\max}}{U_{\min}} = \text{NEDEFINIRAN}$$



$$\Gamma' = e^{-2\alpha l} e^{-j2\beta l} \Gamma$$

$$|\Gamma'| = |\Gamma| e^{-2\alpha l}$$

LOG. ENOTE

PRILAGAJENOST  
RETURN LOSS

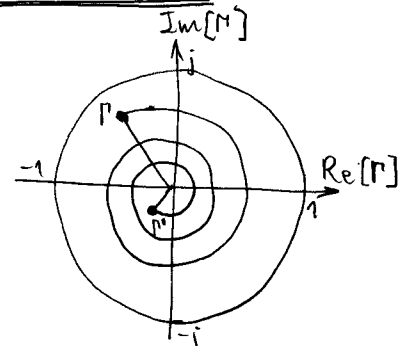
$$\Gamma_{dB} = 20 \log_{10} |\Gamma|$$

$$\Gamma'_{dB} = \Gamma_{dB} - \frac{20}{\ln 10} (2\alpha l)$$

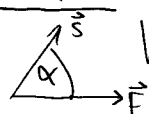
$$\Gamma'_{dB} = \Gamma_{dB} - 2\alpha_{dB}$$

$$\Gamma_{NP} = \ln |\Gamma| = \ln \frac{U_0}{U_N}$$

$$\alpha_{dB} = 10 \log_{10} \frac{P(0)}{P(l)} = 20 \log_{10} \frac{U(0)}{U(l)} = \frac{20}{\ln 10} \alpha_{NP} \quad \alpha_{dB}/l = \frac{20}{\ln 10} \alpha$$



FIZIKA



$$W = |\vec{F}| |\vec{S}| \cos \alpha = \vec{F} \cdot \vec{S}$$



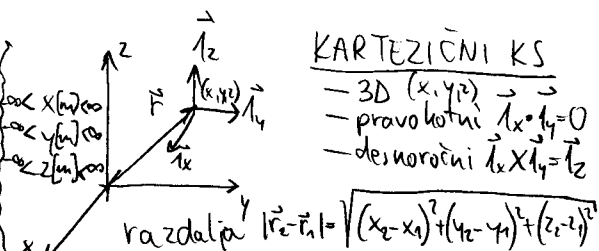
$$|\vec{v}| = |\vec{r}| |\vec{\omega}| \sin \alpha$$

$$\vec{v} = d\vec{r}, \vec{\omega} = d\vec{\omega}$$

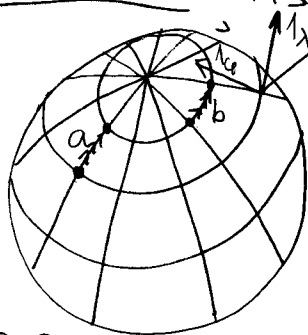
$$\vec{v} = \vec{\omega} \times \vec{r}$$

KARTEZIČNI KS

- 3D  $(x, y, z) \rightarrow$
- pravokotni  $\vec{i}_x \cdot \vec{i}_y = 0$
- desnoručni  $\vec{i}_x \times \vec{i}_y = \vec{i}_z$



ZEMLJEPISMI KS  $(\lambda, \varphi, h)$



$$a = h_\varphi \Delta \varphi \quad 0 \leq \lambda [^\circ] \leq 360^\circ$$

$$-90^\circ \leq \varphi [^\circ] \leq +90^\circ$$

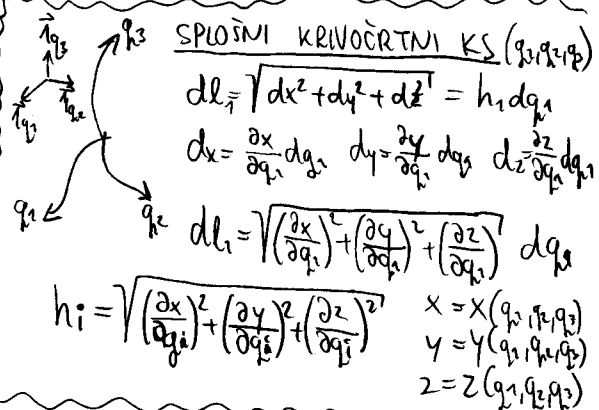
$$h_\varphi = \frac{40000 \text{ km}}{360^\circ} = 111 \text{ km/}^\circ$$

$$6378000 \text{ m} < h [\text{m}] < \infty$$

$$b = h_\lambda \Delta \lambda$$

$$h_\lambda = \frac{40000 \text{ km}}{360^\circ} \cos \varphi = 111 \text{ km/}^\circ \cdot \cos \varphi$$

SPLOŠNI KRIVOČRTNI KS  $(q_1, q_2, q_3)$



$$dl = \sqrt{dx^2 + dy^2 + dz^2} = h_i dq_i$$

$$dx = \frac{\partial x}{\partial q_1} dq_1 \quad dy = \frac{\partial y}{\partial q_1} dq_1 \quad dz = \frac{\partial z}{\partial q_1} dq_1$$

$$dl_i = \sqrt{\left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2} dq_i$$

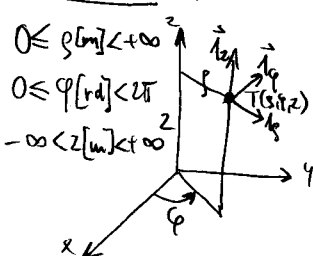
$$h_i = \sqrt{\left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2}$$

$$x = x(q_1, q_2, q_3)$$

$$y = y(q_1, q_2, q_3)$$

$$z = z(q_1, q_2, q_3)$$

VALJNI KS  $(\rho, \varphi, z)$



$$0 \leq \rho [\text{m}] < +\infty$$

$$0 \leq \varphi [\text{rad}] < 2\pi$$

$$-\infty < z [\text{m}] < +\infty$$

$$\rho = \sqrt{x^2 + y^2}$$

$$\varphi = \arctg \frac{y}{x} \text{ (kvadrant!)}$$

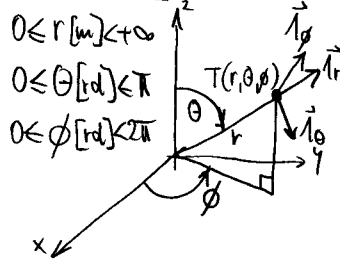
$$z = z$$

$$x = \rho \cos \varphi \quad h_\rho = 1$$

$$y = \rho \sin \varphi \quad h_\varphi = \rho$$

$$z = z \quad h_z = 1$$

KROGLJNI KS  $(r, \theta, \phi)$



$$0 \leq r [\text{m}] < +\infty$$

$$0 \leq \theta [\text{rad}] \leq \pi$$

$$0 \leq \phi [\text{rad}] < 2\pi$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \arccos \frac{z}{r}$$

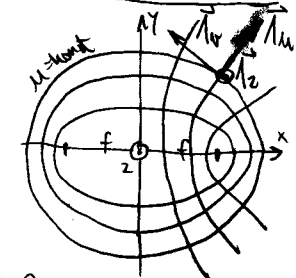
$$\phi = \arctg \frac{y}{x} \text{ (kvadrant!)}$$

$$x = r \sin \theta \cos \phi \quad h_r = 1$$

$$y = r \sin \theta \sin \phi \quad h_\theta = r$$

$$z = r \cos \theta \quad h_\phi = r \sin \theta$$

VALJNI-ELIPTIČNI KS  $(u, v, z)$



$$0 \leq u [ ] < +\infty$$

$$0 \leq v [\text{rad}] < 2\pi$$

$$-\infty < z [\text{m}] < +\infty$$

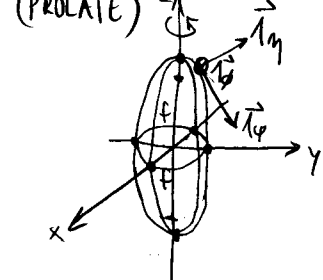
podatke f [m]

$$x = f \operatorname{ch} u \cos v \quad h_z = 1$$

$$y = f \operatorname{sh} u \sin v$$

$$z = z \quad h_u = h_v = f \sqrt{\operatorname{sh}^2 u \pm \sin^2 v}$$

PODOLGOVATI KROGLJNI-ELIPTIČNI KS  $(\eta, \psi, \phi)$



(PROLATE)

$$0 \leq \eta [ ] < +\infty$$

$$0 \leq \psi [\text{rad}] \leq \pi$$

$$0 \leq \phi [\text{rad}] < 2\pi$$

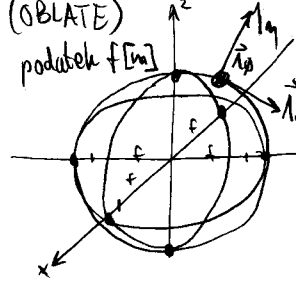
podatke f [m]

$$x = f \operatorname{sh} \eta \sin \psi \cos \phi$$

$$y = f \operatorname{sh} \eta \sin \psi \sin \phi$$

$$z = f \operatorname{ch} \eta \cos \psi$$

SPLOŠČENI KROGLJNI-ELIPTIČNI KS  $(\eta, \psi, \phi)$



(OBLATE)

$$0 \leq \eta [ ] < +\infty$$

$$0 \leq \psi [\text{rad}] \leq \pi$$

$$0 \leq \phi [\text{rad}] < 2\pi$$

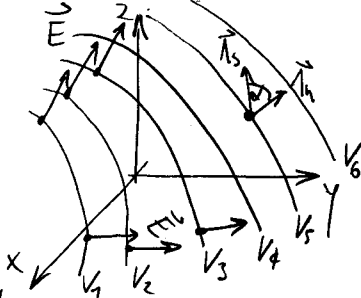
podatke f [m]

$$x = f \operatorname{ch} \eta \sin \psi \cos \phi$$

$$y = f \operatorname{ch} \eta \sin \psi \sin \phi$$

$$z = f \operatorname{sh} \eta \cos \psi$$

SMERNI ODVOD = GRADIENT  $V(x, y, z)$



$$\max \frac{\partial V}{\partial s} = \frac{\partial V}{\partial n}$$

$$\frac{\partial V}{\partial s} = \vec{i}_s \cdot \vec{i}_n \frac{\partial V}{\partial n} = \frac{\partial V}{\partial n} \cos \alpha$$

$$\vec{i}_n \frac{\partial V}{\partial n} = \operatorname{grad} V$$

$$(x, y, z) \rightarrow \vec{i}_x \cdot \operatorname{grad} V = \frac{\partial V}{\partial x}, \vec{i}_y \cdot \operatorname{grad} V = \frac{\partial V}{\partial y}, \vec{i}_z \cdot \operatorname{grad} V = \frac{\partial V}{\partial z} \rightarrow \operatorname{grad} V = \vec{i}_x \frac{\partial V}{\partial x} + \vec{i}_y \frac{\partial V}{\partial y} + \vec{i}_z \frac{\partial V}{\partial z}$$

$$\vec{\nabla} = \vec{i}_x \frac{\partial}{\partial x} + \vec{i}_y \frac{\partial}{\partial y} + \vec{i}_z \frac{\partial}{\partial z} \rightarrow \operatorname{grad} V = \vec{\nabla} V$$

$$(q_1, q_2, q_3) \rightarrow \vec{i}_{q_1} \cdot \operatorname{grad} V = \frac{\partial V}{\partial q_1}, \vec{i}_{q_2} \cdot \operatorname{grad} V = \frac{\partial V}{\partial q_2}, \vec{i}_{q_3} \cdot \operatorname{grad} V = \frac{\partial V}{\partial q_3} \rightarrow \operatorname{grad} V = \vec{i}_{q_1} \frac{1}{h_1} \frac{\partial V}{\partial q_1} + \vec{i}_{q_2} \frac{1}{h_2} \frac{\partial V}{\partial q_2} + \vec{i}_{q_3} \frac{1}{h_3} \frac{\partial V}{\partial q_3}$$

$$(\rho, \varphi, z) \rightarrow \operatorname{grad} V = \vec{i}_\rho \frac{\partial V}{\partial \rho} + \vec{i}_\varphi \frac{1}{\rho} \frac{\partial V}{\partial \varphi} + \vec{i}_z \frac{\partial V}{\partial z}$$

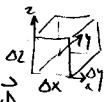
$$(r, \theta, \phi) \rightarrow \operatorname{grad} V = \vec{i}_r \frac{\partial V}{\partial r} + \vec{i}_\theta \frac{1}{r} \frac{\partial V}{\partial \theta} + \vec{i}_\phi \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi}$$

# Elektrodinamika 5/11/2012

## IZVORNOST

$$\int_V \rho \, dv = Q = \oint_A \vec{D} \cdot d\vec{A} \rightarrow \rho = \lim_{\Delta V \rightarrow 0} \frac{\oint \vec{D} \cdot d\vec{A}}{\Delta V} = \text{div } \vec{D}$$

$$\text{div } \vec{D} = \frac{1}{h_1 h_2 h_3} \left( \frac{\partial}{\partial q_1} (h_2 h_3 D_1) + \frac{\partial}{\partial q_2} (h_1 h_3 D_2) + \frac{\partial}{\partial q_3} (h_1 h_2 D_3) \right)$$



$$\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta y \Delta z \Delta x + \Delta x \Delta z \Delta y + \Delta x \Delta y \Delta z}{\Delta x \Delta y \Delta z} = \frac{\partial A_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial C_z}{\partial z} = \vec{\nabla} \cdot \vec{D}$$

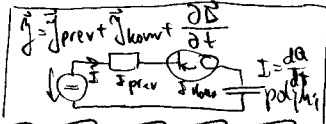
$$\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta(l_1 l_2 l_3 D_1) + \Delta(l_1 l_3 D_2) + \Delta(l_2 l_3 D_3)}{\Delta l_1 \Delta l_2 \Delta l_3} = \frac{\partial(h_1 h_2 D_1)}{\partial q_1} + \frac{\partial(h_1 h_3 D_2)}{\partial q_2} + \frac{\partial(h_2 h_3 D_3)}{\partial q_3}$$



## VRTIČNOST

$$\oint_A \vec{H} \cdot d\vec{l} = I = \oint_S \vec{H} \cdot d\vec{l} \rightarrow \vec{H} = \lim_{\Delta A \rightarrow 0} \frac{\oint \vec{H} \cdot d\vec{l}}{\Delta A} = \vec{i}_z \cdot \text{rot } \vec{H}$$

$$\vec{H} = \text{rot } \vec{H}$$



$$\vec{H} = \lim_{\Delta A \rightarrow 0} \frac{\Delta y H_1 - \Delta x H_2}{\Delta x \Delta y} = \frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y} \rightarrow \vec{H} = \vec{\nabla} \times \vec{H} = \begin{vmatrix} \vec{i}_x & \vec{i}_y & \vec{i}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$\vec{H} = \lim_{\Delta A \rightarrow 0} \frac{\Delta l_2 H_2 - \Delta l_1 H_1}{\Delta l_1 \Delta l_2} = \frac{1}{h_1 h_2} \left( \frac{\partial(h_2 H_2)}{\partial q_1} - \frac{\partial(h_1 H_1)}{\partial q_2} \right) \rightarrow \vec{H} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 h_2 \vec{i}_1 & h_1 h_3 \vec{i}_2 & h_2 h_3 \vec{i}_3 \\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3} \\ h_1 H_1 & h_2 H_2 & h_3 H_3 \end{vmatrix}$$

## ME v d.f. obliki:

- ①  $\text{rot } \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$
- ②  $\text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- ③  $\text{div } \vec{D} = \rho$

## Harmonske veličine

$$\vec{E}(x, y, z, t) = \vec{E}_0(x, y, z) e^{j\omega t}$$

$$\frac{\partial}{\partial t} = j\omega$$

- ①  $\text{rot } \vec{H} = \vec{j} + j\omega \vec{D}$
- ②  $\text{rot } \vec{E} = -j\omega \vec{B}$
- ③  $\text{div } \vec{D} = \rho$

## Snov: $\epsilon, \mu$ (skalarni konstanti)

- ①  $\text{rot } \vec{H} = \vec{j} + j\omega \epsilon \vec{E}$
- ②  $\text{rot } \vec{E} = -j\omega \mu \vec{H}$
- ③  $\text{div}(\epsilon \vec{E}) = \rho$

## EM naloga

izvor:  $(\rho, \vec{j}) \rightarrow$  polje  $(\vec{E}, \vec{H})$

$$\Delta \vec{E} + \omega^2 \mu \epsilon \vec{E} = \text{grad} \left( \frac{\rho}{\epsilon} \right) + j\omega \vec{j}$$

$$\Delta \vec{H} + \omega^2 \mu \epsilon \vec{H} = -\text{rot } \vec{j}$$

## Sestavljenе operacije

$$\text{rot}(\text{grad } V) = \vec{\nabla} \times (\vec{\nabla} V) = 0$$

$$\text{div}(\text{rot } \vec{F}) = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0$$

$$\text{rot}(\text{rot } \vec{F}) = \text{grad}(\text{div } \vec{F}) - \Delta \vec{F}$$

$$\text{div}(\text{grad } V) = \vec{\nabla} \cdot (\vec{\nabla} V) = \Delta V$$

## Potenciali

OE I  $\rightarrow \omega = 0, \vec{E} = -\text{grad } V, \Delta V = -\frac{\rho}{\epsilon}$

OE II  $\rightarrow \omega = 0, \vec{j} = 0, \vec{H} = -\text{grad } V_m, \Delta V_m = 0$

## VEKTORSKI POTENCIAL $\vec{B} = \text{rot } \vec{A}$

$$\text{② } \text{rot } \vec{E} = -j\omega \vec{B} = -j\omega \text{rot } \vec{A} \rightarrow \text{rot}(\vec{E} + j\omega \vec{A}) = 0 \rightarrow \vec{E} + j\omega \vec{A} = -\text{grad } V, \vec{E} = -j\omega \vec{A} - \text{grad } V$$

$$\text{① } \text{rot } \vec{H} = \text{rot} \left( \frac{1}{\mu} \text{rot } \vec{A} \right) = \vec{j} + j\omega \epsilon \vec{E} \rightarrow \text{rot}(\text{rot } \vec{A}) = \text{grad}(\text{div } \vec{A}) - \Delta \vec{A} = \mu \vec{j} + j\omega \mu \epsilon (-j\omega \vec{A} - \text{grad } V)$$

$$\Delta \vec{A} + \omega^2 \mu \epsilon \vec{A} = -\mu \vec{j} + \text{grad}(\text{div } \vec{A} + j\omega \mu \epsilon V) \quad \text{Columb: } \text{div } \vec{A} = 0 \quad \text{Lorentz: } \text{div } \vec{A} = -j\omega \mu \epsilon V$$

$$\Delta \vec{A} + \omega^2 \mu \epsilon \vec{A} = -\mu \vec{j}$$

$$\text{③ } \text{div}(\epsilon \vec{E}) = \rho \rightarrow \frac{\rho}{\epsilon} = -j\omega \text{div } \vec{A} - \text{div}(\text{grad } V)$$

$$\Delta V + \omega^2 \mu \epsilon V = -\frac{\rho}{\epsilon}$$

OE I,  $\omega=0$   $W_e = \frac{1}{2} \int_V \vec{E} \cdot \vec{D} \, dv$   $\operatorname{div}(\epsilon V \operatorname{grad} V) = \epsilon \operatorname{grad} V \cdot \operatorname{grad} V + \epsilon V \Delta V = \vec{E} \cdot \vec{D} - \rho V$

$\Delta V = -\frac{\rho}{\epsilon}$   $\int_V \operatorname{div}(\epsilon V \operatorname{grad} V) \, dv = \oint_A \epsilon V \operatorname{grad} V \cdot \vec{n} \, dA = \int_V \vec{E} \cdot \vec{D} \, dv - \int_V \rho V \, dv \rightarrow W_e = \frac{1}{2} \int_V \rho V \, dv$

OE II,  $\omega=0$   $W_m = \frac{1}{2} \int_V \vec{H} \cdot \vec{B} \, dv$   $\operatorname{div}(\vec{H} \times \vec{A}) = \vec{\nabla} \cdot (\vec{H} \times \vec{A}) = \vec{A} \cdot \operatorname{rot} \vec{H} - \vec{H} \cdot \operatorname{rot} \vec{A} = \vec{j} \cdot \vec{A} - \vec{H} \cdot \vec{B}$

$\int_V \operatorname{div}(\vec{H} \times \vec{A}) \, dv = \oint_A (\vec{H} \times \vec{A}) \cdot \vec{n} \, dA = 0 = \int_V \vec{j} \cdot \vec{A} \, dv - \int_V \vec{H} \cdot \vec{B} \, dv \rightarrow W_m = \frac{1}{2} \int_V \vec{j} \cdot \vec{A} \, dv$

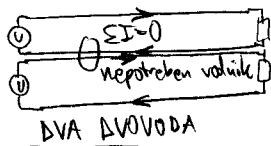
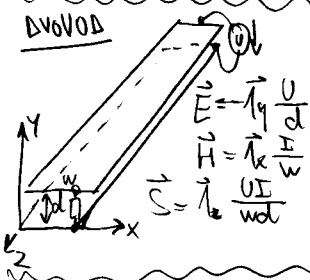
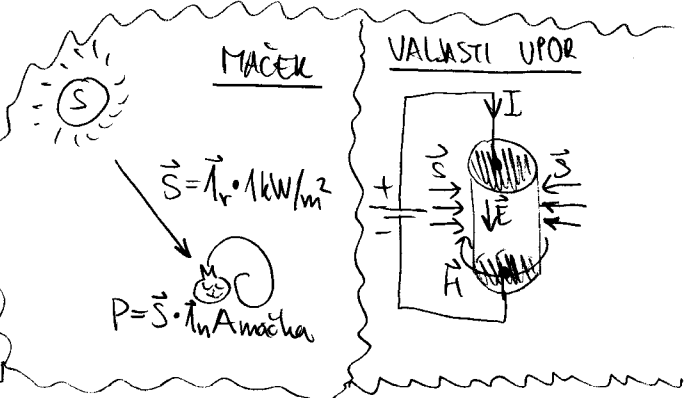
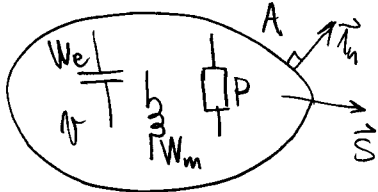
POYNTING  $W = \frac{1}{2} \int_V \vec{E} \cdot \vec{D} \, dv + \frac{1}{2} \int_V \vec{H} \cdot \vec{B} \, dv$ ,  $\frac{\partial \vec{D}}{\partial t} = \operatorname{rot} \vec{H} - \vec{j}$ ,  $\frac{\partial \vec{B}}{\partial t} = -\operatorname{rot} \vec{E}$ ,  $P = \int_V \vec{j} \cdot \vec{E} \, dv$

$\frac{dW}{dt} = \frac{1}{2} \int_V [2\vec{E} \cdot (\operatorname{rot} \vec{H} - \vec{j}) - 2\vec{H} \cdot \operatorname{rot} \vec{E}] \, dv = -\int_V \vec{j} \cdot \vec{E} \, dv + \int_V [\vec{E} \cdot \operatorname{rot} \vec{H} - \vec{H} \cdot \operatorname{rot} \vec{E}] \, dv =$

$= -\int_V \vec{j} \cdot \vec{E} \, dv + \int_V \operatorname{div}(\vec{H} \times \vec{E}) \, dv = -P - \oint_A (\vec{E} \times \vec{H}) \cdot \vec{n} \, dA \rightarrow \oint_A (\vec{E} \times \vec{H}) \cdot \vec{n} \, dA = \oint_A \vec{S} \cdot \vec{n} \, dA = -P - \frac{dW}{dt}$

$\vec{S} = \vec{E} \times \vec{H}$  [ $W/m^2$ ]

$\vec{E}$  [ $V/m$ ]  
 $\vec{H}$  [ $A/m$ ]



$\operatorname{div}(U \operatorname{grad} V - V \operatorname{grad} U) = U \Delta V - V \Delta U$  GREEN

$\oint_A (U \operatorname{grad} V - V \operatorname{grad} U) \cdot \vec{n} \, dA = \int_V (U \Delta V - V \Delta U) \, dv$

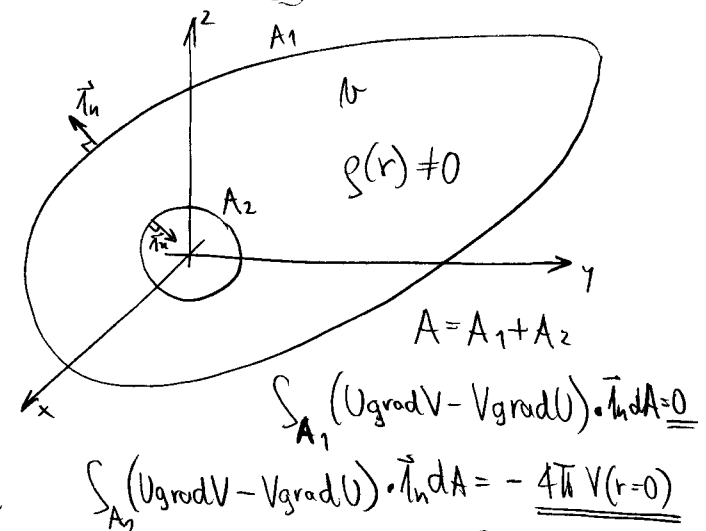
$\omega=0 \rightarrow U = \frac{1}{r}$ ;  $\operatorname{grad} U = -\vec{r} \frac{1}{r^2}$ ;  $\Delta U = 0$

$\Delta V = -\frac{\rho}{\epsilon}$ ;  $\int_V (U \Delta V - V \Delta U) \, dv = -\int_V \frac{\rho}{\epsilon r} \, dv$

$\omega \neq 0 \rightarrow U = \frac{e^{ikr}}{r}$ ;  $\operatorname{grad} U = -\vec{r} (\frac{1}{r} + ik) U$ ;  $\Delta U = -k^2 U$

$\Delta V = -\frac{\rho}{\epsilon} - k^2 V$ ;  $\int_V (U \Delta V - V \Delta U) \, dv = -\int_V \frac{\rho}{\epsilon} \frac{e^{ikr}}{r} \, dv$

$\omega=0 \rightarrow V(r=0) = \frac{1}{4\pi\epsilon} \int_V \frac{\rho}{r} \, dv$ ;  $\omega \neq 0 \rightarrow V(r=0) = \frac{1}{4\pi\epsilon} \int_V \rho \frac{e^{ikr}}{r} \, dv$



POTENCIAL V POCSUBNI TOČKI

$V(\vec{r}) = \frac{1}{4\pi\epsilon} \int_{V'} \rho(\vec{r}') \frac{e^{-ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \, do'$

$V(\vec{r}) = \frac{1}{4\pi\epsilon} \sum_i Q_i \frac{e^{-ik|\vec{r}-\vec{r}_i|}}{|\vec{r}-\vec{r}_i|}$

VEKTORSKI POTENCIAL

$\Delta \vec{A} + k^2 \vec{A} = -\mu \vec{j}$

$\Delta \vec{A} = \vec{r}_x A_x + \vec{r}_y A_y + \vec{r}_z A_z \quad (x, y, z)$

$\Delta A_x + k^2 A_x = -\mu j_x$

$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_{V'} \vec{j}(\vec{r}') \frac{e^{-ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \, do'$

$A_x(\vec{r}) = \frac{\mu}{4\pi} \int_{V'} j_x(\vec{r}') \frac{e^{-ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \, do'$

Poljuben KS

PONOVI TEV ME,  $\frac{\partial}{\partial t} = j\omega$  ①  $\text{rot } \vec{H} = \vec{j} + j\omega \epsilon \vec{E} \xrightarrow{\text{div } + ③} \text{div}(\text{rot } \vec{H}) = 0 = \text{div } \vec{j} + j\omega \text{div}(\epsilon \vec{E})$

②  $\text{rot } \vec{E} = -j\omega \mu \vec{H}$

③  $\text{div}(\epsilon \vec{E}) = \rho$

$0 = \text{div } \vec{j} + j\omega \rho$

Zveznost (kontinuiteta) toka

POTENCIALI

$\vec{B} = \text{rot } \vec{A} = \text{rot } \vec{V}_m$

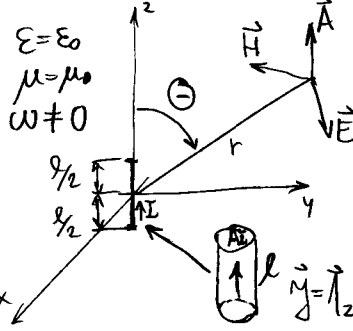
$V = -j\omega \vec{A} - \text{grad } V$

Lorentz-ova izbira

$\Delta \vec{A} + \omega^2 \mu \epsilon \vec{A} = -\mu \vec{j}$   
 $\Delta V + \omega^2 \mu \epsilon V = -\frac{\rho}{\epsilon}$

$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_V \vec{j}(\vec{r}') \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} d\omega'$

$V(\vec{r}) = \frac{1}{4\pi\epsilon} \int_V \rho(\vec{r}') \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} d\omega' = \frac{1}{4\pi\epsilon} \sum_i \frac{Q_i e^{-jk|\vec{r}-\vec{r}_i|}}{|\vec{r}-\vec{r}_i|}$



$\vec{A} = \frac{\mu}{4\pi} \int_V \vec{j}_z \frac{I}{A_z} \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} A_z dz' \approx \vec{j}_z \frac{\mu I l}{4\pi} \frac{e^{-jkr}}{r}$

KAATKA ŽICA

①  $l \ll r \rightarrow \frac{1}{|\vec{r}-\vec{r}'|} \approx \frac{1}{r}$

②  $l \ll \frac{2\pi}{k} \rightarrow e^{-jk|\vec{r}-\vec{r}'|} \approx e^{-jkr}$

$(x, y, z) \rightarrow (r, \theta, \phi): \vec{j}_z = \vec{j}_r \cos\theta - \vec{j}_\theta \sin\theta$

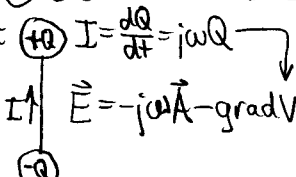
$\vec{A} = (\vec{j}_r \cos\theta - \vec{j}_\theta \sin\theta) \frac{\mu I l}{4\pi} \frac{e^{-jkr}}{r}$

$\vec{H} = \frac{1}{\mu} \text{rot } \vec{A}$

$\vec{H} = \frac{1}{\mu} \frac{1}{r^2 \sin\theta} \begin{vmatrix} \vec{j}_r & \vec{j}_\theta & \vec{j}_\phi \sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \frac{\mu I l}{4\pi} \frac{e^{-jkr}}{r} \cos\theta & -\frac{\mu I l}{4\pi} \frac{e^{-jkr}}{r} \sin\theta & 0 \end{vmatrix} = \vec{j}_\phi \frac{I l}{4\pi} e^{-jkr} \left( \frac{jk}{r} + \frac{1}{r^2} \right) \sin\theta$

SEVANJE BIOT-SAVART

Zveznost toka



② ME @  $\vec{j} = 0$

$\vec{E} = \frac{1}{j\omega\epsilon} \text{rot } \vec{H} = \frac{1}{j\omega\epsilon} \frac{1}{r^2 \sin\theta} \begin{vmatrix} \vec{j}_r & \vec{j}_\theta & \vec{j}_\phi \sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & r \cdot 0 & r \sin\theta \frac{I l}{4\pi} e^{-jkr} \left( \frac{jk}{r} + \frac{1}{r^2} \right) \sin\theta \end{vmatrix}$

$\vec{E} = \frac{Ql}{4\pi\epsilon} e^{-jkr} \left[ \vec{j}_r \left( \frac{jk}{r^2} + \frac{1}{r^3} \right) 2\cos\theta + \vec{j}_\theta \left( -\frac{k^2}{r} + \frac{jk}{r^2} + \frac{1}{r^3} \right) \sin\theta \right]$

PRIMERJAVA

	f=50MHz	f=900MHz	lambda=0,5um
k	10^6 rd/m	193rd/m	1,3 * 10^7 rd/m
k^2	10^12 rd^2/m^2	380rd^2/m^2	1,7 * 10^14 rd^2/m^2
r = l/k	1000km	5,3cm	80nm

TOČKASTI STAT. DIPOLO

SEVANJE

TOČKASTI STAT. DIPOLO

$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^*$

$k = \omega \sqrt{\mu \epsilon}$

$Z_0 = \sqrt{\frac{\mu}{\epsilon}}$

$\frac{1}{\omega \epsilon} = \frac{Z_0}{k}$

$\vec{S} = \frac{1}{2} \frac{I l}{j\omega 4\pi \epsilon} e^{-jkr} \left[ \vec{j}_r \left( \frac{jk}{r^2} + \frac{1}{r^3} \right) 2\cos\theta + \vec{j}_\theta \left( -\frac{k^2}{r} + \frac{jk}{r^2} + \frac{1}{r^3} \right) \sin\theta \right] \times \vec{j}_\phi \frac{I l}{4\pi} e^{+jkr} \left[ -\frac{jk}{r} + \frac{1}{r^2} \right] \sin\theta$

$\vec{S} = \frac{|I l|^2 \epsilon^2 Z_0}{32\pi^2} \left[ \vec{j}_r \left( \frac{k^2}{r^2} - \frac{j}{kr^3} \right) \sin^2\theta + \vec{j}_\theta \left( \frac{j}{kr^3} + \frac{j}{kr^3} \right) 2\cos\theta \sin\theta \right]$

$P = \oint \vec{S} \cdot \vec{j}_r r^2 \sin\theta d\theta d\phi$

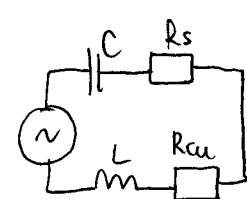
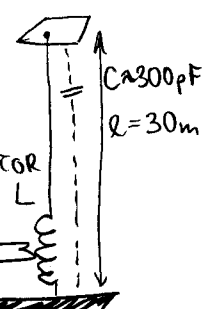
$P = \frac{|I l|^2 \epsilon^2 Z_0}{32\pi^2} \int_0^{2\pi} \int_0^\pi \left( \frac{k^2}{r^2} - \frac{j}{kr^3} \right) \sin^2\theta \sin\theta d\theta = \frac{|I l|^2 \epsilon^2 Z_0 k^2}{16\pi} \int_{-1}^1 (1-u^2) du = \frac{|I l|^2 \epsilon^2 Z_0 k^2}{12\pi} = \frac{1}{2} |I l|^2 R_s$

$R_s = \frac{l^2 Z_0^2}{6\pi} = \frac{2}{3} \pi Z_0 \left( \frac{l}{\lambda} \right)^2$

f=30kHz -> lambda=10km

R\_s approx 7,2m Ohm

TESLOV TRANSFORMATOR ~1900 L



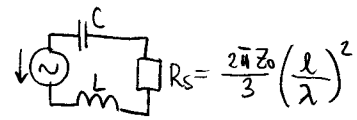
$R_{cu} = \frac{\omega L}{Q} = \frac{1}{\omega C Q} \approx 60 \Omega$

Q approx 300

eta = R\_s / (R\_s + R\_cu) approx 0,012%

$\omega \neq 0$   
 $\mu = \mu_0$   
 $\epsilon = \epsilon_0$   
 $I = j\omega Q$

PONOVI TEV:  
 $\vec{A} = (\vec{r}_r \cos \theta - \vec{r}_\theta \sin \theta) \frac{\mu I l}{4\pi} \frac{e^{-jkr}}{r}$   
 $\vec{H} = \vec{r}_\phi \frac{I l}{4\pi} e^{-jkr} \left( \frac{jk}{r} + \frac{1}{r^2} \right) \sin \theta$   
 $\vec{E} = \frac{Q l}{4\pi \epsilon} e^{-jkr} \left[ \vec{r}_r \left( \frac{jk}{r^2} + \frac{1}{r^3} \right) 2 \cos \theta + \vec{r}_\theta \left( -\frac{jk}{r} + \frac{jk}{r^2} + \frac{1}{r^2} \right) \sin \theta \right]$



POENOSTAVITVE ZA SEVANJE

$r \gg \frac{1}{k} \rightarrow \frac{\partial}{\partial r} \approx -jk$   
 $r \gg \frac{\lambda}{2\pi} \rightarrow \frac{\partial}{\partial \theta} \rightarrow 0$   
 $\frac{\partial}{\partial \phi} \rightarrow 0$

$\text{rot } \vec{A} = \frac{1}{r \sin \theta} \begin{vmatrix} \vec{r}_r & r \vec{r}_\theta & r \sin \theta \vec{r}_\phi \\ -jk & 0 & 0 \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix} = \vec{\nabla} \times \vec{A} = \vec{r}_\theta jk A_\phi - \vec{r}_\phi jk A_\theta$

$\vec{\nabla} = \vec{r}_r (-jk)$      $\omega \mu = k z_0$

$\vec{H} \approx \vec{r}_\phi \frac{jk}{4\pi} I l \frac{e^{jkr}}{r} \sin \theta$      $\vec{E} = \frac{1}{j\omega \epsilon} \vec{\nabla} \times (\frac{1}{\mu} \vec{\nabla} \times \vec{A}) \approx -j\omega (\vec{A} - \vec{r}_r (\vec{r}_r \cdot \vec{A})) = \vec{r}_\theta \frac{jk z_0}{4\pi} I l \frac{e^{jkr}}{r} \sin \theta$

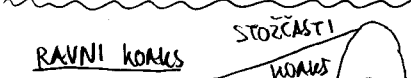
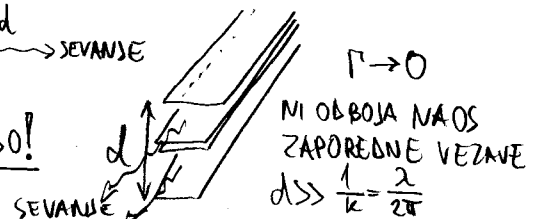
LASTNOSTI SEVANJA TEM  $\vec{H} \perp \vec{r}_r; \vec{E} \perp \vec{r}_r; \vec{E} \perp \vec{H}; |\vec{E}| = z_0 |\vec{H}|; \vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \vec{r}_r \frac{|\vec{E}|^2}{2z_0} = \text{DELOVNA MOČ}$

TRAKASTI DVOVOD

$\epsilon = \epsilon_0$      $\mu = \mu_0$   
 $\omega \neq 0$   
 $\vec{E} = -\vec{r}_r \frac{U}{d} = -\vec{r}_r \frac{U_0}{d} e^{j\beta z}$   
 $\vec{H} = \vec{r}_\phi \frac{I}{w} = \vec{r}_\phi \frac{I_0}{w} e^{j\beta z}$   
 $\vec{E} \perp \vec{r}_r, \vec{H} \perp \vec{r}_r, \vec{E} \perp \vec{H}, |\vec{E}| = z_0 |\vec{H}|$

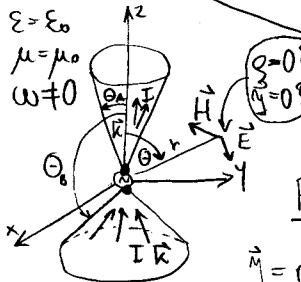


$w, d \gg \frac{1}{k}$   
 NI ODPORA NA OS!  $\Gamma \rightarrow 0$   
 (OS = OBRTE SPONKE)



Ugiban resitev:  $\vec{E} = \vec{r}_\theta \frac{C}{r \sin \theta} e^{-jkr}$

Preverim ③ ME:  $\rho = \text{div}(\epsilon \vec{E}) = \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (r \sin \theta \epsilon \frac{C}{r \sin \theta} e^{-jkr}) = 0 \checkmark$



Izračunam  $\vec{H}$  iz ② ME:

$\vec{H} = \frac{j}{\omega \mu} \text{rot } \vec{E} = \frac{j}{\omega \mu} \frac{1}{r^2 \sin \theta} \begin{vmatrix} \vec{r}_r & r \vec{r}_\theta & r \sin \theta \vec{r}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & r \frac{C}{r \sin \theta} e^{-jkr} & 0 \end{vmatrix} = \vec{r}_\phi \frac{C/z_0}{r \sin \theta} e^{-jkr}$

Preverim ① ME:  $\vec{J} = \text{rot } \vec{H} - j\omega \epsilon \vec{E} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \vec{r}_r & r \vec{r}_\theta & r \sin \theta \vec{r}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & r \sin \theta \frac{C/z_0}{r \sin \theta} e^{-jkr} & 0 \end{vmatrix} - j\omega \epsilon \vec{r}_\theta \frac{C}{r \sin \theta} e^{-jkr} = \vec{r}_\theta \frac{j\omega \epsilon C}{r \sin \theta} e^{-jkr} - \vec{r}_\theta \frac{j\omega \epsilon C}{r \sin \theta} e^{-jkr} = 0 \checkmark$

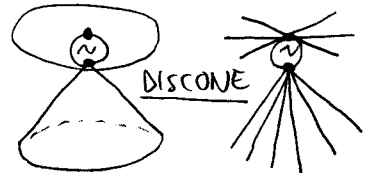
Izračunam tok I:

$d\vec{s} = \vec{r}_\phi r \sin \theta d\phi$   
 $I = \oint \vec{H} \cdot d\vec{s} = \int_0^{2\pi} \vec{r}_\phi \frac{C/z_0}{r \sin \theta} e^{-jkr} \cdot \vec{r}_\phi r \sin \theta d\phi = \frac{2\pi C}{z_0} e^{-jkr}$

$Z_k = \frac{U}{I} = \frac{z_0}{2\pi} \ln \left( \frac{tg(\theta/2)}{tg(\theta/4)} \right) \approx 60 \Omega \ln \left( \frac{tg(\theta/2)}{tg(\theta/4)} \right)$

Izračunam napetost U:

$d\vec{s} = \vec{r}_\theta r d\theta$   
 $U = \int_A^B \vec{E} \cdot d\vec{s} = \int_{\theta_A}^{\theta_B} \vec{r}_\theta \frac{C}{r \sin \theta} e^{-jkr} \cdot \vec{r}_\theta r d\theta = C e^{-jkr} \int_{\theta_A}^{\theta_B} \frac{d\theta}{\sin \theta} = C e^{-jkr} \ln \left( \frac{tg(\theta_B/2)}{tg(\theta_A/2)} \right)$



MASHNA ZANKA (xy)

$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_V \vec{J}(\vec{r}') \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dV' = \frac{\mu}{4\pi} \int_0^{2\pi} \int_0^\pi \vec{r}'_\phi \frac{I}{A z} \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} A z \sin \theta' d\theta' d\phi' = \frac{\mu I a}{4\pi} \int_0^{2\pi} \int_0^\pi \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} d\theta' d\phi'$

$|\vec{r}-\vec{r}'| = \sqrt{(r \sin \theta \cos \phi - a \cos \theta')^2 + (r \sin \theta \sin \phi - a \sin \theta')^2 + (r \cos \theta - a \cos \theta')^2} = \sqrt{r^2 + a^2 - 2ar \sin \theta \cos(\theta - \theta')}$

$|\vec{r}-\vec{r}'| \approx r - a \sin \theta \cos(\theta - \theta')$ ;  $\frac{1}{|\vec{r}-\vec{r}'|} \approx \frac{1}{r} \left( 1 + \frac{a}{r} \sin \theta \cos(\theta - \theta') \right)$ ;  $e^{-jk|\vec{r}-\vec{r}'|} \approx e^{-jkr} \left( 1 + jk a \sin \theta \cos(\theta - \theta') \right)$

①  $a \ll r$   
 ②  $a \ll \frac{1}{k}$

$\vec{A}(\vec{r}) \approx \frac{\mu I a}{4\pi} \frac{e^{-jkr}}{r} \int_0^{2\pi} \int_0^\pi (-\vec{r}'_x \sin \theta' + \vec{r}'_y \cos \theta') \left( 1 + \frac{a}{r} \sin \theta \cos(\theta - \theta') \right) \left( 1 + jk a \sin \theta \cos(\theta - \theta') \right) d\theta' d\phi'$

$\vec{r}'_\phi = -\vec{r}'_x \sin \theta' + \vec{r}'_y \cos \theta'$

$Q=0 \rightarrow \text{grad } V=0$      $\vec{r}_\theta \pi a \sin \theta$      $\omega \mu = \omega \sqrt{\mu \epsilon} \sqrt{\frac{\mu}{\epsilon}} = k z_0$

$\vec{A}(\vec{r}) \approx \vec{r}_\phi \frac{\mu}{4\pi} I \pi a^2 \frac{e^{-jkr}}{r} \left( jk + \frac{1}{r} \right) \sin \theta$      $\vec{E} = -j\omega \vec{A} - \text{grad } V = \vec{r}_\theta \frac{-jk z_0}{4\pi} I a^2 e^{-jkr} \left( \frac{jk}{r} + \frac{1}{r^2} \right) \sin \theta$      $\vec{H} = \frac{j}{\omega \mu} \text{rot } \vec{E}$



# Elektrodinamika #9 3.12.2012

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0$$

## VALOVNA ENAČBA BREZ IZVOROV

$$\omega^2 \mu \epsilon = k^2$$

$$\Delta \vec{E} + \omega^2 \mu \epsilon \vec{E} = 0 \quad (x, y, z) \rightarrow \Delta \vec{E} = \vec{1}_x \Delta E_x + \vec{1}_y \Delta E_y + \vec{1}_z \Delta E_z$$

$$\Delta \vec{H} + \omega^2 \mu \epsilon \vec{H} = 0$$

$$\Delta E_x = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}$$

$$E_x(x, y, z) = X(x) Y(y) Z(z)$$

$$\frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} + \frac{1}{Z} \frac{d^2 Z}{dz^2} + k^2 = 0$$

$$\Delta \vec{E} = \text{grad}(\text{div} \vec{E}) - \text{rot}(\text{rot} \vec{E})$$

STOJNI VAL

ODBITI

MAPREDOJOČI

$$k_z^2 > 0 \rightarrow Z(z) = C_1 \cos k_z z + C_2 \sin k_z z = C_3 e^{jk_z z} + C_4 e^{-jk_z z}$$

$$k_z^2 < 0 \rightarrow Z(z) = C_5 \cosh |k_z| z + C_6 \sinh |k_z| z = C_7 e^{|k_z| z} + C_8 e^{-|k_z| z}$$

exp. usihanje

Zgled: preveri  $\text{div}(\epsilon \vec{E}) = 0$   
 izračunaj  $\vec{H} = \frac{1}{\omega \mu} \text{rot} \vec{E}$   
 preveri  $\vec{\nabla} \cdot \vec{H} - j\omega \epsilon \vec{E} = 0$   
 izračunaj  $\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^*$

## VALOVNI VEKTOR

$$\vec{k} = \vec{1}_x k_x + \vec{1}_y k_y + \vec{1}_z k_z$$

$$\begin{cases} E_x(x, y, z) = E_{x0} e^{-ik_x x} e^{-ik_y y} e^{-ik_z z} \\ E_y(x, y, z) = E_{y0} e^{-ik_x x} e^{-ik_y y} e^{-ik_z z} \\ E_z(x, y, z) = E_{z0} e^{-ik_x x} e^{-ik_y y} e^{-ik_z z} \end{cases}$$

$$\vec{E}(x, y, z) = \vec{E}_0 e^{-j\vec{k} \cdot \vec{r}}$$

$$|\vec{k}| = k = \omega \sqrt{\mu \epsilon}$$

$$\vec{k} = \vec{1}_k \omega \sqrt{\mu \epsilon} = \vec{1}_k k$$

$$\text{div} \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = -jk_x E_x - jk_y E_y - jk_z E_z = -j\vec{k} \cdot \vec{E} \rightarrow \text{FIZICALNA REŠITEV } \rho = 0 \rightarrow \vec{k} \perp \vec{E}$$

$$\text{rot} \vec{E} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\vec{k} \times \vec{E} \rightarrow \vec{H} = \frac{\vec{k} \times \vec{E}}{\omega \mu} = \frac{\vec{1}_k \times \vec{E}}{Z_0} \rightarrow \vec{E} \perp \vec{H}; \vec{H} \perp \vec{k}; |\vec{H}| = \frac{|\vec{E}|}{Z_0}$$

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \vec{E}_{\text{eff}} \times \vec{H}_{\text{eff}}^* = \vec{1}_s \frac{|\vec{E}|^2}{2 Z_0} = \vec{1}_s \frac{|\vec{H}|^2 Z_0}{2}; \vec{1}_s = \vec{1}_k$$

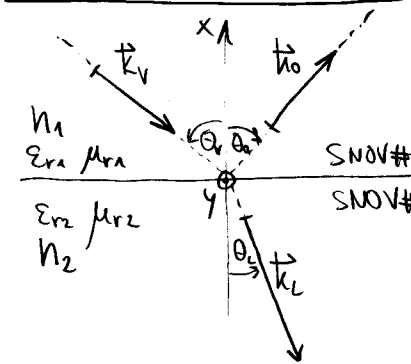
## SNOV $\epsilon_r \neq 1, \mu_r \neq 1$

$$\text{Lomnikoličnik } n = \sqrt{\mu_r \epsilon_r}$$

$$\vec{k} = \vec{1}_k k = \vec{1}_k \omega \sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0} = \vec{1}_k \frac{\omega}{c_0} \sqrt{\mu_r \epsilon_r} = \vec{1}_k \frac{\omega}{c_0} n$$

$$c = \frac{c_0}{n}$$

## ODBOS IN LOM VALOVANJA



$$\vec{k}_V = \vec{1}_x k_{Vx} + \vec{1}_z \beta$$

$$\vec{k}_0 = \vec{1}_x k_{0x} + \vec{1}_z \beta_0$$

$$k_{Vx}^2 + \beta^2 = k_{0x}^2 + \beta_0^2 = k_1^2 = n_1 \omega \sqrt{\mu_0 \epsilon_0}$$

$$\sin \theta_V = \frac{\beta_0}{k_1} \quad \sin \theta_L = \frac{\beta_0}{k_2}$$

isti pojav kjer koli na meji snovi

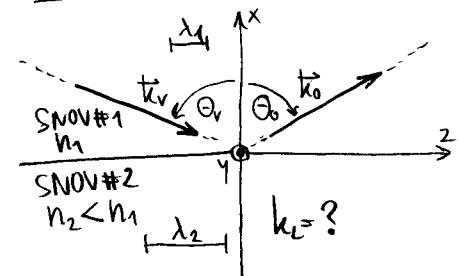
isti  $\beta = k_z$

$$\vec{k}_L = \vec{1}_x k_{Lx} + \vec{1}_z \beta$$

$$k_{Lx}^2 + \beta^2 = k_2^2 = n_2 \omega \sqrt{\mu_0 \epsilon_0}$$

$$\sin \theta_L = \frac{\beta_0}{k_2} \rightarrow n_2 \sin \theta_L = n_1 \sin \theta_V \text{ Snell-ov lomnikoličnik}$$

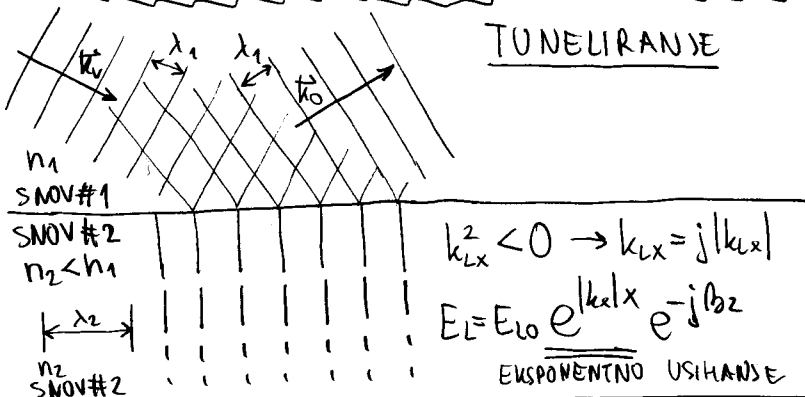
## POPOLNI ODBOS



$$\theta_L = \arcsin\left(\frac{n_1}{n_2} \sin \theta_V\right)$$

$$\frac{n_1}{n_2} \sin \theta_V > 1?$$

## TUNELIRANJE



$$k_{Lx}^2 < 0 \rightarrow k_{Lx} = j|k_{Lx}|$$

$$E_L = E_{L0} e^{|k_{Lx}| x} e^{-j\beta z}$$

EKSPONENTNO USIHANJE

SNOV #1  $n_1$

TUNELIRANJE VALOVANJA  $\vec{k}_T = \vec{k}_V$

# Elektrodinamika #10 10.12.2012

Ponovitev  $\Delta \vec{E} + k^2 \vec{E} = 0 \rightarrow \vec{E} = \vec{E}_0 e^{i\vec{k} \cdot \vec{r}}$ ,  $\vec{k} \perp \vec{E}$ ,  $\vec{E} \perp \vec{H}$ ,  $\vec{H} \perp \vec{k}$ ,  $\frac{|\vec{E}|}{|\vec{H}|} = Z_0 = \sqrt{\frac{\mu}{\epsilon}}$   
 POTUJOČI VAL

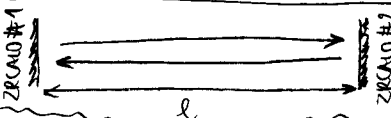
## 1D STOJNI VAL

$\vec{E} = \vec{E}_x C \cos kz = \vec{E}_x \frac{C}{2} (e^{ikz} + e^{-ikz})$   
 STAJNI VAL

$\vec{H} = \frac{1}{\omega \mu} \text{rot} \vec{E} = \frac{1}{\omega \mu} \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 0 \end{vmatrix} = \vec{e}_y \frac{-ikC}{\omega \mu} \sin kz = \vec{e}_y \frac{1}{Z_0} C \sin kz$   
 ODBITNI VAL    NAPREDOJOČI VAL

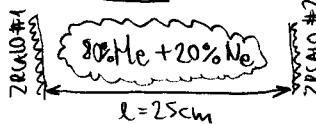
$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \vec{e}_z \frac{1}{2} \frac{|C|^2}{Z_0} \cos kz \sin kz$   
 JALOVA MOČ

## FABRY-PEROT-OV REZONATOR

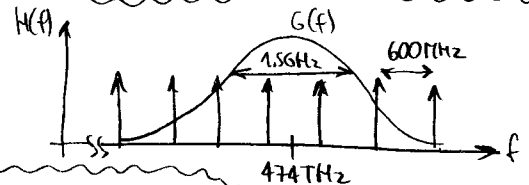


$kl = m\pi$      $f = \frac{c_0}{2l} m$ ,  $m=1,2,3,4,...$   
 $k = \frac{2\pi f}{c_0}$     TEM<sub>00m</sub> PRIMERJAVNA STRUJNO!

## HeNe laser



$f_0 \approx 474 \text{ THz}$      $c_0 = 3 \cdot 10^8 \text{ m/s}$   
 Doppler  $\pm 1.5 \cdot 10^{-6}$  (toplotno gibanje  $v \approx 450 \text{ m/s}$ )  
 $\Delta f = \frac{c_0}{2l} = 600 \text{ MHz}$

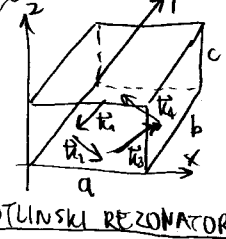
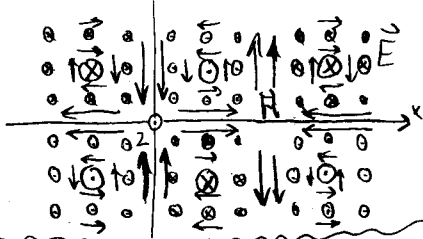


## 2D STOJNI VAL

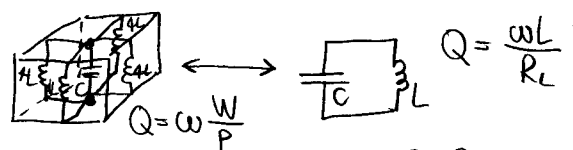
$\vec{E} = \vec{E}_x C \sin k_x x \sin k_y y = \vec{E}_x \frac{C}{4} (e^{ik_x x} - e^{-ik_x x})(e^{ik_y y} - e^{-ik_y y}) = \vec{E}_x \frac{C}{4} [e^{i(k_x x + k_y y)} - e^{i(-k_x x + k_y y)} - e^{i(k_x x - k_y y)} + e^{i(-k_x x - k_y y)}]$

$\vec{H} = \frac{1}{\omega \mu} \text{rot} \vec{E} = \frac{1}{\omega \mu} \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & C \sin k_x x \sin k_y y \end{vmatrix} = \vec{e}_x \frac{jk_y}{\omega \mu} C \sin k_x x \cos k_y y - \vec{e}_y \frac{jk_x}{\omega \mu} C \cos k_x x \sin k_y y$

$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \text{jalov!}$   
 kvadratura  $\vec{E}, \vec{H}$



TM<sub>110</sub>:  $f = \frac{c_0}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$



TM<sub>mno</sub>  $\rightarrow f_{mn} = \frac{c_0}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$     Q  $\uparrow \Rightarrow m, n = \text{LASER visokm}$

3D STOJNI VAL  $f_{lmn} = \frac{c_0}{2} \sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2}$   
 8 valovnih veljivosti

## PREPREČEVANJE VIŠJIM ROBOV

$C < \frac{a}{2}, \frac{b}{2}$

## DIELEKTRIK

$\sqrt{\epsilon_r} = n$

$f_{lmn} = \frac{c_0}{2n} \sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2} = \frac{c_0}{2\sqrt{\epsilon_r}} \sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2}$

## 1D STOJNI (y) + 1D POTUJOČI (z)

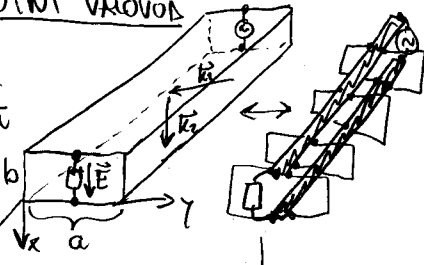
$\vec{E} = \vec{E}_x C \sin k_y y e^{-\beta z}$   
 $k_y^2 + \beta^2 = k^2 = \omega^2 \mu \epsilon$

$\vec{H} = \frac{1}{\omega \mu} \text{rot} \vec{E} = \frac{1}{\omega \mu} \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & C \sin k_y y e^{-\beta z} & 0 \end{vmatrix} = \vec{e}_y \frac{\beta C}{\omega \mu} \sin k_y y e^{-\beta z} - \vec{e}_z \frac{jk_y}{\omega \mu} C \cos k_y y e^{-\beta z}$

$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \frac{1}{2} \vec{e}_x C \sin k_y y e^{-\beta z} \times \left[ \vec{e}_y \frac{\beta C}{\omega \mu} \sin k_y y e^{-\beta z} + \vec{e}_z \frac{jk_y}{\omega \mu} C \cos k_y y e^{-\beta z} \right] = \vec{e}_x \frac{\beta |C|^2}{2 \omega \mu} \sin^2 k_y y - \vec{e}_z \frac{jk_y |C|^2}{\omega \mu} \sin k_y y \cos k_y y$

## PRAVOKOTNI VLOVON

TE<sub>01</sub>  
 $k_y = \frac{\pi}{a}$



TE<sub>01</sub>  $\rightarrow \beta_z = \sqrt{\omega^2 \mu \epsilon - \left(\frac{\pi}{a}\right)^2}$

(TE<sub>0m</sub>  $\rightarrow \beta_z = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$ )

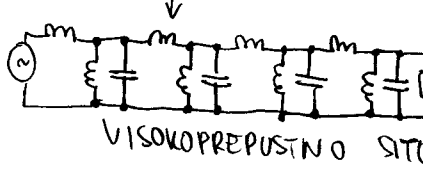
$\vec{E} = \vec{E}_x C \sin k_y y e^{-\beta z}$

$\omega^2 \mu \epsilon > \left(\frac{\pi}{a}\right)^2 \rightarrow \beta_z = \text{realen (potujoči val)}$   
 $\omega^2 \mu \epsilon < \left(\frac{\pi}{a}\right)^2 \rightarrow \beta_z = \text{imaginaren (exponentno uhanje)}$

$\omega^2 \mu \epsilon = \left(\frac{\pi}{a}\right)^2$

$\omega_c = \frac{\pi c_0}{a} \rightarrow f_c = \frac{c_0}{2a}$

$\vec{E} = \vec{E}_x C \sin k_y y e^{-\beta z}$

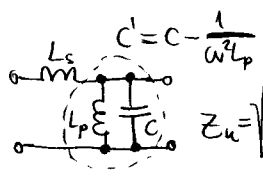
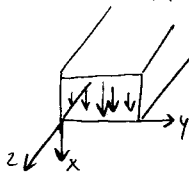


# Elektrodinamika #11 17.12.2012

## PRAVOKOTNI VALOVOD

$$\vec{E} = \vec{1}_x C \sin ky e^{-\beta z}$$

$$k_y^2 + \beta^2 = k^2 = \omega^2 \mu \epsilon \rightarrow \beta = \sqrt{\omega^2 \mu \epsilon - k_y^2} = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$



$$C' = C - \frac{1}{\omega L_p}$$

$$Z_u = \sqrt{\frac{L_s}{C'}} = \sqrt{\frac{L_s}{C - \frac{1}{\omega L_p}}}$$

$$\omega_u = \frac{1}{\sqrt{\mu \epsilon}} \left(\frac{m\pi}{a}\right) = m \frac{\pi c_0}{a}$$

$\beta = \text{realen}$   $\omega > \omega_u \rightarrow Z_u = \text{realen} = \text{preput}$

$\beta = \text{imaginaran}$   $\omega < \omega_u \rightarrow Z_u = \text{imaginaran} = \text{zapora}$

## FAZNA HITROST $\vec{E} = \text{Re}[\vec{1}_x C \sin ky e^{i(\omega t - \beta z)}$

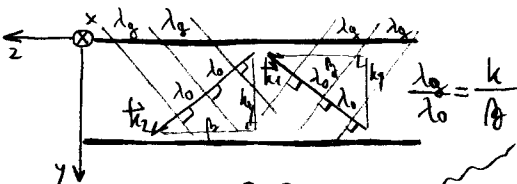
$$\varphi = \omega t - \beta z = \text{konst} \rightarrow \frac{d\varphi}{dt} = 0$$

$$\omega - \beta \frac{dz}{dt} = 0 \rightarrow v_f = \frac{dz}{dt} = \frac{\omega}{\beta}$$

$$v_f = \frac{\omega}{\beta} = \frac{c_0}{\sqrt{1 - \frac{k_y^2}{\omega^2 \mu \epsilon}}} = \frac{c_0}{\sqrt{1 - \left(\frac{\omega_u}{\omega}\right)^2}} \geq c_0$$

### VALOVNA VALOVNA DOŽINA

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\omega^2 \mu \epsilon - k_y^2}} = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\omega_u}{\omega}\right)^2}} \geq \lambda_0$$



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \text{Re}[\vec{1}_x \sin ky (C_1 e^{i\varphi_1} + C_2 e^{i\varphi_2})] \quad \Delta\varphi = \varphi_1 - \varphi_2$$

## SKUPINSKA HITROST

Vsota  $\vec{E}$  dveh frekvenc  $\omega_1, \omega_2$

$$\vec{E}_1 = \text{Re}[\vec{1}_x \sin ky e^{i(\omega_1 t - \beta_1 z)}] = \text{Re}[\vec{1}_x C_1 \sin ky e^{i\varphi_1}]$$

$$\vec{E}_2 = \text{Re}[\vec{1}_x \sin ky e^{i(\omega_2 t - \beta_2 z)}] = \text{Re}[\vec{1}_x C_2 \sin ky e^{i\varphi_2}]$$

$$\varphi_1 = \omega_1 t - \beta_1 z$$

$$\varphi_2 = \omega_2 t - \beta_2 z$$

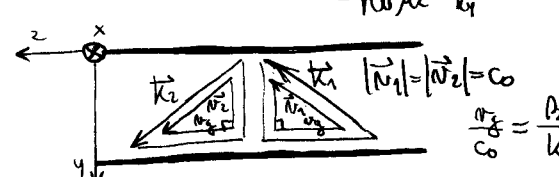
$$\vec{E} = \vec{1}_x \sin ky \text{Re}\left[e^{i\frac{\varphi_1 + \varphi_2}{2}} (C_1 e^{i\frac{\varphi_1 - \varphi_2}{2}} + C_2 e^{-i\frac{\varphi_1 - \varphi_2}{2}})\right]$$

amplituda  $|\vec{E}| = \sin ky |C_1 e^{i\Delta\varphi} + C_2 e^{-i\Delta\varphi}|$

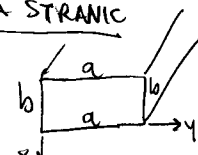
ovojnico sledim  $\Delta\varphi = \text{konst} = \Delta\omega t - \Delta\beta z \rightarrow \frac{d\Delta\varphi}{dt} = 0$

$$\frac{dz}{dt} = v_g = \frac{\Delta\omega}{\Delta\beta} = \frac{d\omega}{d\beta} \quad \boxed{v_g \geq c_0 \geq v_{g0}}$$

$$v_g = \frac{d\omega}{d\beta} = \frac{1}{2} \frac{2\omega \mu \epsilon}{\sqrt{\omega^2 \mu \epsilon - k_y^2}} = \frac{1}{c_0 \sqrt{1 - \left(\frac{\omega_u}{\omega}\right)^2}} \rightarrow v_g = c_0 \sqrt{1 - \left(\frac{\omega_u}{\omega}\right)^2} \leq c_0$$

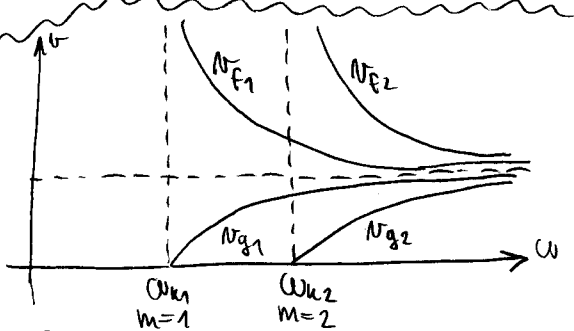


## IZBIRA STRANIC

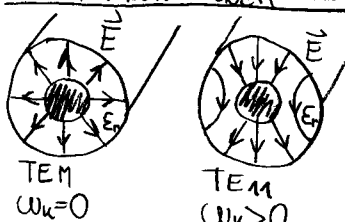


$$b \leq \frac{a}{2} \Leftrightarrow \omega_{kTE10} \geq \omega_{kTE02}$$

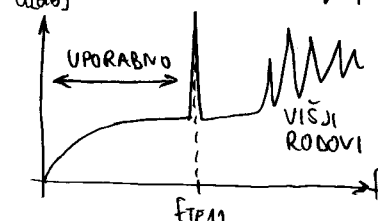
$$\omega_{kTE02} = 2 \omega_{kTE01}$$



## ROBNOVI V KOAKSIALNEM KABLU



$$f_{kTEM} \approx \frac{c_0}{\pi(R_2^2 + R_0^2)\sqrt{\epsilon_r}}$$



## VOTLINSKI REZONATOR (R,P,Z)

$$\Delta \vec{E} + k^2 \vec{E} = 0 \quad \vec{1}_z \cdot \Delta \vec{E} = \Delta E_z$$

$$\Delta E_z = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial E_z}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 E_z}{\partial \varphi^2} + \frac{\partial^2 E_z}{\partial z^2}$$

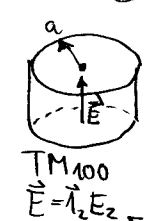
$$E_z(\rho, \varphi, z) = R(\rho) F(\varphi) Z(z)$$

$$\frac{1}{\rho R} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial R}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 F}{\partial \varphi^2} + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} + k^2 = 0$$

$$\text{Zgled: } \frac{\partial}{\partial \varphi} = \frac{\partial}{\partial z} = 0 \quad -m^2 \quad \pm l^2$$

$$\frac{1}{\rho R} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial R}{\partial \rho} \right) + k^2 = 0 \rightarrow R(\rho) = J_0(k\rho)$$

$$k a = 2.405 \rightarrow f_{100} = \frac{2.405 c_0}{2\pi a} = \frac{114.8 \text{ MHz} \cdot \text{m}}{a}$$



## DIELEKTRIČNI REZONATOR

$$\epsilon_r \gg 1 \rightarrow \vec{H}(\rho, a) \approx 0$$

$$\vec{H} = \vec{1}_z C J_0(k\rho)$$

$$TE_{100} \quad \Delta \vec{H} + k^2 \vec{H} = 0$$

$$f_{100} \approx \frac{2.405 c_0}{2\pi a \sqrt{\epsilon_r}}$$

$$f_{100} \approx \frac{114.8 \text{ MHz} \cdot \text{m}}{a \sqrt{\epsilon_r}}$$

# Elektrodinamika #12 7/1/2013

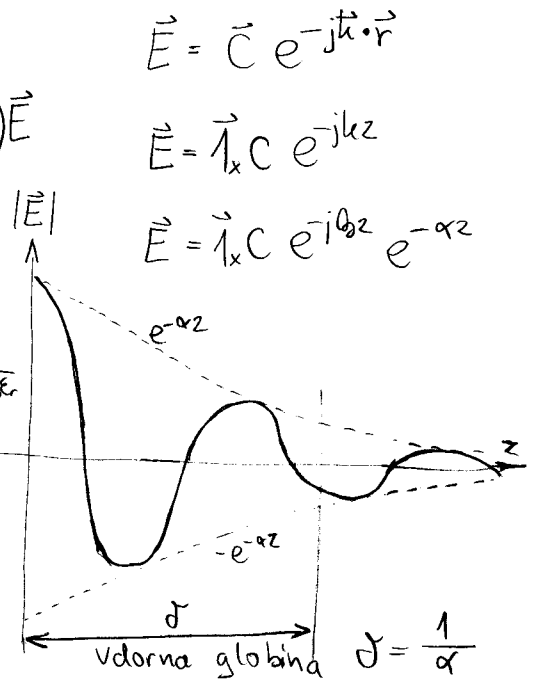
## VALOVANJE V IZGOBNI SNOVI

① ME:  $\text{rot } \vec{H} = \vec{j} + j\omega \epsilon \vec{E} = (\gamma + j\omega \epsilon_0 \epsilon_r) \vec{E} = j\omega \epsilon_0 \left( \epsilon_r - \frac{j\gamma}{\omega \epsilon_0} \right) \vec{E}$   
 $\vec{j} = \gamma \vec{E} \quad \epsilon = \epsilon_0 \epsilon_r$

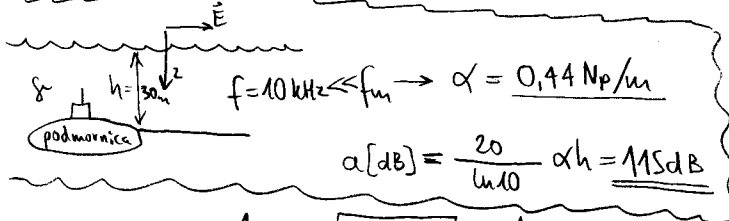
$\Delta \vec{E} + k^2 \vec{E} = 0 \rightarrow k = \omega \sqrt{\mu \epsilon_0 \left( \epsilon_r - \frac{j\gamma}{\omega \epsilon_0} \right)} = \beta - j\alpha$

baker $\gamma = 56 \cdot 10^6 \text{ S/m}$ $\epsilon_r = 1$	morška voda $\gamma = 5 \text{ S/m}$ $\epsilon_r = 80$	SiO <sub>2</sub> steklo $\epsilon_r = 3,9$
$f_m \approx 10^{18} \text{ Hz}$	$f_m = 1,13 \cdot 10^9 \text{ Hz}$	$f_m = 3 \cdot 10^{10} \text{ Hz}$
dober prevodnik $\alpha = \beta$	vmesni primer $0 < \alpha < \beta$	dober dielektrik $\alpha \rightarrow 0$

$f_m = \frac{\gamma}{2\pi \epsilon_0 \epsilon_r}$



Dober prevodnik  $\frac{\gamma}{\omega \epsilon_0} \gg \epsilon_r \quad k = \omega \sqrt{\mu \left( -\frac{j\gamma}{\omega} \right)} = \sqrt{-j} \sqrt{\omega \mu \gamma} = \beta - j\alpha \rightarrow \alpha = \beta = \sqrt{\frac{\omega \mu \gamma}{2}}$



Kovina  $\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \mu \gamma}} = \frac{1}{\beta}$

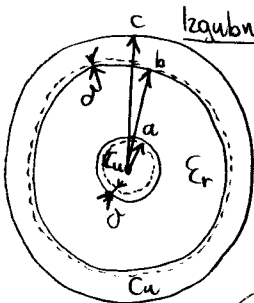
$\vec{K} = \int_0^\infty \vec{j} dz = \gamma \int_0^\infty \vec{E}_0 e^{j\beta z} e^{-\alpha z} dz = \frac{\gamma \vec{E}_0}{(\alpha + j\beta)}$

$\vec{E}_0 = Z_p \vec{K} \rightarrow Z_p = R_p + jX_p = \sqrt{\frac{\omega \mu}{2\gamma}} + j \sqrt{\frac{\omega \mu}{2\gamma}}$

f	$\delta_{Cu}$	$\delta_{Fe}$
1Hz	67mm	6,7mm
100Hz	6,7mm	0,67mm
10kHz	0,67mm	67µm
1MHz	67µm	6,7µm
100MHz	6,7µm	0,67µm
10GHz	0,67µm	67nm

$\omega=0 \quad R = \frac{\rho}{\pi r^2}$

$\omega>0 \quad R_w = \frac{\rho}{2\pi r} R_p = \frac{\rho}{2\pi r} \sqrt{\frac{\omega \mu}{2\gamma}}$



izračun koeficientnega kabla

$L/l = \frac{\mu_0}{2\pi} \ln \frac{b}{a}$

$C/l = \frac{2\pi \epsilon_0 \epsilon_r}{\ln \frac{b}{a}}$

$Z_k = \sqrt{\frac{L/l + \frac{R/l}{j\omega} + \frac{jX/l}{\omega}}{C/l}} \approx \sqrt{\frac{L/l}{C/l}} = \frac{Z_0}{\sqrt{\epsilon_r}} \ln \frac{b}{a}$

$k = \beta - j\alpha = \sqrt{\omega \left( L/l + \frac{R/l}{j\omega} + \frac{jX/l}{\omega} \right) \omega C/l} \approx \omega \sqrt{\left( L/l + \frac{R/l}{j\omega} \right) C/l}$

$\beta \approx \omega \sqrt{L/l C/l} \quad \alpha \approx \frac{R/l}{2Z_k}$

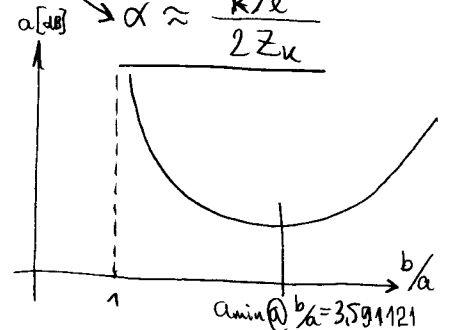
$a[db]/l = \frac{20}{\ln 10} \alpha = \frac{10}{\ln 10} \frac{R/l}{Z_k} = \frac{10}{\ln 10} \frac{\sqrt{\frac{\omega \mu}{2\gamma}}}{Z_0 \sqrt{\epsilon_r}} \frac{1}{b} \ln \frac{b}{a}$

$R/l = R_s/l + R_0/l = \frac{R_p}{2\pi a} + \frac{R_p}{2\pi b} = \frac{R_p}{2\pi} \left( \frac{1}{a} + \frac{1}{b} \right)$

$R_p = \sqrt{\frac{\omega \mu}{2\gamma}} = 5,94 \text{ m}\Omega$   
 $f = 500 \text{ MHz}, \gamma = 56 \cdot 10^6 \text{ S/m}$

$a = 1 \text{ mm}$   
 $b = 3,5 \text{ mm}$   
 $\epsilon_r = 2$

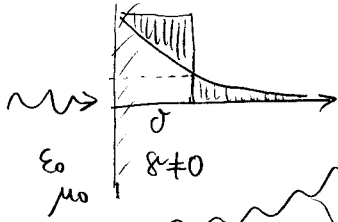
$Z_k = 53,2 \Omega \quad a[db]/l = \frac{10}{\ln 10} \frac{5,94 \text{ m}\Omega}{377 \Omega} \sqrt{2} \frac{1}{3,5 \cdot 10^{-3} \text{ m}} \frac{3,5+1}{\ln 3,5} = 0,0993 \text{ dB/m} = 99,3 \text{ dB/km}$



Ponovitev: DOBER PREVODNIK  $\sigma \gg \omega \epsilon$  (kovine  $f < \text{rentgen}$ )

$$J = \sqrt{\frac{2}{\omega \mu \sigma}}$$

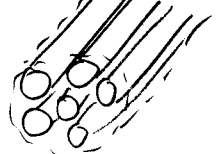
$$R_p = R_o = \sqrt{\frac{\omega \mu}{2 \sigma}}$$



$$R = \rho \frac{l}{A} = \frac{l}{2\pi r \delta \sigma} = \frac{l}{2\pi r} R_p$$

okrogel vodnik  $\rightarrow$  VF pletenica

$$Q = \frac{\omega l}{R} < 100$$



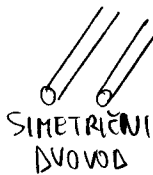
Ponovitev:



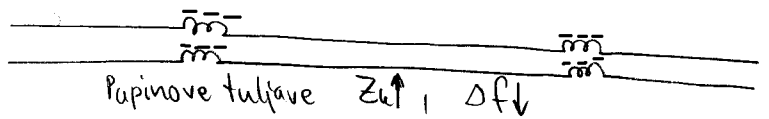
$$\alpha/l = \frac{10}{\ln 10} \frac{R/l}{Z_0}$$

$$\alpha/l = \frac{10}{\ln 10} \frac{\sqrt{\epsilon_r} R_p}{Z_0} \frac{1}{b} \left( \frac{b/a + 1}{\ln b/a} \right)$$

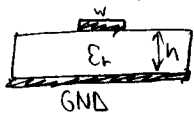
$\min @ b/a = 3,591121 \approx 3,6$   
 $\alpha/l \approx 0,1 \text{ dB/m @ } 500 \text{ MHz}$



$Z_0 \approx (2 \dots 6) Z_{\text{koaks}}$   
 $R/l \approx 2 \times (R/l)_{\text{koaks}}$

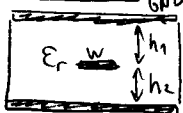


**MIKROSTRIP**



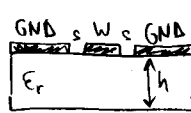
MIKROTRANASTI

**STRIPLINE**

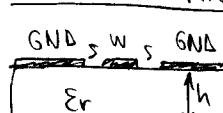


TRAKASTI

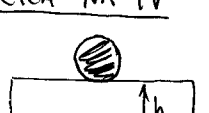
**KOPLANARNI**



**KOPLANARNI Z MASO**



**ZICA NA TV**



GND

**MIKROSTRIP BREZ STRESANJA**

zelo grob približek (TEM)



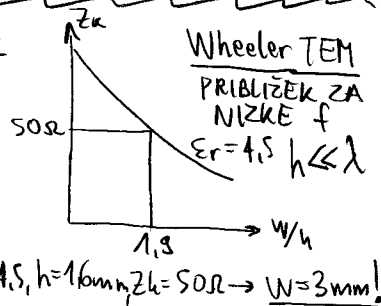
$$l/\lambda = \mu_0 \frac{h}{w}$$

$$c/l = \epsilon_0 \epsilon_r \frac{w}{h}$$

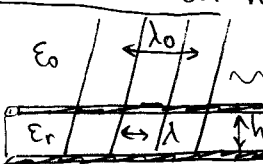
$$Z_0 \approx \frac{Z_0}{\sqrt{\epsilon_r}} \frac{h}{w}$$

$\epsilon_r = 4,5, h = 1,6 \text{ mm}, Z_0 = 50 \Omega \rightarrow w = 6 \text{ mm}?$

**MIKROSTRIP S STRESANJEM**

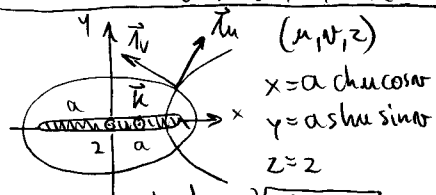


**HIBRIDNI ROBOVI**



$E_z \neq 0$  TOČNA PRIZKUPNA  
 $H_z \neq 0$  SLIKA  
 $U = \text{nedefiniran}$   
 $Z_0 = \text{nedefiniran}$

**OSAMLJEN KOVINSKI TRAK**

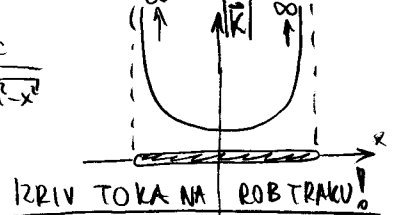


Ista inducirani  $\vec{E} \rightarrow \vec{A} = \vec{A}_2 A_2(\omega)$

$$\vec{H} = \frac{1}{\mu_0} \text{rot } \vec{A} = \frac{1}{\mu_0} \frac{\partial A_2}{\partial x} \hat{y} - \frac{\partial A_2}{\partial z} \hat{z}$$

$$\vec{H} = \frac{1}{\mu_0} \frac{1}{\sqrt{a^2 - x^2}} \frac{\partial A_2}{\partial x} \hat{y} - \frac{\partial A_2}{\partial z} \hat{z}$$

TRAK:  $M=0 \rightarrow \vec{H} = \vec{H}_0 \frac{c}{a \sin \alpha} = \vec{H}_0 \frac{c}{\sqrt{a^2 - x^2}}$   
 $\cos \alpha = \frac{x}{a} \quad \sin \alpha = \sqrt{1 - (x/a)^2}$   
 $\vec{k} = \vec{k}_0 \frac{2|\vec{H}|}{\sqrt{a^2 - x^2}}$



IZRIV TOKA NA ROB TRAKU!

**IZGUBE MIKROTRAKA STEGA VODA**

- 1) IZRIV TOKA NA POVRŠINO  $\rightarrow \delta!$
- 2) IZRIV TOKA NA ROB TRAKU!
- 3) HRPAVOST BAKRA ZA LEPLJENJE!

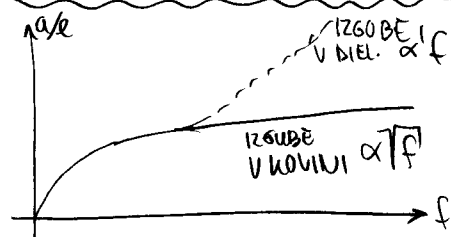
$(\alpha/l)_{\text{mikrostrip}} > 10 \times (\alpha/l)_{\text{koaks}}$  (isti dielektrik, izmere, f)

**IZGUBE V BAKRU / DIELEKTRIKU**

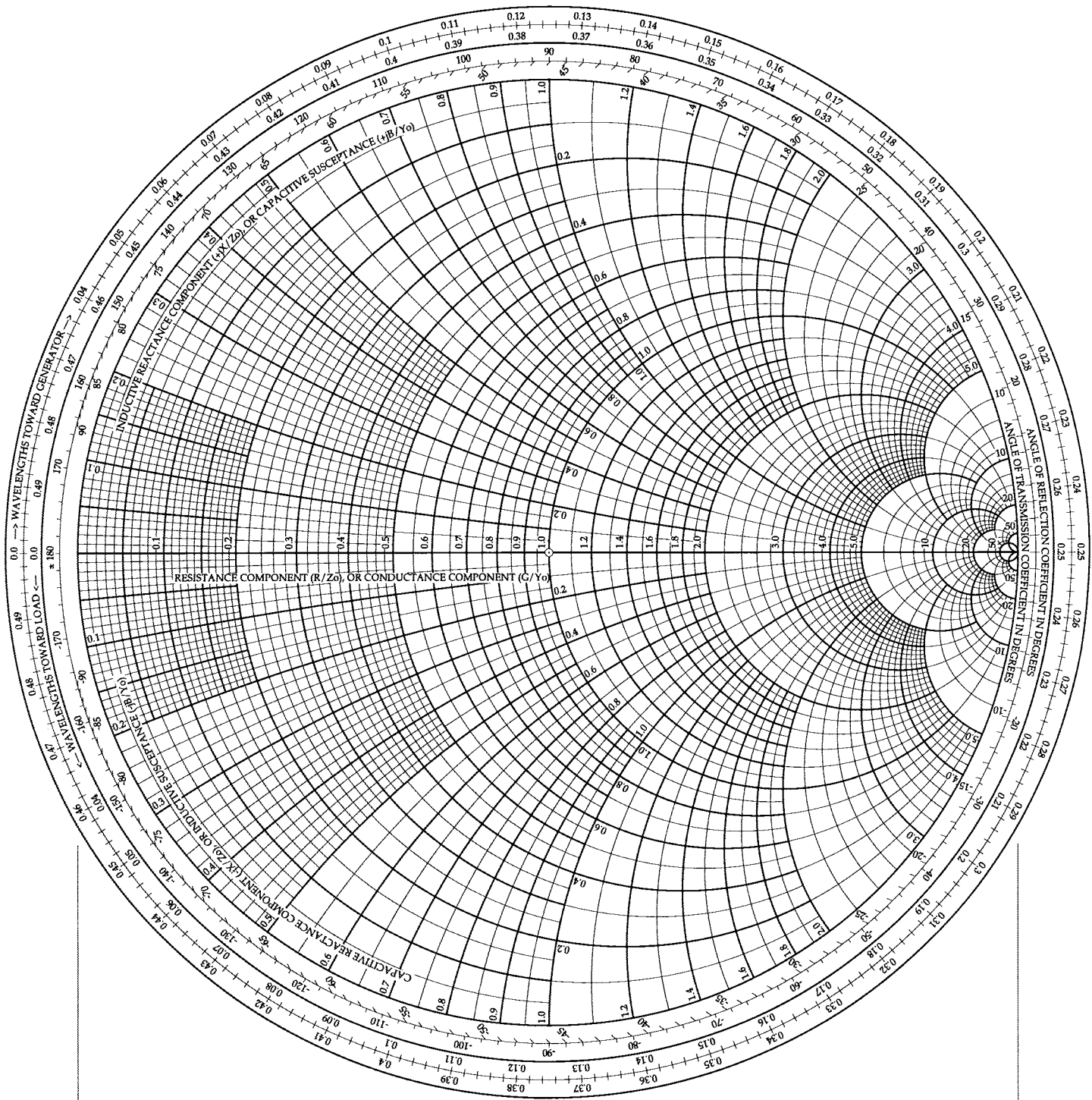
KOVINA:  $R_p = \sqrt{\frac{\omega \mu}{2 \sigma}} = \alpha \sqrt{f}$

**DIELEKTRIK Z UMAZNANJO:**

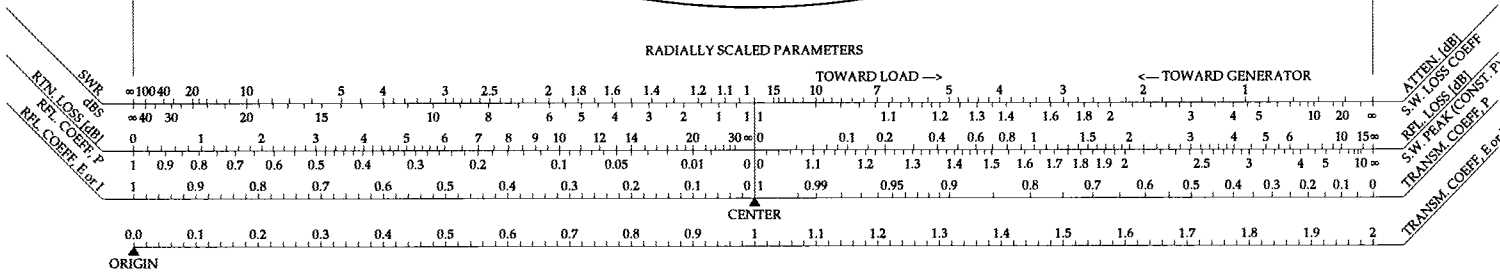
$\frac{1}{R} \rightarrow \alpha \sqrt{f}$



# Smith-ov diagram: impedanca/admitanca v merilu odbojnosti

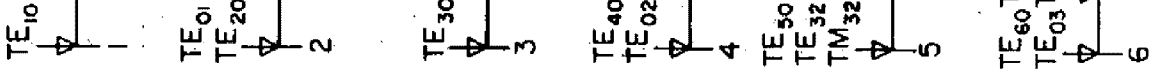
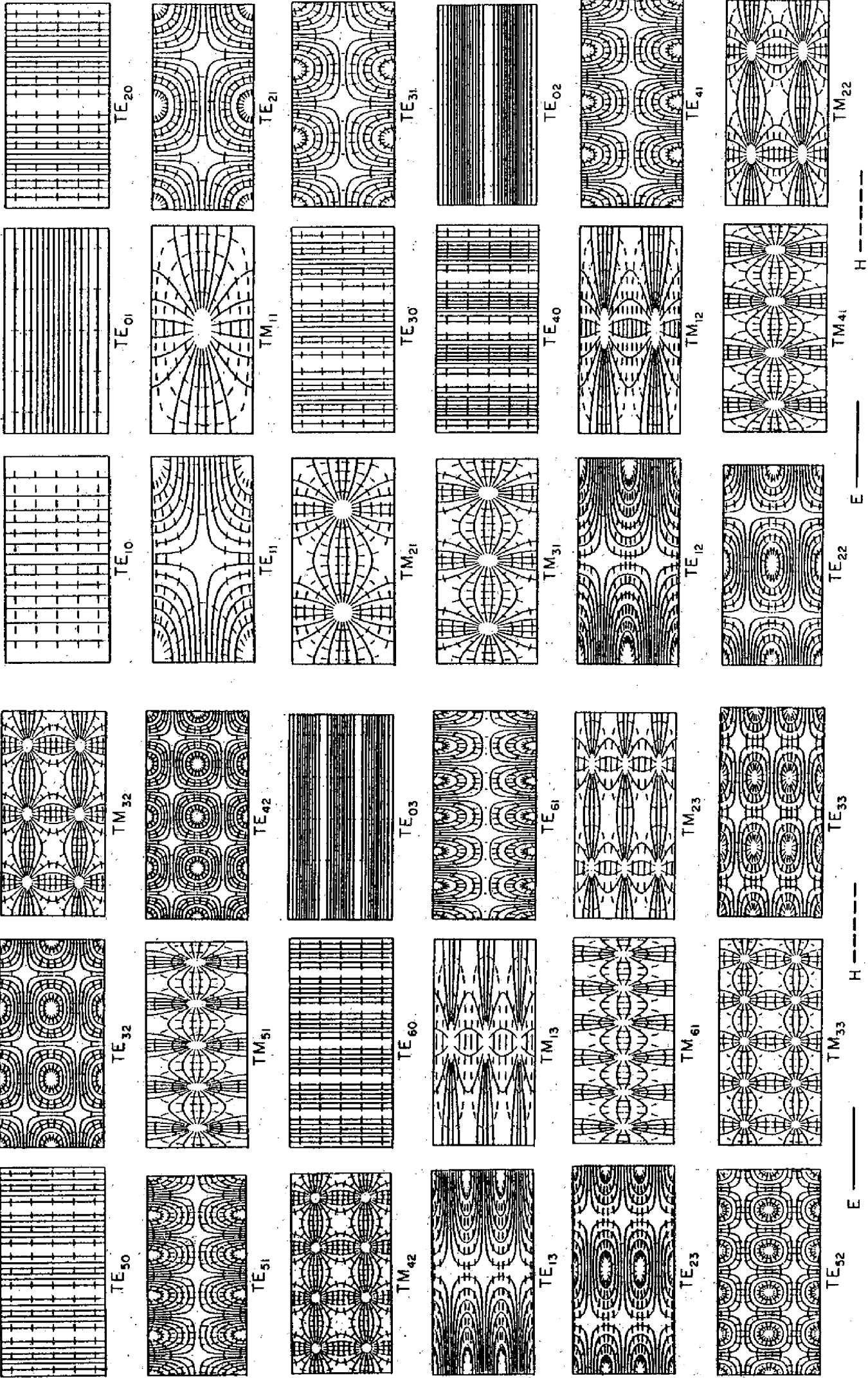
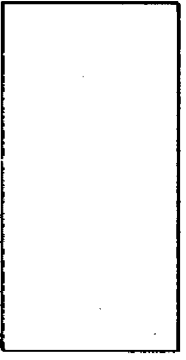


## RADIALLY SCALED PARAMETERS

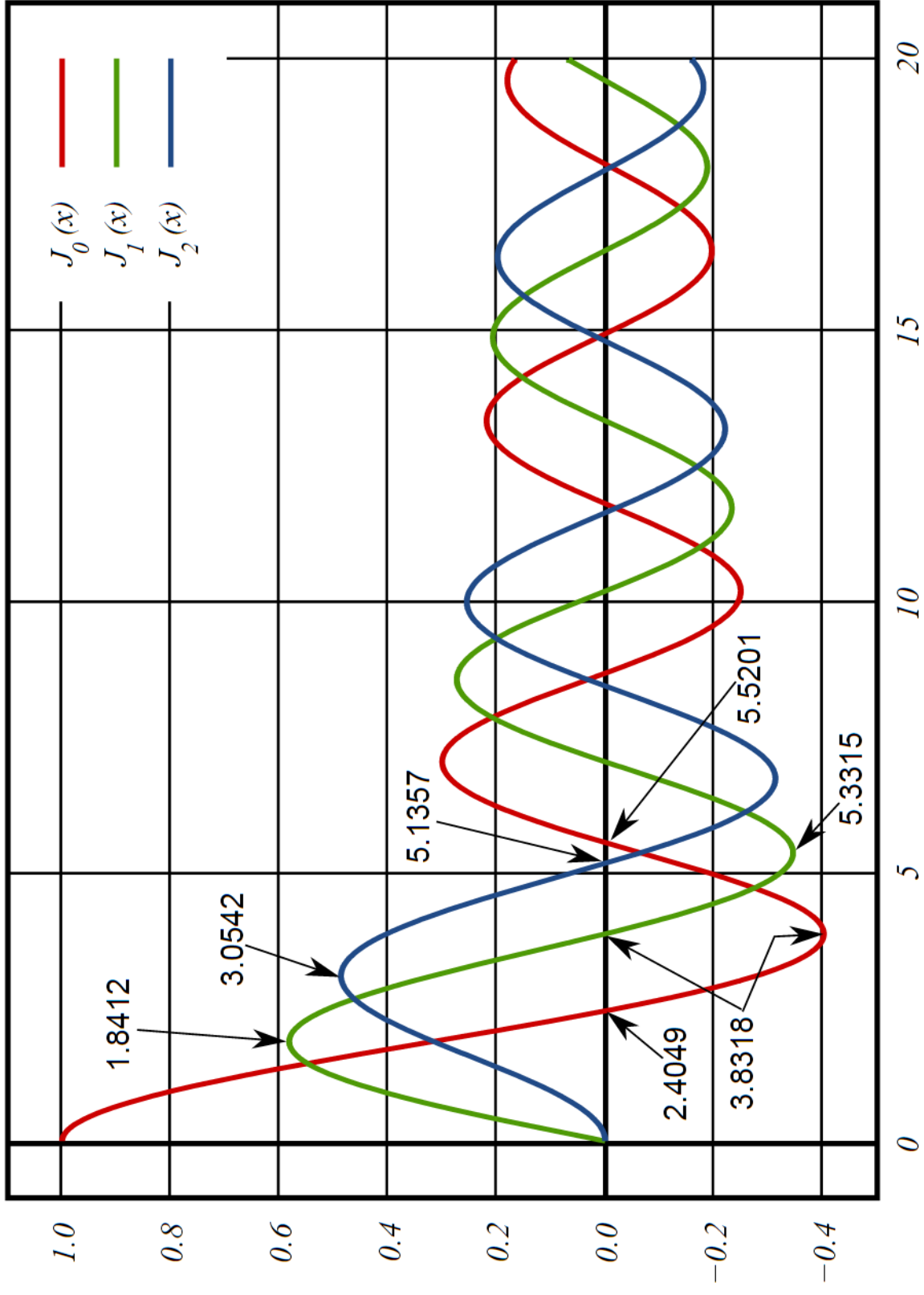


# Pravokotni valovod

$a/2$

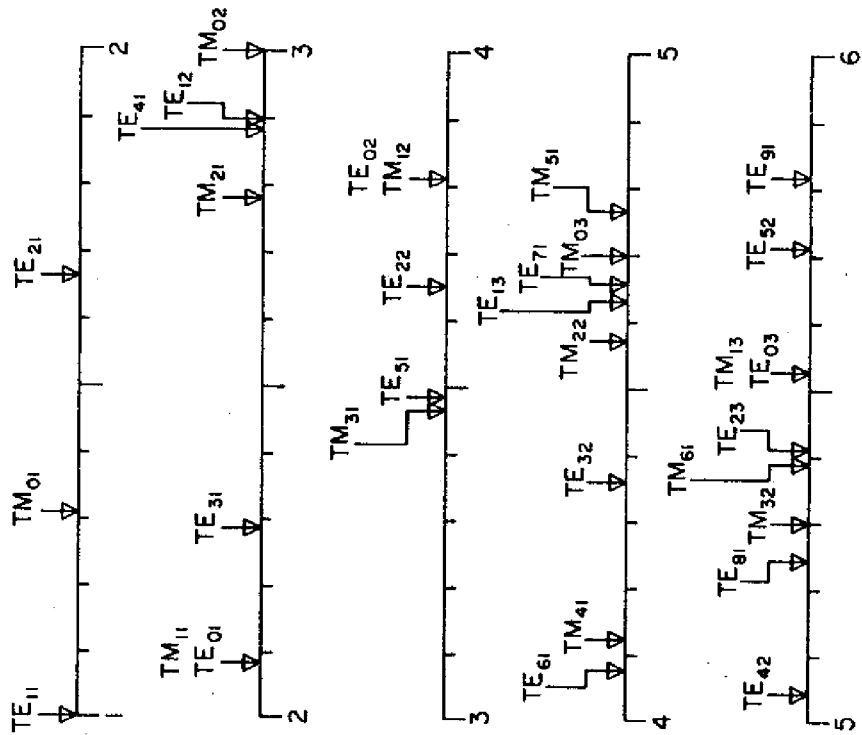
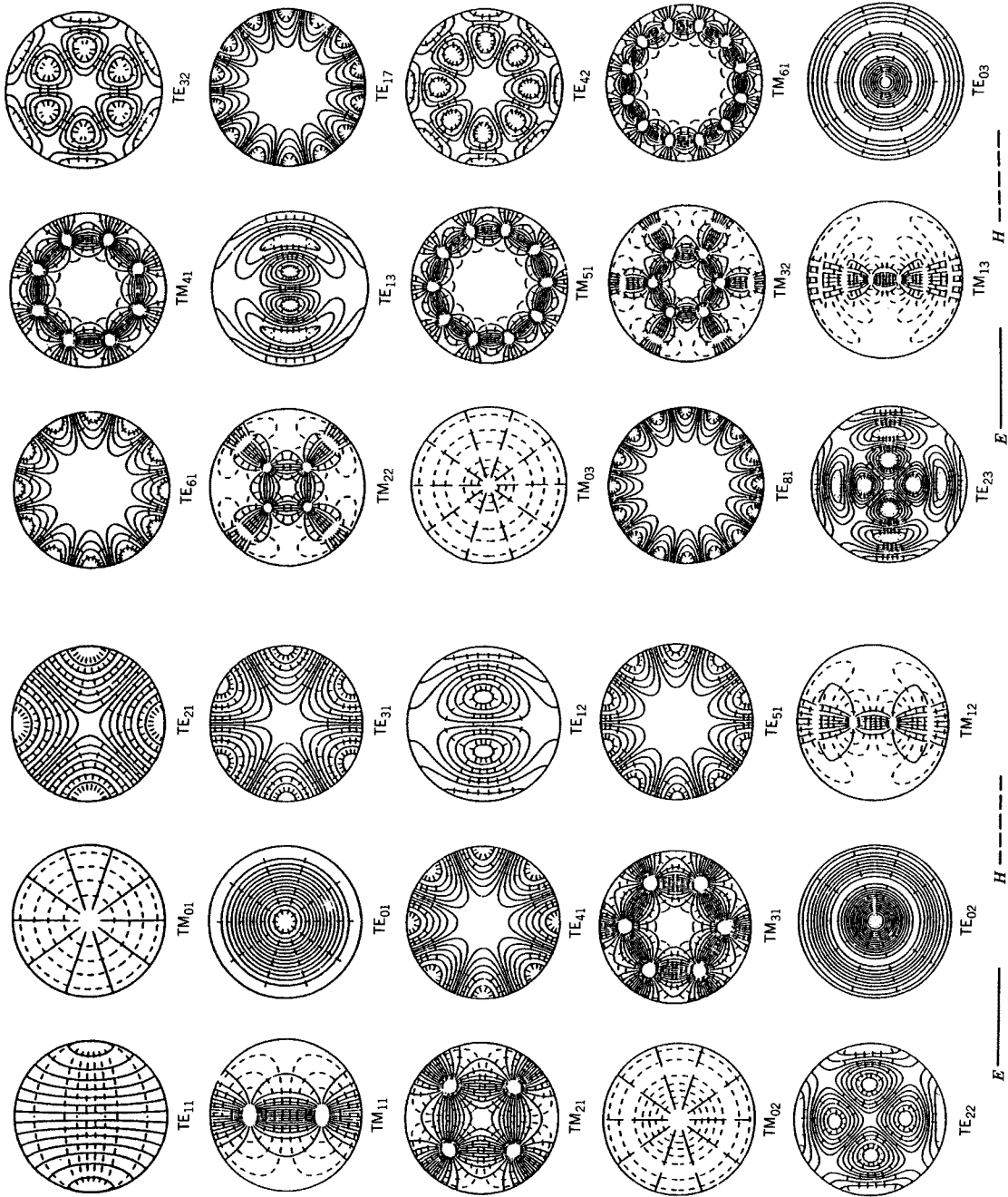
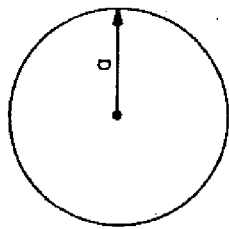


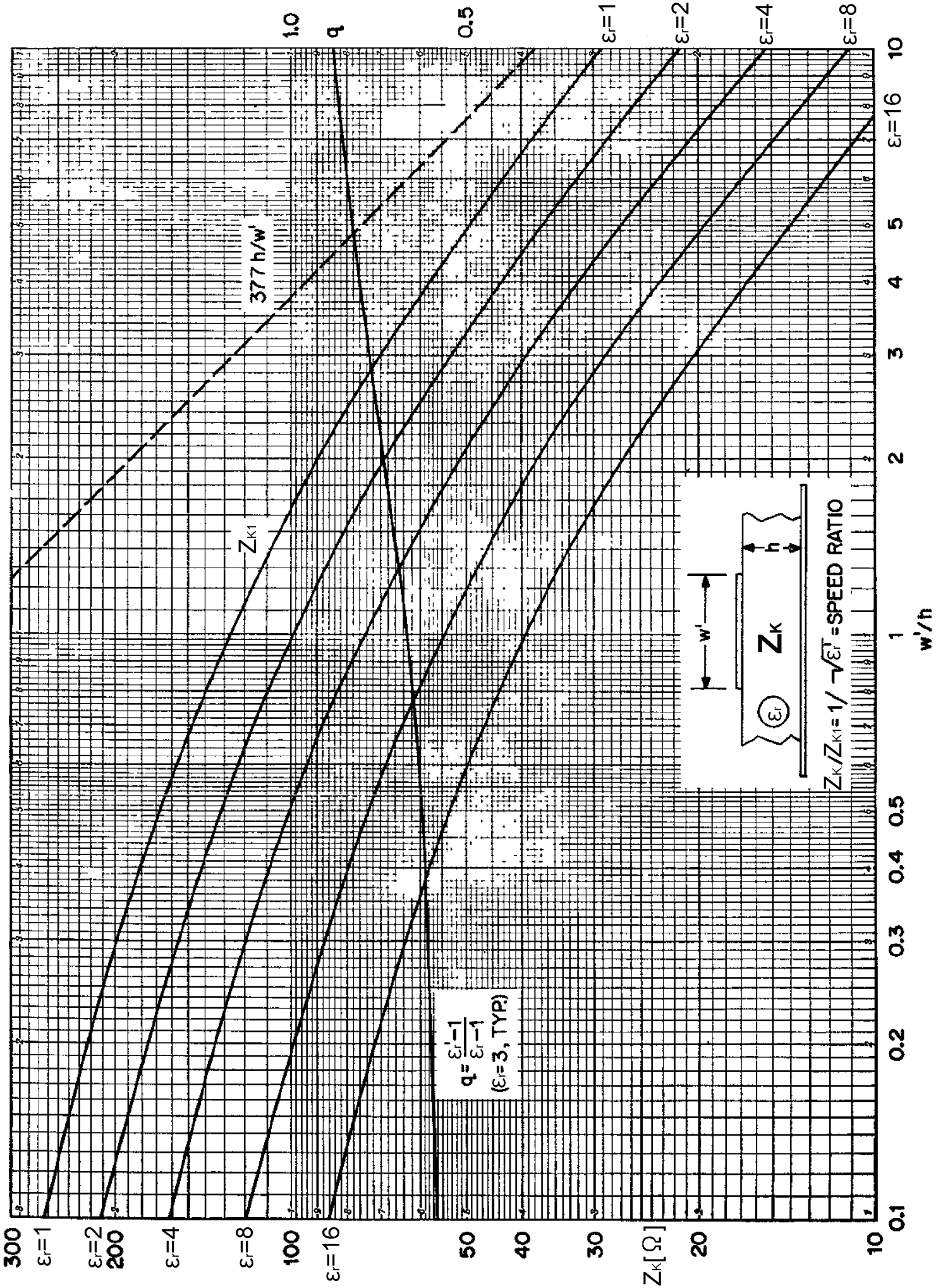
# Bessel-ove funkcije





# Krožni valovod





Karakteristična impedanca mikrotrakastega voda (TEM približek Wheeler 1977)