

Antene in razširjanje valov #1 1/10/2013

Fizikalne veličine:

- merske enote
- skalarji in vektorji

Skalarni produkt:

$$W[\text{J}] = \vec{F}[\text{N}] \cdot \vec{s}[\text{m}]$$

$$W = |\vec{F}| |\vec{s}| \cos \alpha$$

Vektorski produkt:

$$\vec{\omega}[\text{m/s}] = \vec{\omega}[\text{rd/s}] \times \vec{r}[\text{m}]$$

$$|\vec{\omega}| = |\vec{\omega}| |\vec{r}| \sin \alpha$$

smerni $\vec{\omega}$?

Koordinatni sistem:

- 1) 3D
- 2) PRAVOKOTNI $\vec{i}_x \cdot \vec{i}_y = 0$
- 3) DESNOROČNI $\vec{i}_x \times \vec{i}_y = \vec{i}_z$

Kartezijski KS (x, y, z):

$$\vec{r} = x\vec{i}_x + y\vec{i}_y + z\vec{i}_z$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

$$\vec{r} \cdot \vec{s} = r_x s_x + r_y s_y + r_z s_z$$

$$\vec{\omega} \times \vec{r} = \begin{vmatrix} \vec{i}_x & \vec{i}_y & \vec{i}_z \\ \omega_x & \omega_y & \omega_z \\ r_x & r_y & r_z \end{vmatrix}$$

Krogelni KS (r, θ, φ):

$$0 \leq r[\text{m}] < \infty$$

$$0 \leq \theta[\text{rad}] \leq \pi$$

$$0 \leq \phi[\text{rad}] < 2\pi$$

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

$$h_r = 1$$

$$h_\theta = r$$

$$h_\phi = r \sin \theta$$

$$\textcircled{1} l_1 = h_\theta \Delta \theta ; h_\theta = r = 111 \text{ km/}^\circ$$

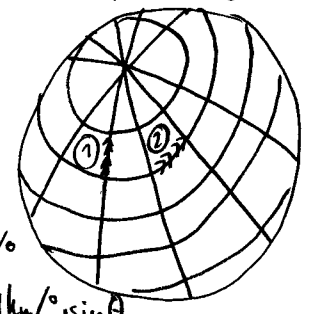
$$\textcircled{2} l_2 = h_\phi \Delta \phi ; h_\phi = r \sin \theta = 111 \text{ km/}^\circ \cdot \sin \theta$$

Krivočrtni KS (q₁, q₂, q₃):

$$dl_i = h_i dq_i$$

$$h_i = \sqrt{\left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2}$$

Zemljepis:
λ = φ, φ = π/2 - θ



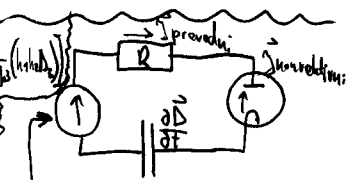
Gauss:



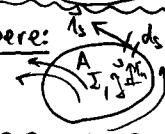
$$\oint_A \vec{D} \cdot \vec{n} dA = Q = \int_V \rho dv$$

Izvorovost

$$\text{div } \vec{D} = \lim_{r \rightarrow 0} \frac{\oint_A \vec{D} \cdot \vec{n} dA}{V} \Rightarrow \text{div } \vec{D} = \rho$$



Ampere:

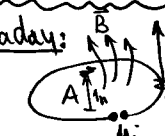


$$\oint_S \vec{H} \cdot \vec{l} ds = I = \int_A \vec{j} \cdot \vec{n} dA$$

Vrtinčenost

$$\text{rot } \vec{H} = \vec{j} = \int_{\text{prevodni}} \vec{j} + \int_{\text{konvekivni}} \frac{\partial \vec{D}}{\partial t}$$

Faraday:



$$\oint_S \vec{E} \cdot \vec{l} ds = - \frac{d\Phi}{dt} = - \frac{\partial}{\partial t} \int_A \vec{B} \cdot \vec{n} dA$$

rot E = - dB/dt

$$\text{rot } \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

ME:

- 1) $\text{rot } \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} = \vec{j} + j\omega \vec{D}$
- 2) $\text{rot } \vec{E} = - \frac{\partial \vec{B}}{\partial t} = -j\omega \vec{B}$
- 3) $\text{div } \vec{D} = \rho$

SNOV / VAKUUM (μ₀, ε₀, γ=0)

$$\vec{D} = \epsilon \vec{E} = \epsilon_r \epsilon_0 \vec{E}$$

$$\vec{B} = \mu \vec{H} = \mu_r \mu_0 \vec{H}$$

$$\vec{j} = \gamma \vec{E}$$

- 1) $\text{rot } \vec{H} = \vec{j} + j\omega \epsilon \vec{E}$
- 2) $\text{rot } \vec{E} = -j\omega \mu \vec{H}$
- 3) $\text{div} (\epsilon \vec{E}) = \rho$

Antenska naloga
znani $\vec{j}(\vec{r}), \rho(\vec{r})$
 $\vec{E}(\vec{r}) = ? \vec{H}(\vec{r}) = ?$

Potenciali:

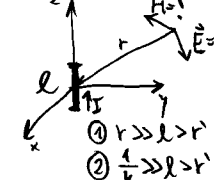
$$\vec{B} = \text{rot } \vec{A} \quad \text{ME + Lorentz}$$

$$\vec{E} = -j\omega \vec{A} - \text{grad } V$$

$$\Delta V + \omega^2 \mu \epsilon V = - \frac{\rho}{\epsilon} \rightarrow V(\vec{r}) = \frac{1}{4\pi \epsilon} \int_V \rho(\vec{r}') \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} d\omega'$$

$$\Delta \vec{A} + \omega^2 \mu \epsilon \vec{A} = -\mu \vec{j} \rightarrow \vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_V \vec{j}(\vec{r}') \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} d\omega'$$

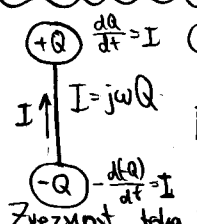
Sevanje žice:



$$\vec{A} = \frac{\mu}{4\pi} \int_V \frac{\vec{j}}{|\vec{r}-\vec{r}'|} d\omega' = \vec{i}_z \frac{\mu I l}{4\pi} \frac{e^{-jk r}}{r} = (\vec{i}_r \cos \theta - \vec{i}_\theta \sin \theta) \frac{\mu I l}{4\pi} \frac{e^{-jk r}}{r}$$

$$\vec{H} = \frac{1}{\mu} \text{rot } \vec{A} = \frac{1}{\mu r^2 \sin \theta} \begin{vmatrix} \vec{i}_r & r \vec{i}_\theta & r \sin \theta \vec{i}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & 0 \end{vmatrix} = \frac{1}{4\pi} \frac{I l}{r} e^{-jk r} \left(\frac{1}{r} + jk \right) \sin \theta$$

f	k	r
50kHz	10 ³ rad/m	1000km
900kHz	20 rad/m	3cm
600kHz	10 ³ rad/m	~0,1μm



$$\vec{E} = \frac{1}{j\omega \epsilon} \text{rot } \vec{H} \approx \vec{i}_\theta \frac{1}{4\pi} \frac{I l}{r} \frac{e^{-jk r}}{r} \sin \theta$$

SEVANJE RIOT SVARST

SAMO SEVANJE:

$$\frac{\partial}{\partial r} \approx -jk, \frac{\partial}{\partial \theta} \approx \frac{j}{r}$$

Poynting: $\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \vec{i}_r k^2 Z_0 \frac{I l^2 r^2}{2(4\pi)^2} \frac{\sin^2 \theta}{r^2}$

Testov transformator
R_{cu} ≈ 60Ω, R_s = 6mΩ
f = 30kHz, l = 30m
P_s = 1W, P_{cu} = 10kW

$$P_s = \int_0^{2\pi} \int_0^\pi \vec{S} \cdot \vec{i}_r r^2 \sin \theta d\theta d\phi = \frac{I l^2 k^2 Z_0}{12\pi}$$

Ponovitev: sevanje moga el. dipola $l \ll \lambda$

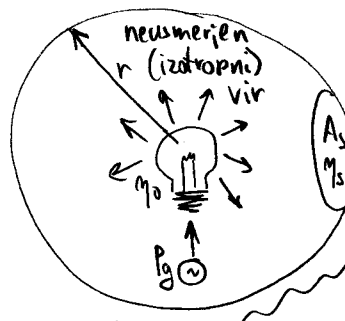
$$\vec{E} = \vec{1}_\theta \frac{j k z_0 I l e^{i k r}}{4 \pi r} \sin \theta$$

$$\vec{H} = \vec{1}_\phi \frac{j k I l e^{i k r}}{4 \pi r} \sin \theta$$

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \vec{1}_r \frac{k z_0 I l^2 \sin^2 \theta}{32 \pi^2 r^2}$$

$$P_s = \oint \vec{S} \cdot \vec{1}_n dA = \frac{k z_0 I l^2}{12 \pi} = \frac{1}{2} I l^2 R_s$$

$$R_s = \frac{k z_0 l^2}{6 \pi} = \frac{2 \pi z_0}{3} \left(\frac{l}{\lambda}\right)^2$$



$$\vec{S} = \vec{1}_r \frac{P_0}{4 \pi r^2} = \frac{P_0 \gamma_0}{4 \pi r^2}$$

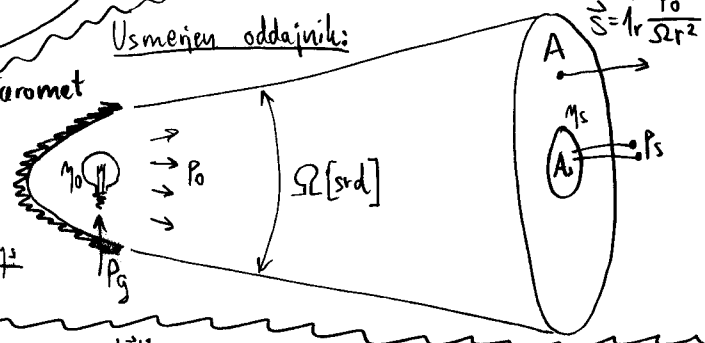
$$\Omega = 4 \pi$$

$$A = 4 \pi r^2$$

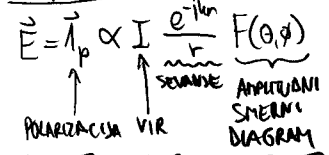
$$P_s = \int_{A_s} \vec{S} \cdot \vec{1}_n dA = |\vec{S}| A_s \gamma_s = \frac{P_0 \gamma_0 A_s \gamma_s}{4 \pi r^2}$$

Smernost (Directivity)

$$D = \frac{|\vec{S}|}{|\vec{S}_i|} = \frac{4 \pi}{\Omega} \geq 1 \quad P_s = \frac{P_0 D \gamma_0 A_s \gamma_s}{4 \pi r^2}$$



Poljuben oddajnik:



$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \vec{1}_r \frac{|\vec{E}|^2}{2 z_0}$$

$$D = \frac{|\vec{S}|}{|\vec{S}_i|} = \frac{4 \pi r^2 \frac{|\vec{E}|^2}{2 z_0}}{\int \frac{|\vec{E}|^2}{2 z_0} r^2 d\Omega} = \frac{4 \pi |F(\theta_{max}, \phi_{max})|^2}{\int_0^\pi \int_0^{2\pi} |F(\theta, \phi)|^2 \sin \theta d\theta d\phi}$$

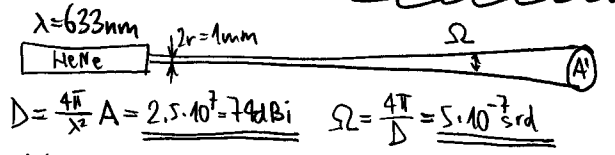
Zgled:

$$F(\theta, \phi) = \sin \theta$$

$$D = 1.5$$

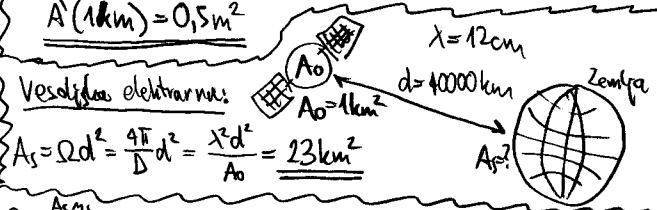
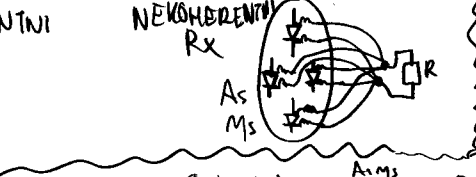
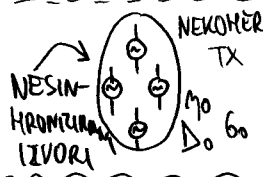
Dobitek (Gain): $G = \eta D$

Log. enote: $G [dBi] = 10 \log_{10} G [lin]$, $D [dBi] = 10 \log_{10} D [lin]$



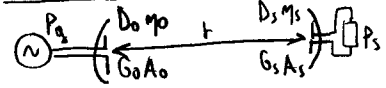
$$D = \frac{4 \pi}{\Omega} A = 2.5 \cdot 10^7 = 74 dBi$$

$$\Omega = \frac{4 \pi}{D} = 5 \cdot 10^{-7} \text{ srad}$$

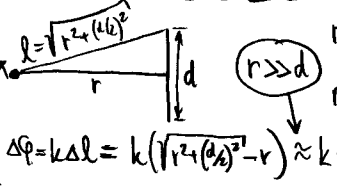
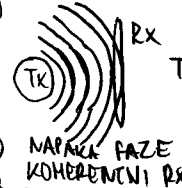
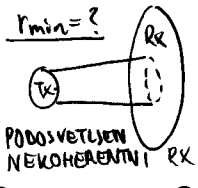


$$A_s = \Omega d^2 = \frac{4 \pi}{D} d^2 = \frac{\lambda^2 d^2}{A_0} = 23 \text{ km}^2$$

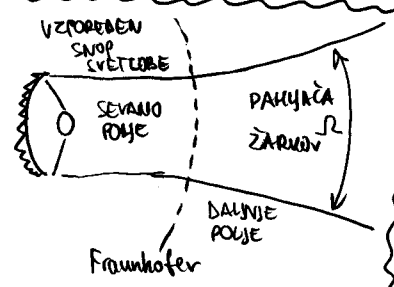
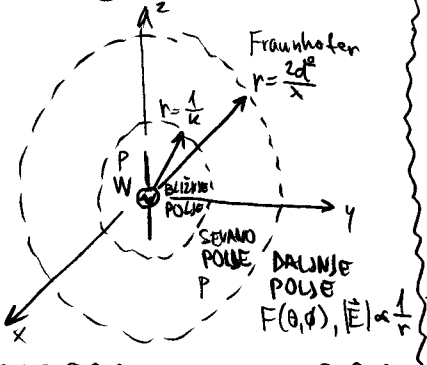
Radialna koherentna zveza:



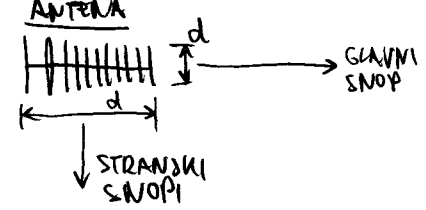
- $P_s = P_0 D_0 \gamma_0 \frac{A_s \gamma_s}{4 \pi r^2} = P_0 G_0 \frac{A_s \gamma_s}{4 \pi r^2} \rightarrow$ RADIODIFUZIJA ~ NEODVISNO OD λ
- $P_s = P_0 D_0 \gamma_0 D_s \gamma_s \left(\frac{\lambda}{4 \pi r}\right)^2 = P_0 G_0 G_s \left(\frac{\lambda}{4 \pi r}\right)^2 \rightarrow$ ZGODOVINA, TELEFONJA λ^2
- $P_s = P_0 \frac{A_0 \gamma_0 A_s \gamma_s}{r^2 \lambda^2} \rightarrow$ CILJANJE? \rightarrow TOČKA-TOČKA $\sim \lambda^{-2}$ $r > r_{min}$



$r_{min} = \frac{k d^2}{2 \Delta \phi}$	$\Delta \phi$	ΔG	r_{min} ccb	$\frac{d}{\lambda} \gg 1$
$r_{min} = \frac{\pi d^2}{4 \Delta \phi}$	π	-4dB	$d^2 / 4 \lambda$	globuska ostrina
	$\pi/2$	-1dB	$d^2 / 2 \lambda$	$\lambda = 0.5 \mu m, d = 1 \mu m \rightarrow r_{min} = 0.5 \mu m$
	$\pi/4$	-0.25dB	d^2 / λ	točne! Fraunhofer meritve anten
	$\pi/8$	-0.0625dB	$2 d^2 / \lambda$	$\lambda = 3 \text{ cm}, d = 1 \text{ cm} \rightarrow r_{min} = 0.8 \text{ m}$



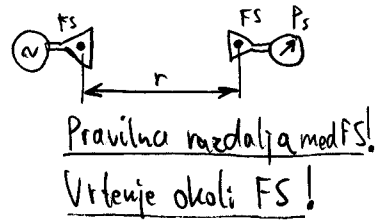
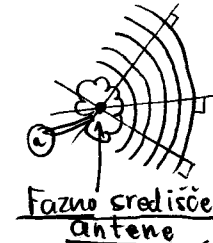
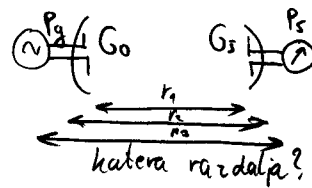
Definicija d: PRECNO NASMER SEVANJA



Ponovitev

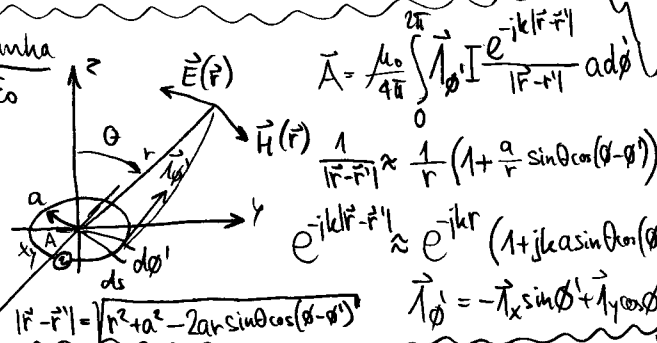
KOM. ZVEZA: $P_s = P_g G_o G_s \left(\frac{\lambda}{4\pi r}\right)^2$

Fraunhofer: $r > r_{min} = \frac{2d^2}{\lambda}$
(antene fokusirane v ∞)



Majhna zanka

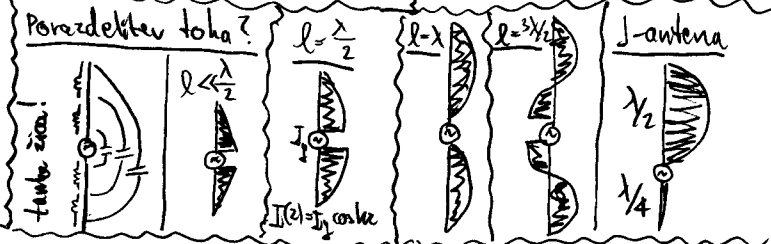
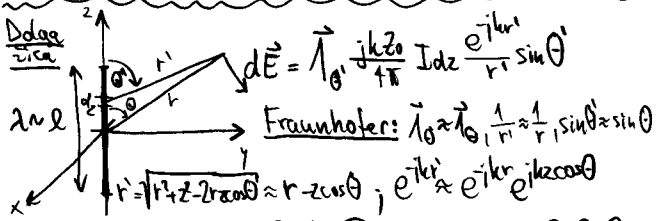
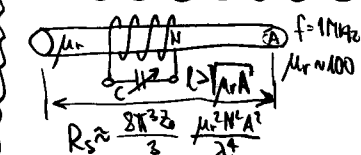
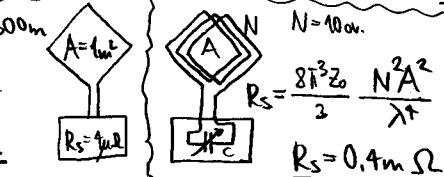
- ① $a \ll r$
- ② $a \ll \lambda$



$F(\theta, \phi) = A(\theta, \phi) e^{j\psi(\theta, \phi)}$
Re konst!

Nekatere antene nimajo FS?

$\vec{S} = \vec{r} \frac{|\vec{E}|^2}{2Z_0} = \vec{r} \frac{k^4 Z_0}{32\pi^2} |I|^2 A^2 \frac{\sin^2 \theta}{r^2}$
 $R_s = \frac{P}{\frac{1}{2}|I|^2} = \frac{k^4 Z_0}{6\pi} A^2 = \frac{8\pi^3 Z_0}{3} \frac{A^2}{\lambda^4} = \frac{8\pi^3 Z_0}{3} \left(\frac{a}{\lambda}\right)^4$

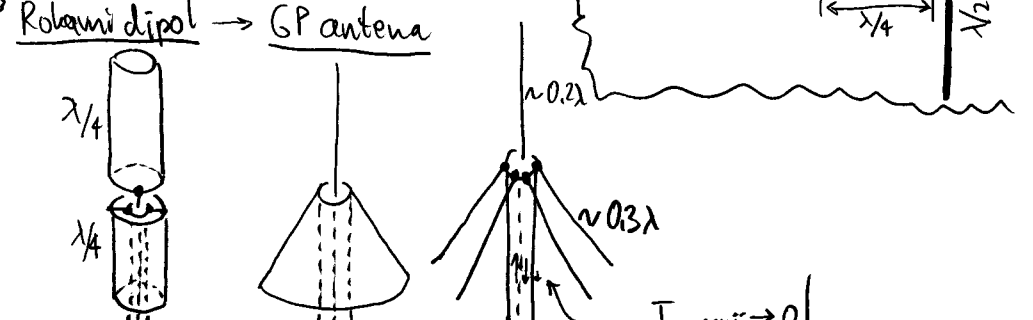
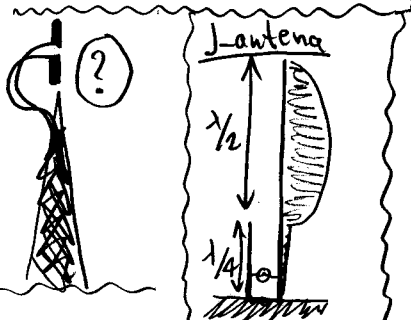
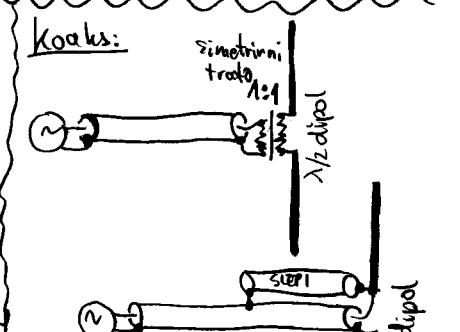
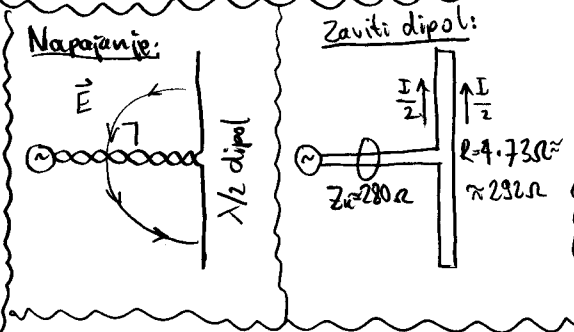
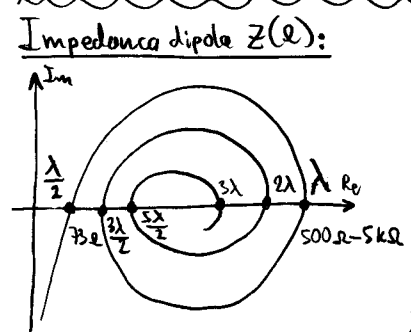


Polvalovni dipol: $\vec{E} = \int_{-\frac{\lambda}{4}}^{+\frac{\lambda}{4}} d\vec{E} = \vec{r} \frac{j k z_0}{4\pi r} \sin \theta \int_{-\frac{\lambda}{4}}^{+\frac{\lambda}{4}} \cos k z e^{j k z \cos \theta} dz = \vec{r} \frac{j z_0}{2\pi} I_0 \frac{e^{-jkr}}{r} \frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta}$
 $F(\theta, \phi) = \frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta}$

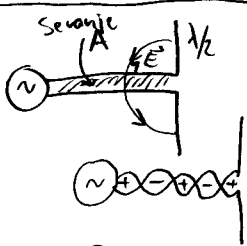
$\vec{S} = \vec{r} \frac{Z_0}{8\pi^2} |I_0|^2 \frac{\cos^4(\frac{\pi}{2} \cos \theta)}{r^2 \sin^2 \theta}$
 $R_s = \frac{1}{2} \frac{|I_0|^2}{|I|^2} \int \vec{S} \cdot \vec{r} r^2 \sin \theta d\theta d\phi = \frac{Z_0}{2\pi} I = 60 \Omega I = 73 \Omega$
 $I = \int_{-1}^{+1} \frac{\cos^2(\frac{\pi}{2} u)}{1-u^2} du = 1.22$

Smernost: $D = \frac{4\pi |F(\theta_m, \phi_m)|^2}{\int |F(\theta, \phi)|^2 d\Omega} = \frac{4\pi}{2\pi \int_0^\pi \frac{\cos^2(\frac{\pi}{2} \cos \theta)}{\sin^2 \theta} \sin \theta d\theta} = \frac{2}{1} = 1.64 = 2.15 \text{ dBi}$

Primerjava s polvalovnim dipolom
 $D[\text{dBi}] = 10 \log_{10} D = D[\text{dBd}] + 2.15 \text{ dB}$



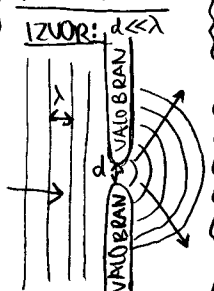
POMNIKTEV: NAPAJANJE DIPOLA



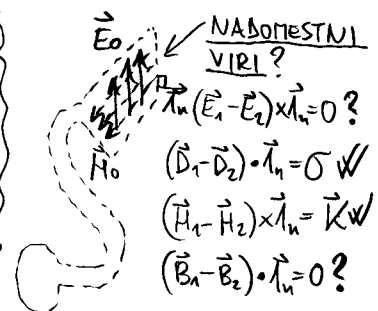
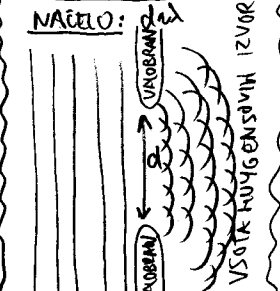
LIJAK IN VALOVNA



HUYGENSOV



HUYGENS-OVO



Razširjene ME:

- 1) $\text{rot } \vec{H} = \vec{j} + j\omega \vec{E}$
- 2) $\text{rot } \vec{E} = -\vec{j}_m - j\omega \mu \vec{H}$
- 3) $\text{div} (\epsilon \vec{E}) = \rho$
- 4) $\text{div} (\mu \vec{H}) = j_m$

Razširjeni prestopai

Pogoji:

$$(\vec{E}_1 - \vec{E}_2) \times \vec{n} = -\vec{K}_m$$

$$(\vec{D}_1 - \vec{D}_2) \cdot \vec{n} = \sigma$$

$$(\vec{H}_1 - \vec{H}_2) \times \vec{n} = \vec{K}$$

$$(\vec{E}_1 - \vec{E}_2) \cdot \vec{n} = 0$$

Reciprocnost Lorentze:

$$\text{rot } \vec{H}_1 = \vec{j}_1 + j\omega \epsilon_1 \vec{E}_1 / \epsilon_2$$

$$\text{rot } \vec{H}_2 = \vec{j}_2 + j\omega \epsilon_2 \vec{E}_2 / \epsilon_1$$

$$\text{rot } \vec{E}_1 = -\vec{j}_{m1} - j\omega \mu_1 \vec{H}_1 / \mu_2$$

$$\text{rot } \vec{E}_2 = -\vec{j}_{m2} - j\omega \mu_2 \vec{H}_2 / \mu_1$$

$\epsilon \equiv$ skalar $\rightarrow \vec{E}_2 \cdot \text{rot } \vec{H}_1 - \vec{E}_1 \cdot \text{rot } \vec{H}_2 = \vec{E}_2 \cdot \vec{j}_1 - \vec{E}_1 \cdot \vec{j}_2$

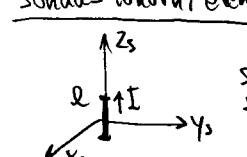
$\mu \equiv$ skalar $\rightarrow \vec{H}_2 \cdot \text{rot } \vec{E}_1 - \vec{H}_1 \cdot \text{rot } \vec{E}_2 = -\vec{H}_2 \cdot \vec{j}_{m1} + \vec{H}_1 \cdot \vec{j}_{m2}$

člena v ∞ enaka!

$$\int_V (\vec{E}_2 \cdot \text{rot } \vec{H}_1 - \vec{H}_1 \cdot \text{rot } \vec{E}_2 + \vec{H}_2 \cdot \text{rot } \vec{E}_1 - \vec{E}_1 \cdot \text{rot } \vec{H}_2) dV = \int_{V \rightarrow \infty} \text{div} (\vec{H}_1 \times \vec{E}_2 + \vec{E}_1 \times \vec{H}_2) dV = \oint_{A \rightarrow \infty} (\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1) \cdot \vec{n} dA = 0$$

$$0 = \int_{V \rightarrow \infty} (\vec{E}_2 \cdot \vec{j}_1 - \vec{E}_1 \cdot \vec{j}_2 - \vec{H}_2 \cdot \vec{j}_{m1} + \vec{H}_1 \cdot \vec{j}_{m2}) dV \rightarrow \int_{V_1} (\vec{E}_2 \cdot \vec{j}_1 - \vec{H}_2 \cdot \vec{j}_{m1}) dV_1 = \int_{V_2} (\vec{E}_1 \cdot \vec{j}_2 - \vec{H}_1 \cdot \vec{j}_{m2}) dV_2$$

Sonda = tokovni element:



$$\vec{E}_s = \vec{a}_z \frac{j k z_0}{4\pi} I_s l_s \frac{e^{-jkr}}{r} \sin \theta_s$$

$$\vec{H}_s = \vec{a}_\phi \frac{j k}{4\pi} I_s l_s \frac{e^{-jkr}}{r} \sin \theta_s$$

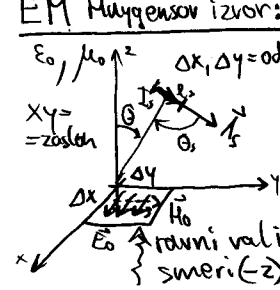
$$\vec{j}_s = \vec{a}_z \frac{I_s}{A_s}$$

$$\vec{j}_{ms} = 0$$

$$\int_{V_s} (\vec{E} \cdot \vec{j}_s - \vec{H} \cdot \vec{j}_{ms}) dV_s = \vec{E} \cdot \vec{a}_z I_s l_s$$

$$\vec{E} \cdot \vec{a}_z = \frac{1}{I_s l_s} \int_V (\vec{E}_s \cdot \vec{j} - \vec{H}_s \cdot \vec{j}_m) dV$$

EM Huygensov izvor:



$$\vec{E}_0 = \vec{a}_x E_0$$

$$\vec{H}_0 = \vec{a}_y \frac{E_0}{z_0}$$

$$\vec{K} = \vec{a}_m \times \vec{H}_0 = -\vec{a}_x \frac{E_0}{z_0}$$

$$\vec{K}_m = \vec{E}_0 \times \vec{a}_m = -\vec{a}_y E_0$$

$$\vec{E} \cdot \vec{a}_z = \frac{1}{I_s l_s} \int_A (\vec{E}_s \cdot \vec{K} - \vec{H}_s \cdot \vec{K}_m) dA = \frac{j k}{4\pi} E_0 \Delta x \Delta y \frac{e^{-jkr}}{r} [\vec{a}_z \cdot (-\vec{a}_x) - \vec{a}_\phi \cdot (-\vec{a}_y)]$$

$$\vec{a}_z \cdot \vec{a}_x = -\cos \theta \cos \phi$$

$$\vec{a}_\phi \cdot \vec{a}_y = \cos \phi$$

$$E_\theta = \frac{j}{2\lambda} E_0 \Delta x \Delta y \frac{e^{-jkr}}{r} (\cos \theta + 1) \cos \phi$$

$$\vec{a}_z \cdot \vec{a}_x = \sin \phi$$

$$\vec{a}_\phi \cdot \vec{a}_y = -\cos \theta \sin \phi$$

$$E_\phi = \frac{j}{2\lambda} E_0 \Delta x \Delta y \frac{e^{-jkr}}{r} (\cos \theta + 1) (-\sin \phi)$$

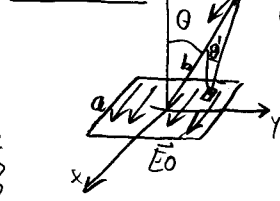
Polje EM Huygens:

$$\vec{E} = (\vec{a}_\theta \cos \phi - \vec{a}_\phi \sin \phi) \frac{j}{2\lambda} E_0 \Delta x \Delta y \frac{e^{jkr}}{r} (\cos \theta + 1)$$

enotin smernik \equiv polarizacija v \vec{a}_x

jednost izvora $F(\theta, \phi)$

Odprtina:



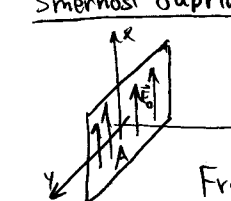
$$d\vec{E} = (\vec{a}_\theta \cos \phi' - \vec{a}_\phi \sin \phi') \frac{j}{2\lambda} E_0(x, y) dx dy \frac{e^{jkr'}}{r'} (\cos \theta')$$

Fraunhofer: $\vec{a}_\theta \approx \vec{a}_\theta$, $\vec{a}_\phi \approx \vec{a}_\phi$, $\theta \approx \theta$, $\phi \approx \phi$, $\frac{1}{r'} \approx \frac{1}{r}$

$$\vec{E} = \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} d\vec{E}$$

$$e^{jkr'} \neq e^{jkr}$$

Smernost odprtine na osi z ($\theta=0$):



$$D_{\text{max}}(\theta=0, \phi) = \frac{|\vec{S}_{\text{max}}|}{|\vec{S}_i|} = \frac{\frac{|\vec{E}(\theta=0)|^2}{2z_0}}{\frac{P_0}{4\pi r^2}} = \frac{4\pi r^2}{2z_0} \left| \int_A \frac{j}{2\lambda} E_0(x, y) \frac{1}{r} 2 dx dy \right|^2 = \frac{4\pi}{\lambda^2} \left| \int_A E_0(x, y) dx dy \right|^2$$

Fraunhofer ($\theta=0$) $\rightarrow e^{-jkr'} \approx e^{jkr} = \text{konst.}$

Zajed: $E_0(x, y) = \text{konst.} \rightarrow \text{max } D!$

Poljuben $E_0(x, y)$:

$$D = \frac{4\pi}{\lambda^2} \frac{|E_0|^2 A^2}{|E_0|^2 A} = \frac{4\pi}{\lambda^2} A$$

$$D = \frac{4\pi}{\lambda^2} A_{\text{eff}} = \frac{4\pi}{\lambda^2} \eta_0 A$$

Izkoristek osvetlitve: $\eta_0 = \frac{|\int_A E_0(x, y) dx dy|^2}{A \int_A |E_0(x, y)|^2 dx dy}$

Efektivna površina: $A_{\text{eff}} = \frac{|\int_A E_0(x, y) dx dy|^2}{\int_A |E_0(x, y)|^2 dx dy}$

Antene in razširjanje valov #5 29. 10. 2013

Huygens-ov izvor:

v ravnini xy, sevanje v smeri +z

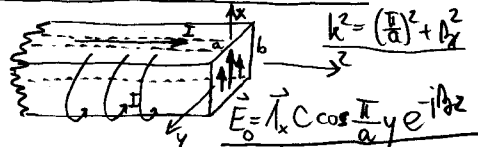
$$d\vec{E} = (\vec{1}_\theta \cos\phi - \vec{1}_\phi \sin\phi) \frac{j}{2\lambda} E_0(x,y) dx dy \frac{e^{jkr}}{r} (\cos\theta + 1) @ \vec{E}_0 = \vec{1}_x E_0$$

$$d\vec{E} = (\vec{1}_\theta \sin\phi + \vec{1}_\phi \cos\phi) \frac{j}{2\lambda} E_0(x,y) dx dy \frac{e^{jkr}}{r} (\cos\theta + 1) @ \vec{E}_0 = \vec{1}_y E_0$$

Max smernost, Aeff, η₀: Aeff = A η₀

$$D = \frac{4\pi}{\lambda^2} \frac{|\int_A E_0(x,y) dx dy|^2}{\int_A |E_0(x,y)|^2 dx dy} = \frac{4\pi}{\lambda^2} A_{eff}$$

Pravokotni kovinski valovod:



$$\lambda_g = \frac{2\pi}{\beta}$$

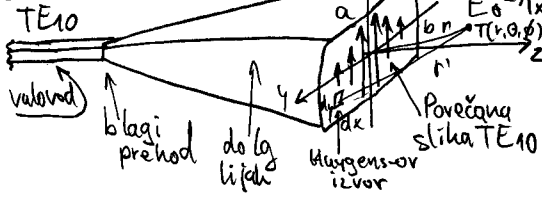
Izkoristek osvetlitve $\vec{E} = \vec{1}_x C \cos \frac{\pi}{a} y$

$$\eta_0 = \frac{|\int_A C \cos \frac{\pi}{a} y dx dy|^2}{A \int_A |C \cos \frac{\pi}{a} y|^2 dx dy} = \frac{|C|^2 a^2 b^2 \frac{4}{\pi^2}}{|C|^2 a^2 b^2 \frac{1}{2}} = \frac{8}{\pi^2}$$

$$\eta_0 \approx 82\%$$

Zgled: $a = \lambda, b = \frac{\lambda}{2} \rightarrow D = \frac{16}{\pi} \approx 5$

Piramidni ližale



$$d\vec{E} = (\vec{1}_\theta \cos\phi - \vec{1}_\phi \sin\phi) \frac{j}{2\lambda} C \cos \frac{\pi}{a} y dx dy \frac{e^{jkr'}}{r} (1 + \cos\theta)$$

Fraunhofer: $r > \frac{2a^2}{\lambda} \rightarrow$ zanemarimo amplitudo θ', ϕ', r'
 $\cos\theta_x = \sin\theta \cos\phi, \cos\theta_y = \sin\theta \sin\phi$ POMEMBNA FAZA $e^{jkr'} \approx e^{jkr} e^{j(kx \cos\theta_x + y \cos\theta_y)}$

$$r' = \sqrt{(r \sin\theta \cos\phi - x)^2 + (r \sin\theta \sin\phi - y)^2 + (r \cos\theta)^2} \approx r - x \cos\theta_x - y \cos\theta_y$$

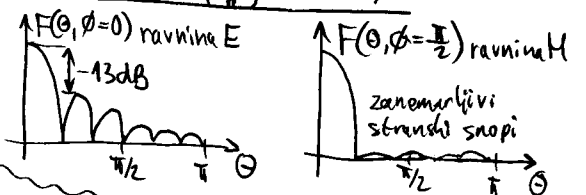
$$\vec{E} = \int_{-\frac{a}{2}}^{+\frac{a}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} d\vec{E} = (\vec{1}_\theta \cos\phi - \vec{1}_\phi \sin\phi) \frac{jC}{2\lambda} \frac{e^{jkr}}{r} \int_{-\frac{a}{2}}^{+\frac{a}{2}} \cos \frac{\pi}{a} y e^{jky \cos\theta_y} dy \int_{-\frac{b}{2}}^{+\frac{b}{2}} e^{-jkx \cos\theta_x} dx (1 + \cos\theta)$$

$$I_x = \int_{-\frac{b}{2}}^{+\frac{b}{2}} e^{-jkx \cos\theta_x} dx = \frac{e^{jkx \cos\theta_x}}{jk \cos\theta_x} \Big|_{-\frac{b}{2}}^{+\frac{b}{2}} = \frac{2 \sin(\frac{kb}{2} \cos\theta_x)}{jk \cos\theta_x} \cdot \frac{b}{2} = b \cdot \frac{\sin(\frac{kb}{2} \sin\theta \cos\phi)}{\frac{kb}{2} \sin\theta \cos\phi}$$

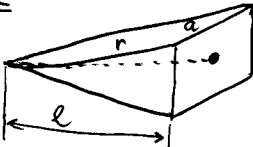
$$I_y = \int_{-\frac{a}{2}}^{+\frac{a}{2}} \cos \frac{\pi}{a} y e^{jky \cos\theta_y} dy = \frac{1}{2} \int_{-\frac{a}{2}}^{+\frac{a}{2}} [e^{i(k \cos\theta_y + \frac{\pi}{a})y} + e^{i(k \cos\theta_y - \frac{\pi}{a})y}] dy = \frac{2 \sin(\frac{ka}{2} \cos\theta_y + \frac{\pi}{2})}{2i(k \cos\theta_y + \frac{\pi}{a})} + \frac{2 \sin(\frac{ka}{2} \cos\theta_y - \frac{\pi}{2})}{2i(k \cos\theta_y - \frac{\pi}{a})}$$

$$= \frac{\cos(\frac{ka}{2} \cos\theta_y)}{k \cos\theta_y + \frac{\pi}{a}} - \frac{\cos(\frac{ka}{2} \cos\theta_y)}{k \cos\theta_y - \frac{\pi}{a}} = \frac{2 \frac{\pi}{a} \cos(\frac{ka}{2} \cos\theta_y)}{(\frac{\pi}{a})^2 - k^2 \cos^2\theta_y} \cdot (\frac{a}{\pi})^2 = a \frac{2}{\pi} \cdot \frac{\cos(\frac{ka}{2} \sin\theta \sin\phi)}{1 - (\frac{ka}{\pi})^2 \sin^2\theta \sin^2\phi}$$

$$F(\theta, \phi) = (1 + \cos\theta) \frac{\sin(\frac{kb}{2} \sin\theta \cos\phi)}{\frac{kb}{2} \sin\theta \cos\phi} \frac{\cos(\frac{ka}{2} \sin\theta \sin\phi)}{1 - (\frac{ka}{\pi})^2 \sin^2\theta \sin^2\phi}$$



kroglaste fronte = kvadratna napaka faze



$$r = \sqrt{l^2 + (\frac{a}{2})^2 + (\frac{b}{2})^2}$$

$$\Delta\varphi = k(r-l) \approx k \frac{a^2 + b^2}{8l} = \frac{\pi(a^2 + b^2)}{4l\lambda}$$

Popravek faze:



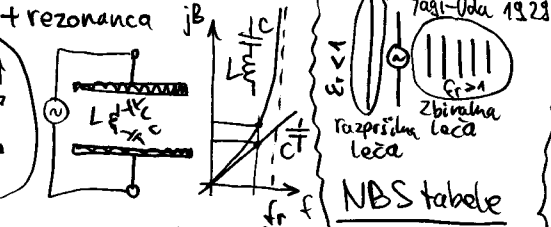
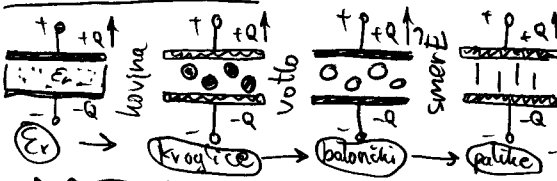
$\Delta\varphi$	ΔG
$\frac{\pi}{2}$	-∞ dB
$\frac{\pi}{4}$	-4 dB
$\frac{\pi}{8}$	-1 dB
$\frac{\pi}{16}$	-0,25 dB

$$\frac{\pi}{2} = \frac{\pi(a^2 + b^2)}{4l\lambda}$$

$$l = \frac{a^2 + b^2}{2\lambda}$$

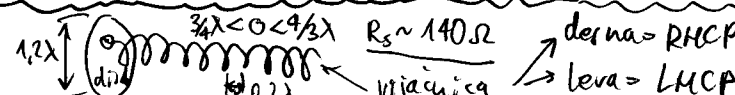
Zgled: $f = 126 \text{ MHz} \rightarrow \lambda = 2,3 \text{ cm}$
 $a = b = 50 \text{ cm}$
 $l = \frac{2500 \text{ cm}^2 + 2500 \text{ cm}^2}{2 \cdot 2,3 \text{ cm}} = 10 \text{ m!}$

Umetni dielektrik:



SLOW-WAVE STRUCTURE
 IIIIII palčke
 XXXXX križci 2x pol
 OOOOO zavlece
 UUUU Uii-vii VVVV
 NNNN
 ●●●●● diski

Vijačna antena z osnim sevanjem

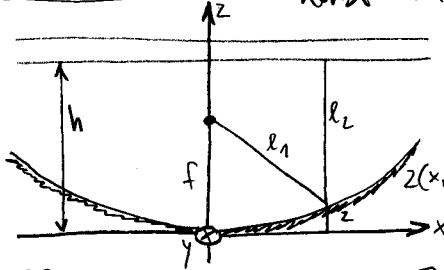


$$R_s \sim 140 \Omega$$

vijačnica

Antene in razširjanje valov #6 5/11/2013

Oblika zrcala:



$$konst = l_1 + l_2 = f + h = \sqrt{x^2 + y^2 + (f-z)^2} + h - z$$

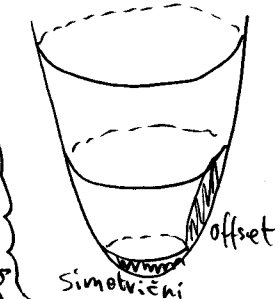
$$l_1 = \sqrt{x^2 + y^2 + (f-z)^2} \quad \sqrt{x^2 + y^2 + (f-z)^2} = f + z$$

$$l_2 = h - z$$

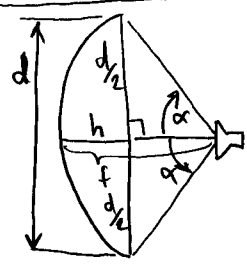
$$x^2 + y^2 + f^2 - 2fz + z^2 = f^2 + 2fz + z^2$$

$$x^2 + y^2 = 4fz \rightarrow z(x, y) = \frac{x^2 + y^2}{4f}$$

1/2 rez



Simetrično zrcalo:



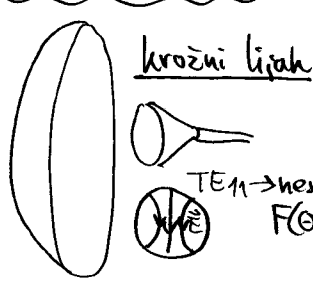
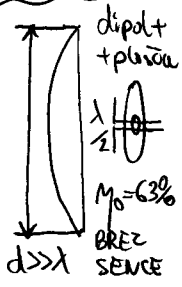
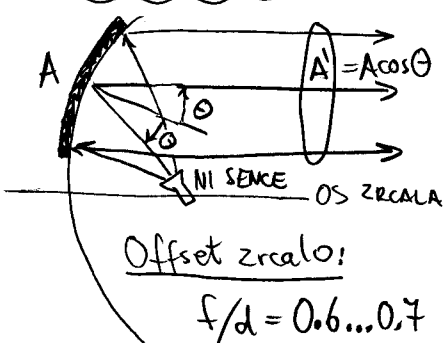
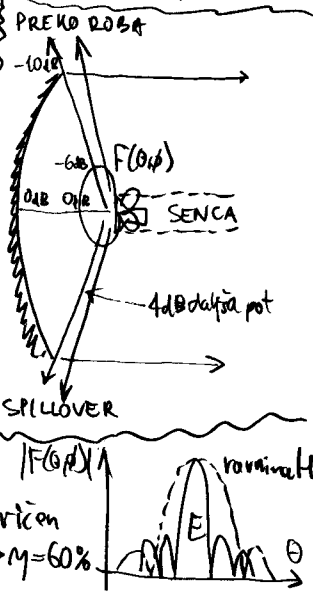
$$f = \frac{d^2}{16h}$$

$$\alpha = \arctg \frac{d/2}{f-h}$$

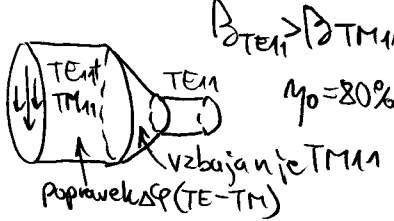
$$\alpha = \arctg \frac{1}{2f/d - \frac{1}{8f/d}}$$

$f/d = 0.3 \dots 0.4$ (zaslonka fotoaparata)
 $f/d = 0.4 \rightarrow \alpha = 64^\circ; 2\alpha = 128^\circ$

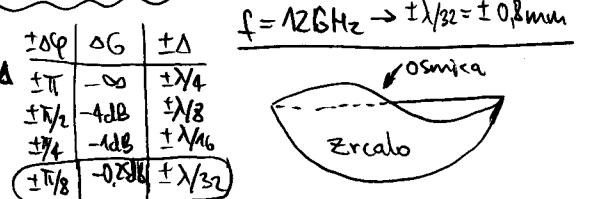
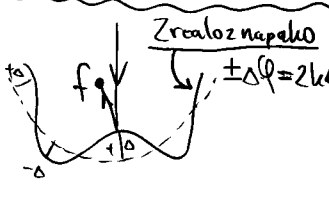
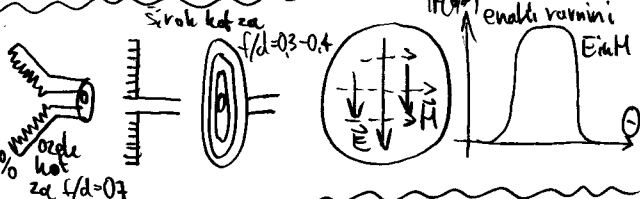
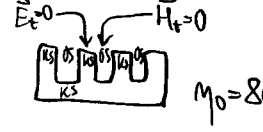
SENCA ŽRILCA $\rightarrow d > 5\lambda$
 OSVETLITEV ROBA
 $\rightarrow -6dB F(\theta, \phi) - 4dB$ daljša pot



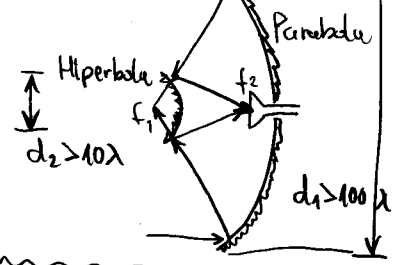
Dvorodovni: lijak TEM + TM11



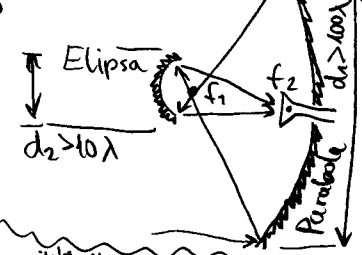
Korugirani lijak



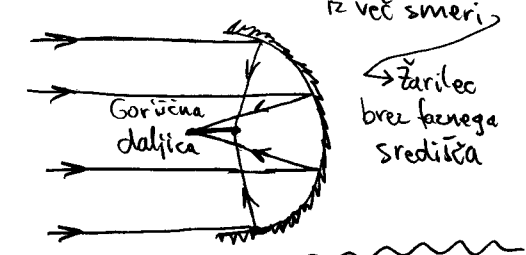
Cassegrain:



Gregorian:



Krogno zrcalo

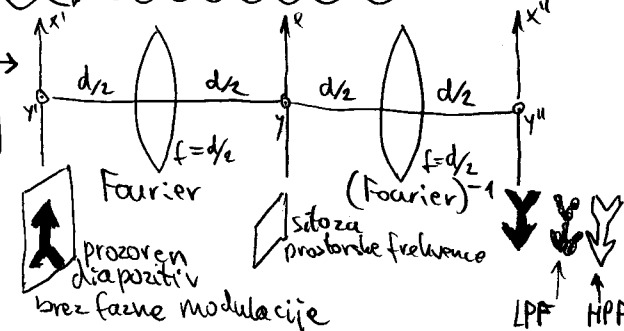


2D-Fourier:

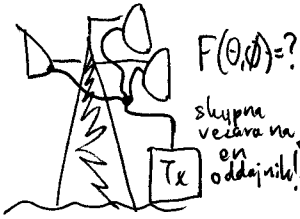
$$dE = \frac{1}{x} E_d(k, y) \frac{e^{-jk(f-F)}}{|F-F|} dx dy$$

$$\frac{1}{|F-F|} \approx \frac{1}{d}$$

$$E = \frac{1}{x} \frac{e^{ikd}}{d} e^{-jk \frac{x^2+y^2}{2d}} \iint E_0(k', y') e^{ik' \frac{x^2+y^2}{2d}} e^{ik \frac{x^2}{d}} e^{ik' y'} dx dy$$



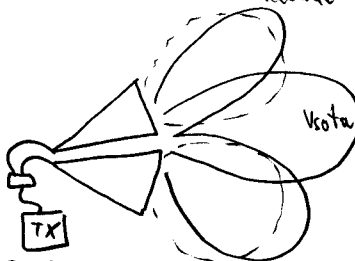
Koherentna skupina



$F(\theta, \phi) = ?$
skupna večava na eni oddajnik!

Sestevanje kazalcev polja!

Realita

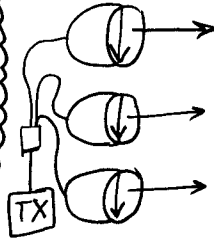


Pravilo o množenju $F(\theta, \phi)$

- ① ENAKE ANTENE
- ② ENAKO ORIENTIRANE
- ③ ENAKO POLARIZIRANE

$$F(\theta, \phi) = F_e(\theta, \phi) \cdot F_s(\theta, \phi)$$

$D \neq D_e \cdot D_s$ NE VELJA!



DVA ZOBOPNA VIRA

Fraunhofer $r > \frac{2h^2}{\lambda}$; $\vec{E}_1 = \vec{E}_2 = \vec{E}$; $\frac{1}{r_1} = \frac{1}{r_2} = \frac{1}{r}$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \vec{E} \alpha I_1 \frac{e^{-jkr_1}}{r_1} + \vec{E} \alpha I_2 \frac{e^{-jkr_2}}{r_2}$$

$$r_1 = \sqrt{r^2 + (h/2)^2 - rh \cos \theta} \approx r - \frac{h}{2} \cos \theta$$

$$r_2 = \sqrt{r^2 + (h/2)^2 + rh \cos \theta} \approx r + \frac{h}{2} \cos \theta$$

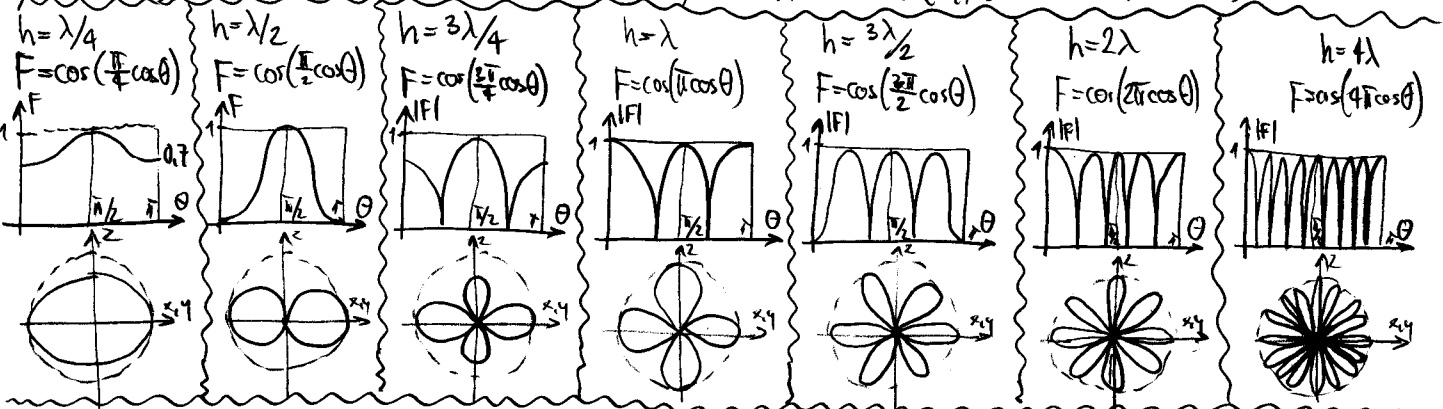
$$\vec{E} = \vec{E} \alpha \frac{e^{-jkr}}{r} (I_1 e^{jk \frac{h}{2} \cos \theta} + I_2 e^{-jk \frac{h}{2} \cos \theta})$$

Naizanimivejši primer: $|I_1| = |I_2| \rightarrow I_1 = I_0 e^{j\frac{\pi}{2}}$; $I_2 = I_0 e^{-j\frac{\pi}{2}}$

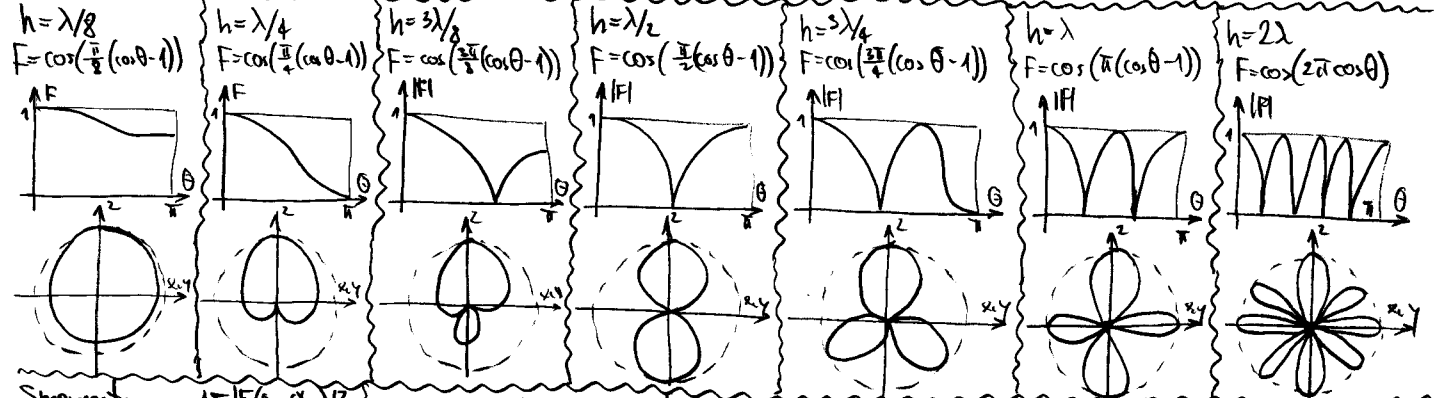
$$\vec{E} = \vec{E} I_0 \alpha \frac{e^{-jkr}}{r} (e^{j(\frac{\pi}{2} + \frac{kh}{2} \cos \theta)} + e^{-j(\frac{\pi}{2} + \frac{kh}{2} \cos \theta)})$$

$$\vec{E} = \vec{E} I_0 \alpha \frac{e^{-jkr}}{r} 2 \cos(\frac{\pi}{2} + \frac{kh}{2} \cos \theta) \rightarrow F(\theta, \phi) = \cos(\frac{\pi}{2} + \frac{kh}{2} \cos \theta)$$

Bočna skupina $\varphi = 0 \rightarrow F(\theta, \phi) = \cos(\frac{kh}{2} \cos \theta)$; $k = \frac{2\pi}{\lambda} \rightarrow F(\theta, \phi) = \cos(\pi \frac{h}{\lambda} \cos \theta)$



Oсна skupina: $\varphi = -kh$ možna izbira $\rightarrow F(\theta, \phi) = \cos(\frac{kh}{2} (\cos \theta - 1)) = \cos(\pi \frac{h}{\lambda} (\cos \theta - 1))$

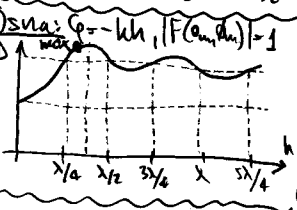
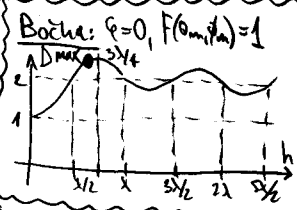


Smernost: $D = \frac{4\pi |F(\theta_m, \phi_m)|^2}{\int |F(\theta, \phi)|^2 d\Omega}$

$$D = \frac{4\pi |F(\theta_m, \phi_m)|^2}{2\pi \int_0^\pi \cos^4(\frac{\pi}{2} + \frac{kh}{2} \cos \theta) \sin \theta d\theta}$$

$$D = \frac{2 |F(\theta_m, \phi_m)|^2}{\int_{-1}^1 (1 + \cos(\varphi + kh u)) du}$$

$$D = \frac{2 |F(\theta_m, \phi_m)|^2}{1 + \frac{\sin kh}{kh} \cos \varphi}$$



$P_s = 4P_{s1}$ $P_g = |I|^2 \text{Re}[Z_{11} + Z_{12}]$

$P_{g1} = \frac{1}{2} |I|^2 \text{Re}[Z_{11}]$

$D = D_e \frac{P_s}{P_{s1}} \frac{P_{g1}}{P_g} = D_e \frac{2 \text{Re}[Z_{11}]}{\text{Re}[Z_{11} + Z_{12}]}$

Približno pravilo: stranski snop $F_s \rightarrow$ ničla F_e !!!

ničla $F_s \rightarrow -3\text{dB } F_e$

$$d = \frac{\lambda/2}{\sin \alpha/2}$$

Ocena točnosti

$|Z_{12}| \ll |Z_{11}|$

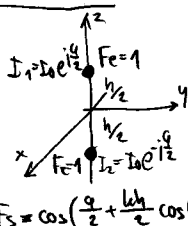
$D \approx D_e \cdot D_s$

Očna max D : $\varphi \rightarrow \pi$ $h \rightarrow 0$ $R_s = ?$

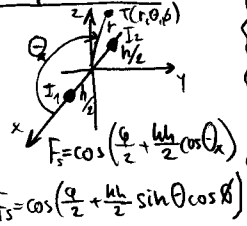
$|F(\theta_m, \phi_m)| \ll 1$

$D_{\text{max}} \rightarrow 4$

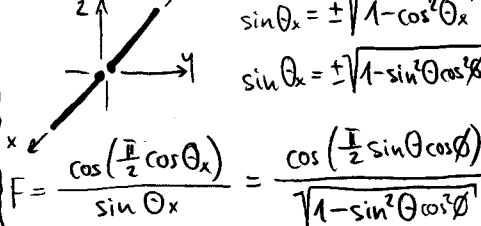
Ponovitev:



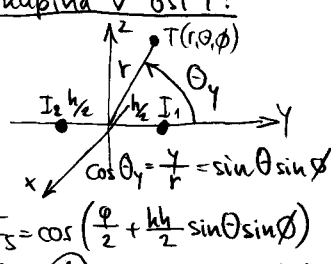
Skupina v osi X:



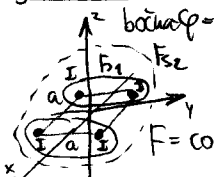
\lambda/2 dipol v osi X:



Skupina v osi Y:

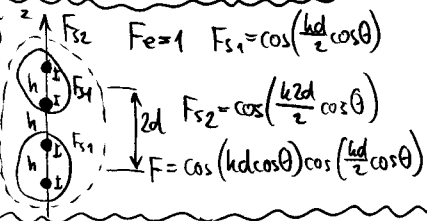


Oglišča kvadrata XY:



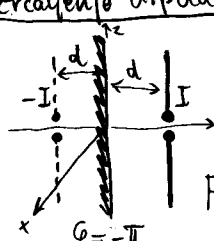
$F_{s1} = \cos(\frac{kx}{2} \sin\theta \cos\phi)$
 $F_{s2} = \cos(\frac{ky}{2} \sin\theta \sin\phi)$
 $F = \cos(\frac{kx}{2} \sin\theta \cos\phi) \cos(\frac{ky}{2} \sin\theta \sin\phi)$

bočina \phi=0



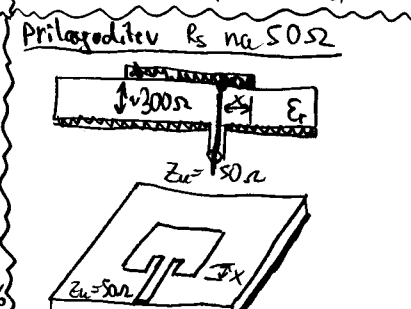
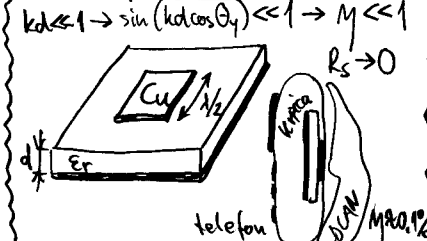
4 dipoli v X na osi Z:
 $F = F_{s2} \cdot F_{s1} \cdot F_e \quad \phi=0$
 $F = \cos(kd \cos\theta) \cos(\frac{kd}{2} \cos\theta) \cdot \frac{\cos(\frac{\pi}{2} \sin\theta \cos\phi)}{\sqrt{1 - \sin^2\theta \cos^2\phi}}$

Zrcaljenje dipola

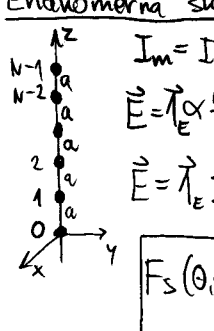


$F_s = \cos(\frac{\omega}{2} + kd \cos\theta_y)$
 $F_s = \cos(-\frac{\omega}{2} + kd \sin\theta \sin\phi)$
 $F = \sin(kd \sin\theta \sin\phi) \frac{\cos(\frac{\pi}{2} \cos\theta)}{\sin\theta}$

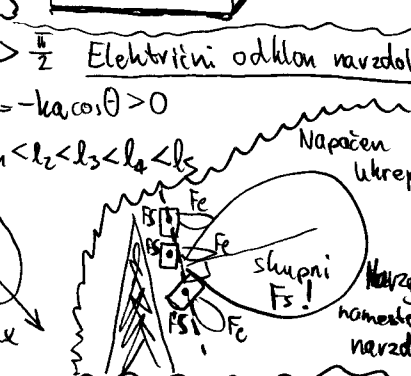
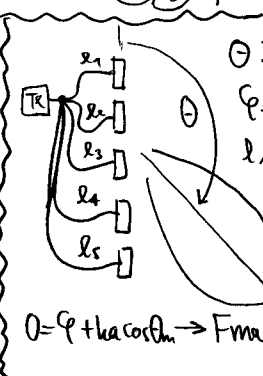
Mikrostrip korpica



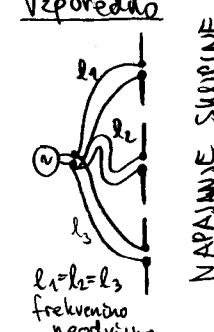
Enakomerna skupina



$\vec{E}_0 = \vec{1}_e \alpha I_0 \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{r}$
 $\vec{E} = \vec{1}_e \alpha \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{r} [I_0 + I_1 e^{i(\phi + ka \cos\theta)} + \dots + I_{N-1} e^{i(N-1)(\phi + ka \cos\theta)}]$
 $\vec{E} = \vec{1}_e I_0 \alpha \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{r} \frac{1 - e^{iN(\phi + ka \cos\theta)}}{1 - e^{i(\phi + ka \cos\theta)}}$
 $F_s(\theta, \phi) = \frac{\sin \frac{N}{2} (\phi + ka \cos\theta)}{\sin \frac{1}{2} (\phi + ka \cos\theta)}$



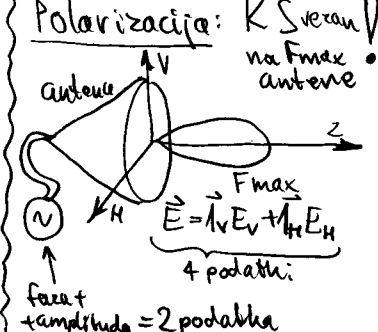
Vzoredno



Zaporedno



Polarizacija: K Svezan!



Razmerje linearnih komponent:

$\frac{E_v}{E_H} \equiv$ dva podabna polarizacije (odvisna od izbire $\vec{1}_v!$)
 $E_v = \vec{1}_v \cdot \vec{E} \quad E_H = \vec{1}_H \cdot \vec{E}$
 Krožna smernika (IEEE): $\vec{1}_L \cdot \vec{1}_L^* = 1; \vec{1}_L \cdot \vec{1}_D^* = 0$
 $\vec{1}_L = \frac{\vec{1}_v + j\vec{1}_H}{\sqrt{2}} \quad E_L = \vec{E} \cdot \vec{1}_L^* \quad E_D = \vec{E} \cdot \vec{1}_D^*$
 $\vec{1}_D = \frac{\vec{1}_v - j\vec{1}_H}{\sqrt{2}} \quad Q = \frac{E_L}{E_D} \equiv$ razmerje krožnih komponent

Oсно razmerje:

$\frac{U_{max}}{U_{min}} = R$
 $1 \leq R = \left| \frac{1+|Q|}{1-|Q|} \right| \quad R_{dB} = 20 \log R$
 R in $|Q|$ neodvisna od $\vec{1}_v!$

Faktor prenosa moči:

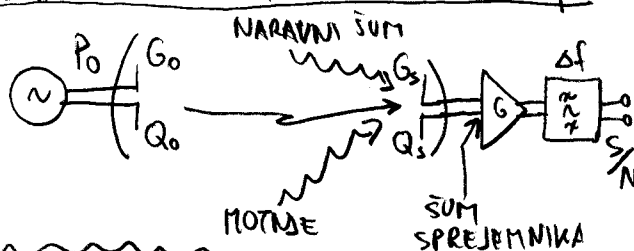
$P_s = P_0 G_0 G_s \left(\frac{\lambda}{4\pi r} \right)^2 \frac{1 + Q_0 Q_s^2}{(1 + |Q_0|^2)(1 + |Q_s|^2)}$
 $0 \leq \eta \leq 1$

Polarizacija	Q	R	Faktor M					
			VP	HP	RHCP	LHCP		
VP	1	∞	1	0	1/2	1/2	1/2	1/2
HP	-1	∞	0	1	1/2	1/2	1/2	1/2
RHCP	0	1	1/2	1/2	1	0	1/2	1/2
LHCP	∞	1	1/2	1/2	0	1	1/2	1/2
PP45	-j	∞	1/2	1/2	1/2	1/2	0	1
PP45	+j	∞	1/2	1/2	1/2	1/2	1	0

Krožno-polarizirane antene

- 90° fazni zamik → zamaknjeni anteni
- 2 anteni pod pravim kotom → napajanje $R+C, R+L$
- dipoli različnih dolžin
- obrezana hrpica
- eliptični valvod $\Delta\beta$
- vijaki pod 45°, ploščica ϵ_r
- dvodomni dielektrik
- vijakna antena z osnim senčjem
- spiralna antena

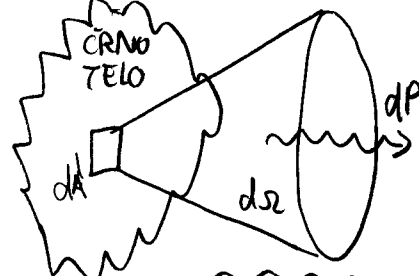
Kohorentna zveza, nekohorentne matrije



Spektralna svetlost

$$B_f = \frac{dP}{df dA' d\Omega}$$

$$B_\lambda = \frac{dP}{d\lambda dA' d\Omega}$$



Planck-ov zakon

(CRNO TELO)

$$B_f = \frac{2hf^3}{c^2} \frac{1}{e^{\frac{hf}{k_B T}} - 1}$$

$h = 6.625 \cdot 10^{-34} \text{ J}\cdot\text{s}$
 $k_B = 1.38 \cdot 10^{-23} \text{ J/K}$
 $c_0 = 3 \cdot 10^8 \text{ m/s}$

Rayleigh-Jeans

Približni $hf \ll k_B T$

$$e^{\frac{hf}{k_B T}} - 1 \approx \frac{hf}{k_B T}$$

$$B_f = \frac{2k_B T f^2}{c^2}$$

$$B_\lambda = \frac{2k_B T}{\lambda^2}$$

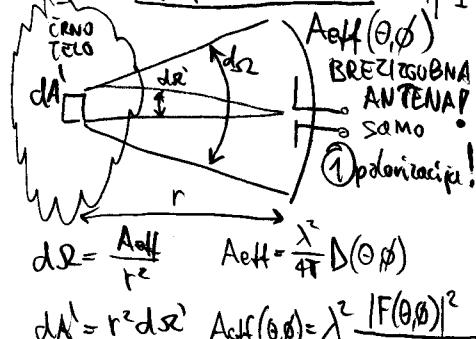
Wien

Približni $hf \gg k_B T$

$$B_f = \frac{2hf^3}{c^2} e^{-\frac{hf}{k_B T}}$$

$f = 100 \text{ GHz}, T = 300 \text{ K}$
 $\frac{hf}{k_B T} = \frac{6.625 \cdot 10^{-34} \text{ J}\cdot\text{s} \cdot 10^5 \text{ s}^{-1}}{1.38 \cdot 10^{-23} \text{ J/K} \cdot 300 \text{ K}} \approx 0.016$

Sprejeta moč šuma



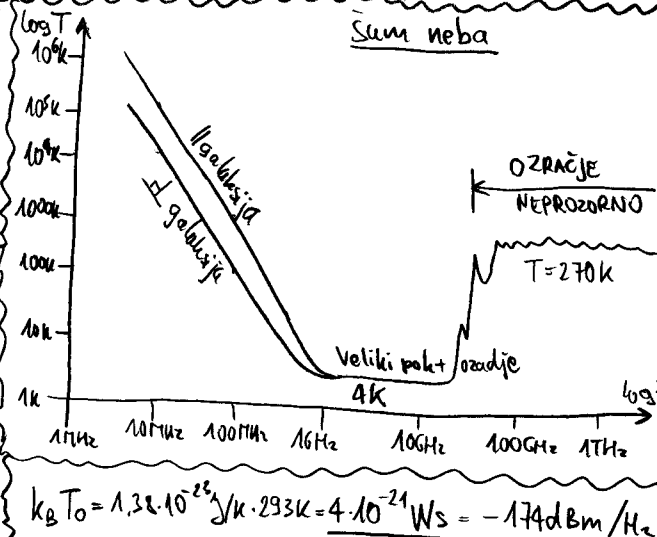
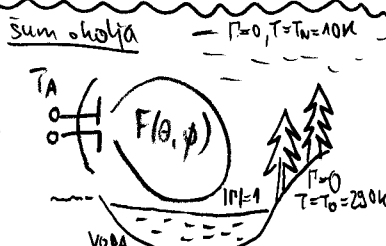
$$P_N = \frac{1}{2} \int B_f df d\Omega dA' = \frac{\Delta f}{2} \int_{4\pi} B_f \frac{A_{eff}}{r^2} r^2 d\Omega = \frac{\Delta f}{2} \int_{4\pi} B_f \lambda^2 \frac{|F(\theta, \phi)|^2}{\int_{4\pi} |F(\theta, \phi)|^2 d\Omega} d\Omega$$

(One polarizacija)

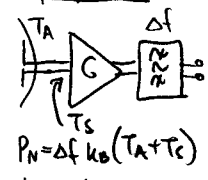
Rayleigh-Jeans: $B_f(\theta, \phi) = \frac{2k_B T}{\lambda^2} T(\theta, \phi) \rightarrow P_N = \Delta f k_B \frac{\int_{4\pi} T(\theta, \phi) |F(\theta, \phi)|^2 d\Omega}{\int_{4\pi} |F(\theta, \phi)|^2 d\Omega}$

$T_A = \frac{\int_{4\pi} T(\theta, \phi) |F(\theta, \phi)|^2 d\Omega}{\int_{4\pi} |F(\theta, \phi)|^2 d\Omega}$; $P_N = \Delta f k_B T_A$

Sevalna upornost



Sprejemnik:

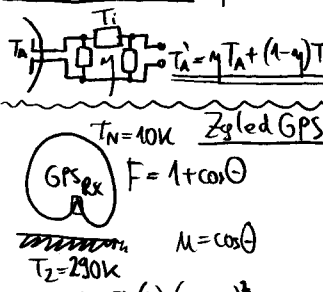


$P_N = \Delta f k_B (T_A + T_n)$
 Polprevodniki $T_s = 30\text{K} - 300\text{K}$
 Cel sprejemnik $T_r = 100\text{K} - 1000\text{K}$

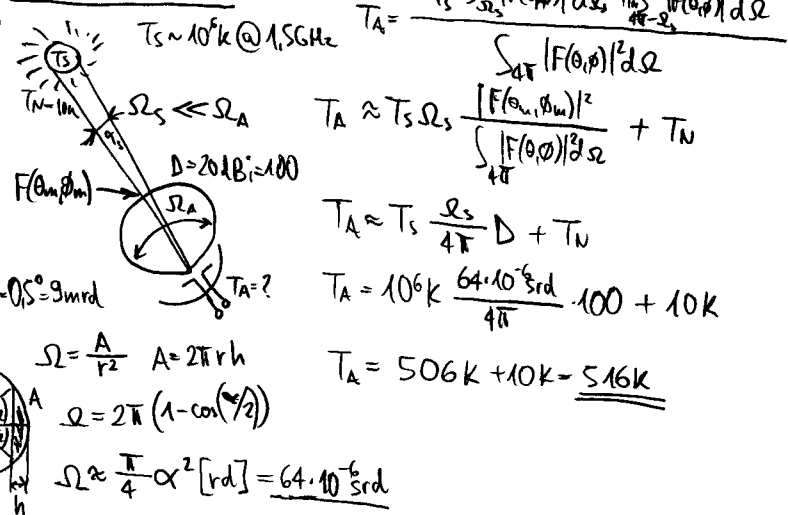
Zgled: GSM telefon

$S/N = 10 \text{ dB}$
 $\Delta f = 200 \text{ kHz}$
 $T_A + T_s = 1000 \text{ K}$
 $P_N = \Delta f \cdot k_B (T_A + T_s) = 2 \cdot 10^5 \text{ s}^{-1} \cdot 1.38 \cdot 10^{-23} \text{ J/K} \cdot 1000 \text{ K} = 2.76 \cdot 10^{-15} \text{ W}$
 $P_s = S/N \cdot P_N = 2.76 \cdot 10^{-14} \text{ W}$
 $P_s [\text{dBm}] = 10 \log \frac{P_s}{1 \text{ mW}} \approx -106 \text{ dBm}$

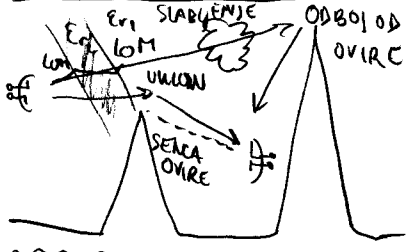
Izračuna antena $\eta < 1$



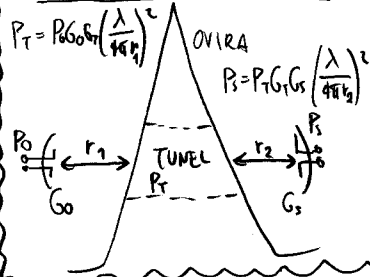
Antena v Sonce:



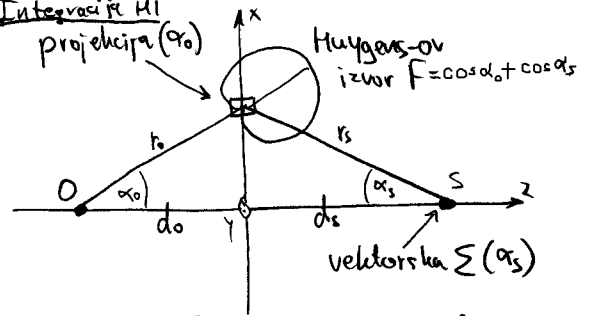
Notnje razširjanja:



2 zaporedni zvezi $\propto r^{-4}$



Integracija H1 projekcija (α_0)

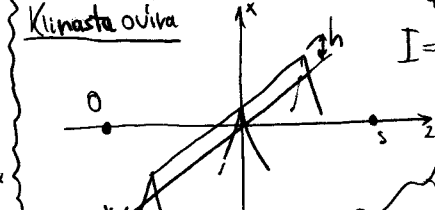
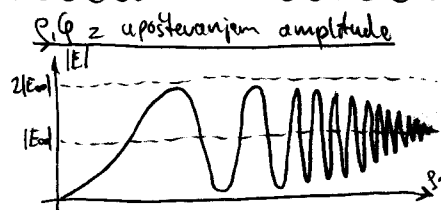
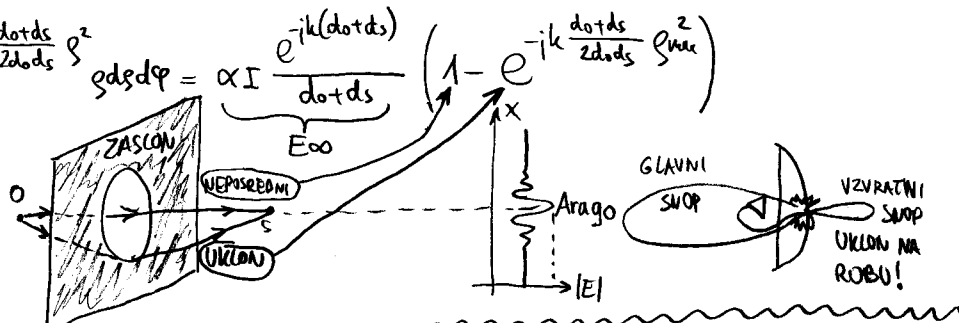
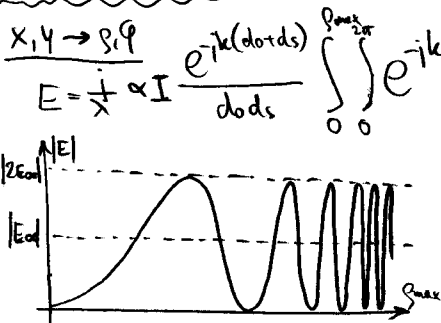


$$E_0 = \alpha I \frac{e^{-jk r_0}}{r_0} \quad dE = \frac{j}{2\lambda} E_0 dx dy \frac{e^{-jk r_s}}{r_s} F(\alpha_0, \alpha_s)$$

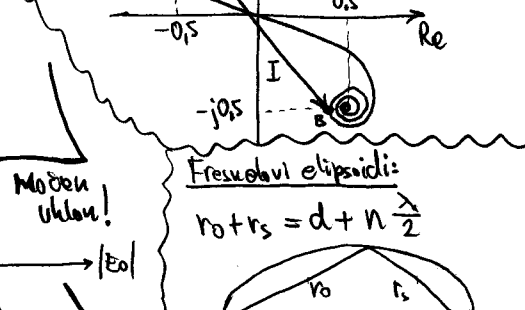
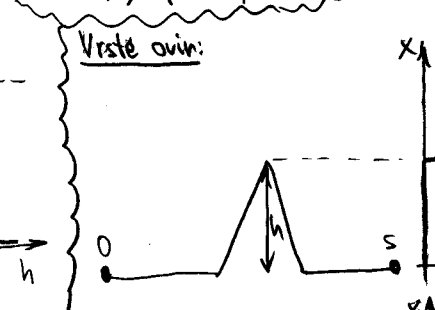
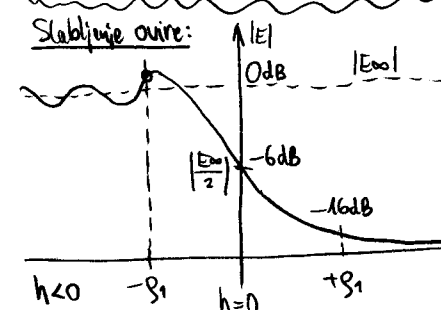
$d_0, d_s \gg x, y \rightarrow$ poenostavim amplitudo: $\frac{1}{r_s} \approx \frac{1}{d_s}, \frac{1}{r_0} \approx \frac{1}{d_0}, F(\alpha_0, \alpha_s) \approx 2$

poenostavim fazo: $e^{-jk r_0} \approx e^{-jk d_0} e^{-jk \frac{x^2+y^2}{2d_0}}, e^{-jk r_s} \approx e^{-jk d_s} e^{-jk \frac{x^2+y^2}{2d_s}}$

$$E = \iint_{x,y} \frac{j}{2\lambda} \alpha I \frac{e^{-jk r_0}}{r_0} dx dy \frac{e^{-jk r_s}}{r_s} F(\alpha_0, \alpha_s) \approx \frac{j}{\lambda} \alpha I \frac{e^{-jk(d_0+d_s)}}{d_0 d_s} \iint_x \left\{ e^{-jk \frac{d_0 d_s}{2d_0 d_s} (x^2+y^2)} dx dy \right.$$



$$I = \int_{-h}^{+h} e^{-jk \frac{d_0 d_s}{2d_0 d_s} x^2} dx \int_{-\infty}^{+\infty} e^{-jk \frac{d_0 d_s}{2d_0 d_s} y^2} dy$$



Slabljene ovire:

$h < 0 \rightarrow -\beta_1$
 $h = 0 \rightarrow a \approx 0dB$
 $h = 0 \rightarrow a = -6dB$
 $h > \beta_1 \rightarrow a \approx -16dB - 20 \log \frac{h}{\beta_1}$

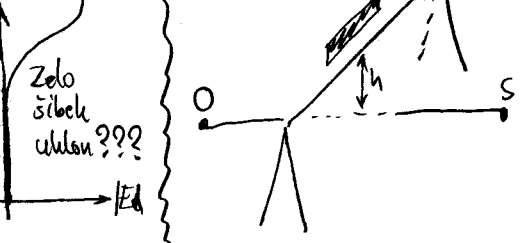
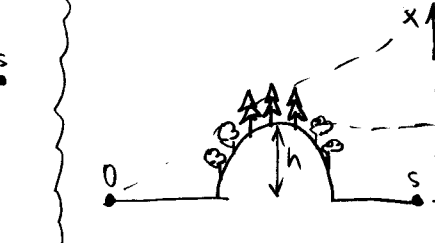


Fresnelovi elipsoidi:

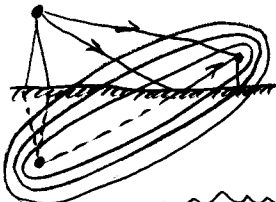
$$r_0 + r_s = d + n \frac{\lambda}{2}$$

Polmeri FC:

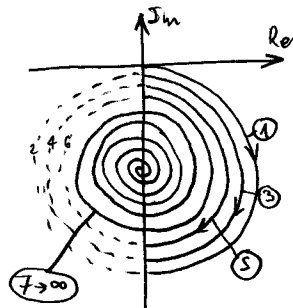
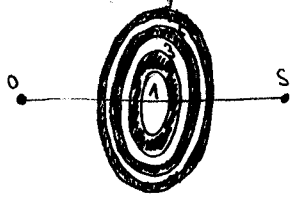
$$r_0 + r_s = d + d_s + n \frac{\lambda}{2}$$

$$R_n = \sqrt{n \lambda \frac{d_0 d_s}{d_0 + d_s}}$$


FC pri odboju:



Fresnel-ova leca



dielektrična leca E_D

$E_F = \sqrt{\epsilon} E_0$

$E_F = \frac{1}{\sqrt{\epsilon}} E_D$ (-10dB)

Fresnel-ova leca s senčenjem

Fresnel-ova dielektrična:



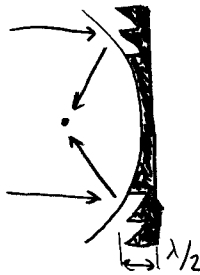
Fresnel-ova zrcalo:

$E_{Fz} = \frac{2}{\pi} E_{Pz}$

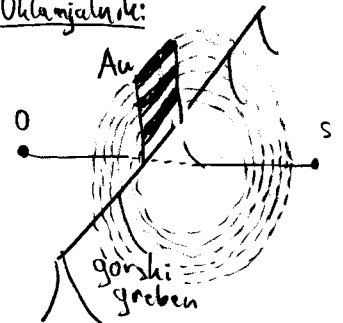
(-4dB)

$\lambda/4$

Fresnel-parabolično:



Uklanjalniki:



Ravno zrcalo:

$P_2 = P_0 G_0 \frac{A_2 \cos \theta}{4\pi r_2^2}$

$P_3 = P_2 G_3 \frac{A_3 \cos \theta}{4\pi r_3^2}$

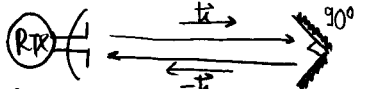
Primenjena zrcala/uklanjalniki:

$P_{sz} = \frac{P_0 G_0 G_s}{(4\pi)^2 r_0^2 r_s^2} A_2^2 \cos^2 \theta$

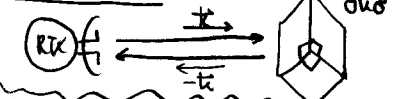
$P_u = \frac{P_0 G_0 G_s}{(4\pi)^2 r_0^2 r_s^2} \frac{A_m^2 \sin^2 \theta}{\pi^2}$

$$\frac{P_{su}}{P_{sz}} = \frac{A_m^2 \tan^2 \theta}{A_z^2 \frac{\pi^2}{\pi^2}}$$

Vogal 2B:



Trirobnik 3B:



Odmema površina σ :

$P_s = P_0 G^2 \frac{\lambda^2 \sigma}{(4\pi)^2 r^4}$

Ravna plošča $\sigma_z = \frac{4\pi}{\lambda^2} A^2$ (trirobnik)

σ velike krogle $a \gg \lambda$:

$A_r = 2\pi a h$

$\sigma_k = \frac{1}{4} \sigma_{1FC}$

$\sigma_k = \frac{1}{4} \frac{4\pi}{\lambda^2} (2\pi)^2 a^2 \left(\frac{\lambda}{4}\right)^2 \left(\frac{2}{\pi}\right)^2$

$\sigma_{1FC} = \frac{4\pi}{\lambda^2} A_1^2 \left(\frac{2}{\pi}\right)^2$

$\sigma_k = \pi a^2$

σ letala:

$\sigma \sim 30 m^2$

$\sigma \sim 0.01 m^2$

$\sigma \sim 100 m^2$

Domet radarja:

$P_0 = 10^6 W = 1 MW$

$P_s = 10^{-12} W = 1 pW$

$G = 40 dB (\sim 10 m^2)$

$\lambda = 0.1 m (3 GHz)$

$r = ?$

$$r = \sqrt[4]{\frac{P_0 G^2 \lambda^2}{P_s (4\pi)^3 \sigma}}$$

$\sigma = 30 m^2 \rightarrow r = 350 km$

$\sigma = 3 m^2 \rightarrow r = 197 km$

$\sigma = 0.01 m^2 \rightarrow r = 47 km$

Doppler

$\Delta f = -2 f_0 \frac{\vec{v} \cdot \vec{r}_r}{c_0}$

$|\vec{v} \cdot \vec{r}_r| > 40 m/s$

$\sigma = 3 km^2 = 3 \cdot 10^6 m^2$

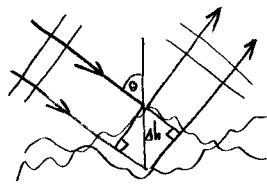
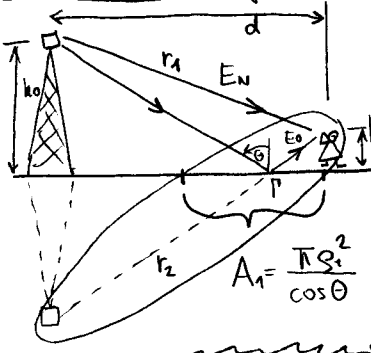
Meritev hitrosti

$f_0 = 24 GHz \rightarrow \lambda = 1.25 cm$

$f_0 = 346 GHz \rightarrow \lambda = 0.86 cm$

$\Delta f = 2 f_0 \frac{v}{c_0}$

1. FC pri odboju

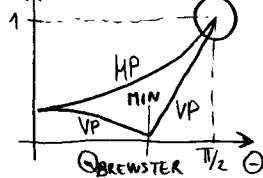


$\Delta \varphi = 2ksh \cos \theta < \frac{\pi}{2}$
za zrcalni odboj -1dB

Rayleigh-ov kriterij za hrpavost: $\Delta h < \frac{\lambda}{8 \cos \theta}$

Odbojnost slabega dielektrika

$\Gamma = -1$ at $\theta = \frac{\pi}{2}$



$E_s \approx \frac{\alpha I}{d} e^{-jk(d + \frac{h_0^2 + h_1^2}{2d})}$

$E_s = E_N + E_O$

$E_s = \alpha I \frac{e^{jkr_1}}{r_1} + \Gamma \alpha I \frac{e^{-jkr_2}}{r_2}$

$E_s \approx \frac{\alpha I}{d} [e^{jkr_1} + \Gamma e^{-jkr_2}]$

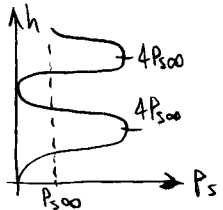
$r_1 = \sqrt{d^2 + (h_0 - h_1)^2} \approx d + \frac{(h_0 - h_1)^2}{2d}$

$r_2 = \sqrt{d^2 + (h_0 + h_1)^2} \approx d + \frac{(h_0 + h_1)^2}{2d}$

$[e^{jk \frac{h_0 h_1}{d}} - e^{-jk \frac{h_0 h_1}{d}}]$

$|E_s| \approx \frac{\alpha I}{d} 2 \sin(k \frac{h_0 h_1}{d})$

$P_s = P_o G_o G_s (\frac{\lambda}{4\pi d})^2 4 \sin^2(k \frac{h_0 h_1}{d})$



$h_0, h_1 \ll d$ VELIKE RAZBAJE

$\sin(k \frac{h_0 h_1}{d}) \approx k \frac{h_0 h_1}{d} = 2\pi \frac{h_0 h_1}{\lambda d}$

$P_s = P_o G_o G_s \frac{h_0^2 h_1^2}{d^4}$

Mestno okolje z ovirami:

$P_s = P_o G_o G_s \frac{h_0^2 h_1^2}{d^N} \propto (\lambda)^{-N}; N=3...5$

ZGLED: $\lambda = 33 \text{ cm}$
 $P_o = 10 \text{ W} = +40 \text{ dBm}$
 $P_s = 10^{-13} \text{ W} = -100 \text{ dBm}$
 $G_o = G_s = 1$
 $h_0 = 1,5 \text{ m}$
 $h_1 = 1,5 \text{ m} / 30 \text{ m}$
 $d = ?$

$d = \sqrt[4]{\frac{P_o}{P_s} G_o G_s h_0^2 h_1^2}$

$h_0 = 1,5 \text{ m} \rightarrow d = 4,74 \text{ km}$

$h_1 = 30 \text{ m} \rightarrow d = 21,2 \text{ km}$

Primerjava prazen prostor:

$d = \sqrt[2]{\frac{P_o}{P_s} G_o G_s (\frac{\lambda}{4\pi})^2} = 263 \text{ km}$

OZRAČJE:

TROPOFERA 0-10km

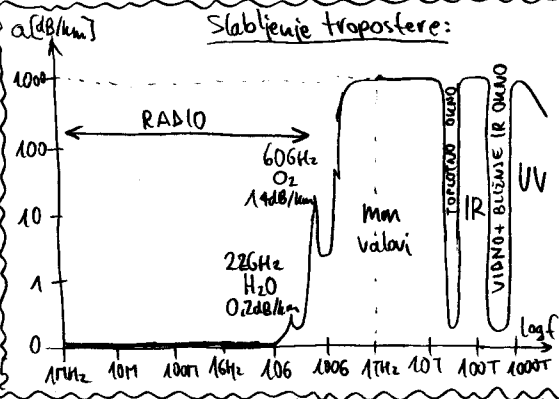
$\epsilon = \epsilon_o \epsilon_r, \gamma \neq 0$

SUHI DEL ($N_2 + O_2$):

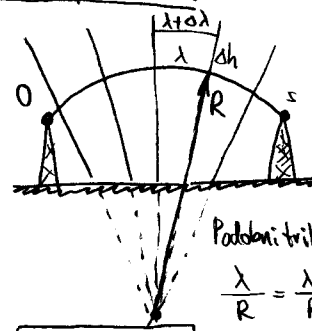
$n = 1 + \Delta n_o e^{-\frac{h}{H}}$

$\Delta n_o = 0,0003$

$H = 8500 \text{ m}$



Krivljenje zarkov:



$\lambda = \frac{\lambda_o}{n}$

$\lambda = \frac{\lambda_o}{1 + \Delta n_o e^{-\frac{h}{H}}}$

$\frac{d\lambda}{dh} = \frac{-\lambda_o}{n^2} \Delta n_o (-\frac{1}{H}) e^{-\frac{h}{H}}$

Podoben trikotniku

$\frac{\lambda}{R} = \frac{1 + \Delta n}{R + \Delta h} = \frac{d\lambda}{dh} \approx \lambda \frac{\Delta n_o}{H} e^{-\frac{h}{H}}$

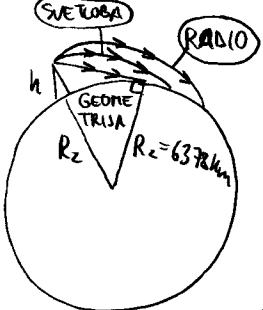
$R = \frac{H}{\Delta n_o} e^{\frac{h}{H}}$

$h=0 \rightarrow R(0) = 28333 \text{ km}$

SUHI + MOVKI DEL

$H_{mokra} = 1,5 \text{ km}$

$\Delta n_{mokra} = f(p, H_2O)$



$n = 1 + \Delta n_{suhi} e^{-\frac{h}{H_{suhi}}} + \Delta n_{mokra} e^{-\frac{h}{H_{mokra}}}$

$R(0) \approx 25000 \text{ km (RADIO)}$

$R(0) \approx 50000 \text{ km (SVETLOBA)}$

Efektivni polmer Zemlje:

$\frac{1}{R_{eff}} = \frac{1}{R_2} - \frac{1}{R}$

$R_{eff} \approx 8600 \text{ km (RADIO)}$

$R_{eff} \approx 7300 \text{ km (SVETLOBA)}$

Zgled: Stolp

$h = 100 \text{ m}$

$d = ?$

$(R_2 + h)^2 = R_2^2 + d^2$

$R_2^2 + 2R_2 h + h^2 = R_2^2 + d^2$

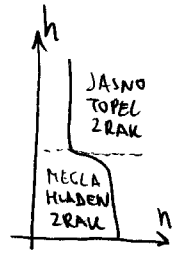
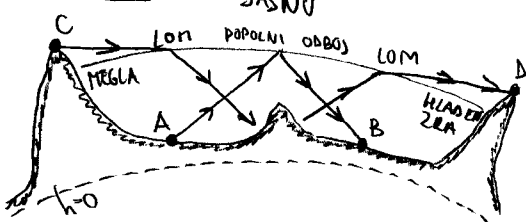
$d = \sqrt{2R_2 h + h^2} \approx \sqrt{2R_2 h}$

Geometrijski domet: $d = 35,7 \text{ km}$ ($R_2 = 6378 \text{ km}$)

Svetlobni domet: $d = 38,2 \text{ km}$ ($R_{eff} = 7300 \text{ km}$)

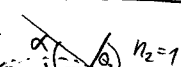
Radialni domet: $d = 41,5 \text{ km}$ ($R_{eff} = 8600 \text{ km}$)

INVERZIJA:



Lom ob sončnem zahodu:

$n_1 = 1,00015$ SVETLOBA



$n_1 \sin \theta_1 = n_2 \sin \theta_2$

$\sin \theta_1 = \frac{n_2}{n_1} = \cos \alpha$

$\alpha = \arccos \frac{1}{n_1}$

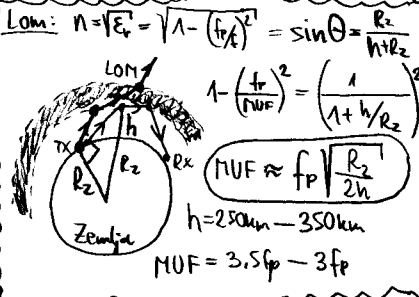
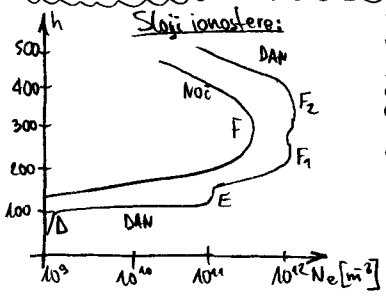
$\alpha \approx 1^\circ$

Ponovitev:
 TROPOSFERA < 10km
 $n = 1 + \Delta n e^{-h}$
 $\Delta n_{dne} = 0,0003$
 $H_{suki} = 8,5km$
 $H_{mudri} = 1,5km$
 $R \approx 25000km @ h=0$
 $R_{eff} \approx 4/3 R_z$

IONOSFERA:
 $h > 60km$
 $\vec{F} = Q\vec{E} = m\vec{a} = m j \omega \vec{r}$
 $\vec{r} = \frac{Q}{j\omega m} \vec{E}$ N[m³]
 $\vec{j} = N Q \vec{r} = \frac{N Q^2}{j\omega m} \vec{E}$
KONVEKTIVNI TOK

delci:
 $m_p \approx 1800 m_e$
 $m_{ion} > m_p$
 samo elektroni!
 $m_e = 9,1 \cdot 10^{-31} kg$
 $Q_e = -1,6 \cdot 10^{-19} A$
 $\vec{j}_y = \frac{N_e Q_e^2}{j\omega m_e} \vec{E}$

$rot \vec{H} = \vec{j} + j\omega \epsilon_0 \vec{E} = \frac{N_e Q_e^2}{j\omega m_e} \vec{E} + j\omega \epsilon_0 \vec{E}$
 $rot \vec{H} = j\omega \epsilon_0 \left(1 - \frac{N_e Q_e^2}{\omega^2 \epsilon_0 m_e}\right) \vec{E}$
 $\epsilon_r = 1 - \frac{N_e Q_e^2}{\omega^2 \epsilon_0 m_e} = 1 - \frac{f_p^2}{f^2}$
 $f_p = \frac{1}{2\pi} \sqrt{\frac{N_e Q_e^2}{\epsilon_0 m_e}} = \begin{cases} \approx 12 MHz DAN \\ \approx 4 MHz NOČ \end{cases}$
Zgled: $f_p = 12 MHz \rightarrow N_e = \frac{\epsilon_0 m_e}{Q_e^2} (2\pi f_p)^2 = 1,8 \cdot 10^{12} \text{elektronov/m}^3$

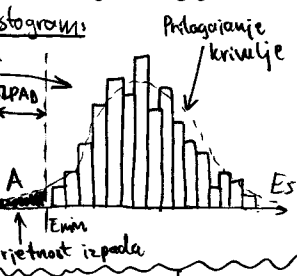
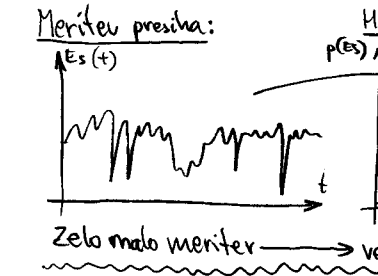
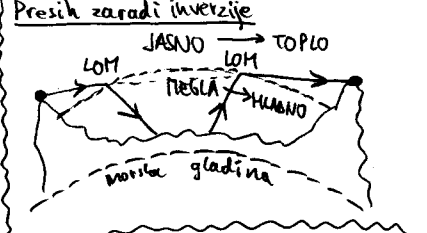
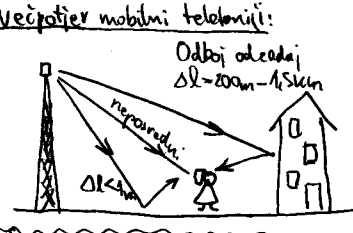
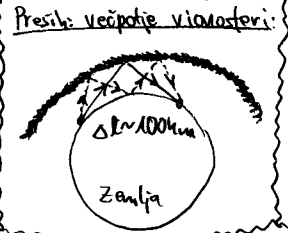


Slabljenje: tli D, E
 LUF! ni točno definiran

	MUF	LUF
DAN	3MHz	5MHz
NOČ	12MHz	-

Fazna in skupinska hitrost:
 $v_f = \frac{\omega}{\beta} = \frac{\omega}{n \frac{\omega}{c_0}} = \frac{c_0}{n} = \frac{c_0}{\sqrt{1 - (f_p/f)^2}}$
 $v_g = \frac{d\omega}{d\beta} = c_0 \sqrt{1 - (f_p/f)^2} < c_0$
Pogreški GPS @ 1575,42MHz >> f_p
 $N_e < c_0 < f_p$ DAN: $|\Delta f| \approx 30m$!

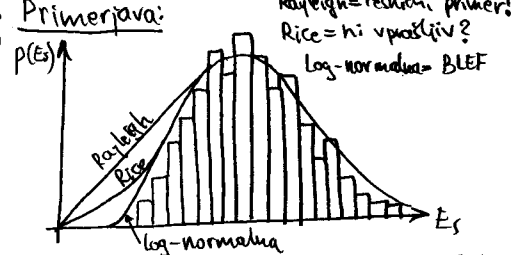
Živomagnetna rezonanca:
 Zemljeno magnetno polje $H_0 \approx 40 A/m$
 $\vec{\omega}_g = \frac{Q_e \mu_0}{m_e} \vec{H}_0$ $f_g \approx 1,4 MHz$
 Visoko slabljenje
 Faraday-eva sukajna polarizacija



Rayleigh: vsota malih kotalcev
 $p(E_x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{E_x^2}{2\sigma^2}}$
 $p(E_y) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{E_y^2}{2\sigma^2}}$
 $p(E_s) = \int p(E_x) p(E_y) E_s d\varphi$
 $E_s = |\vec{E}|$
 $p(E_s) = \frac{E_s}{\sigma^2} e^{-\frac{E_s^2}{2\sigma^2}}$
 $P_{izpada} = \int_0^{E_{min}} p(E_s) dE_s$ Samo en podatek!

Rice: neposredni žarek + vsota malih
 $p(E_s) = \frac{E_s}{\sigma^2} e^{-\frac{E_s^2 + E_0^2}{2\sigma^2}}$
 $I_0(\frac{E_s E_0}{\sigma^2})$
 Bessel-ova funkcija
 dva podatka σ in E_0

Log-normalna:
 $E_{dB} = 20 \log |E/E_{REF}|$
 $p(E_{dB}) = \frac{1}{\sqrt{2\pi} \sigma_{dB}} e^{-\frac{(E_{dB} - m_{dB})^2}{2\sigma_{dB}^2}}$
 dva podatka m_{dB} in σ_{dB}



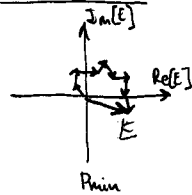
Rayleigh za moč:
 $P_s = \alpha E_s^2 = \alpha (E_x^2 + E_y^2)$
 $\langle P_s \rangle = \alpha (\langle E_x^2 \rangle + \langle E_y^2 \rangle) = \alpha 2\sigma^2$

$P_{izpada} = \int_0^{E_{min}} \frac{E_s}{\sigma^2} e^{-\frac{E_s^2}{2\sigma^2}} dE_s = \int_0^{E_{min}^2} \frac{1}{2\sigma^2} e^{-\frac{E_s^2}{2\sigma^2}} dE_s = \int_0^{\frac{P_{min}}{\alpha}} \frac{1}{\langle P_s \rangle} e^{-\frac{P_s}{\langle P_s \rangle}} dP_s = 1 - e^{-\frac{P_{min}}{\langle P_s \rangle}}$

Zgled: GSM telefon
 $P_{min} = -105 dBm = 31,6 \cdot 10^{-15} W$
 $\langle P_s \rangle = -90 dBm = 10^{-12} W$
 $P_{izpada} = ?$
 $P_{izpada} = 1 - e^{-\frac{P_{min}}{\langle P_s \rangle}} = 1 - e^{-\frac{31,6}{1000}} = 0,0311 = 3,11\%$

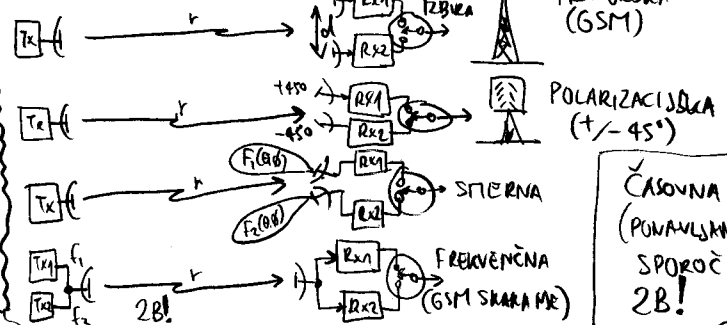
Približek: $P_{min} \ll \langle P_s \rangle$
 $e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \dots \approx 1 - x$
 $P_{izpada} = 1 - e^{-\frac{P_{min}}{\langle P_s \rangle}} \approx \frac{P_{min}}{\langle P_s \rangle} = 3,16\%$

Ponovitev: Rayleigh-ova porazdelitev



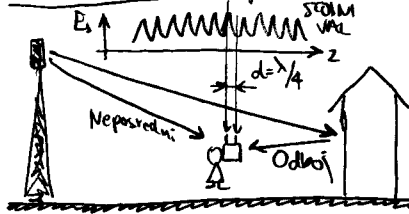
$E_s = |E| = \sqrt{P_s}$
 $p(E) = \frac{2E_s}{\langle E_s^2 \rangle} e^{-\frac{E_s^2}{\langle E_s^2 \rangle}}$
 $p(P_s) = \frac{1}{\langle P_s \rangle} e^{-\frac{P_s}{\langle P_s \rangle}}$
 $P_{izpada} = \int_0^{\infty} p(P_s) dP_s = 1 - e^{-\frac{P_{min}}{\langle P_s \rangle}} = 1 - e^{-\frac{E_{min}^2}{\langle E_s^2 \rangle}}$

Raznolikost: (diversity)

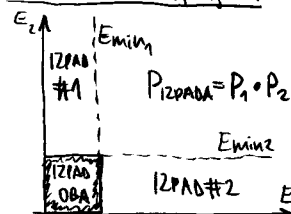


ČASOVNA (POVAJLJAJNE SPOROČILA) 2B!

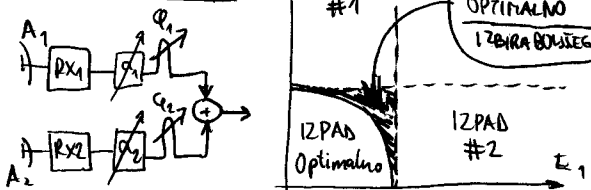
Izharisovane korelacije:



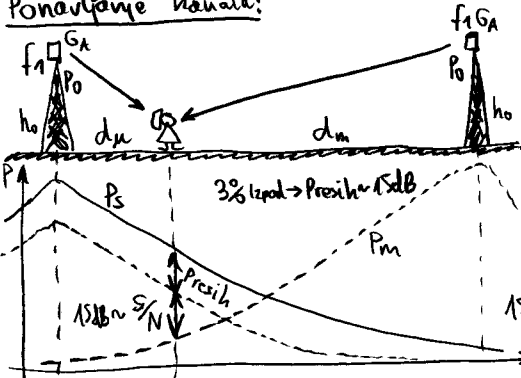
Nekorreliran sprejem:



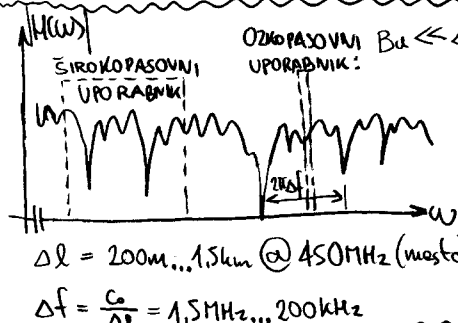
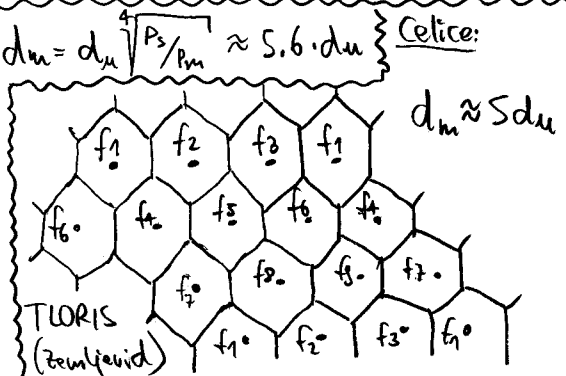
Optimalno sestavljanje:



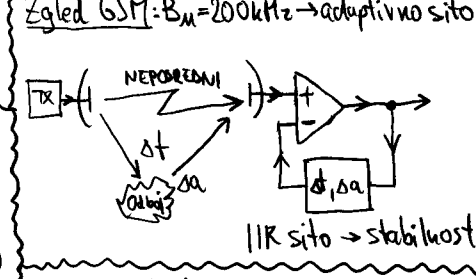
Ponavrtanje kanala:



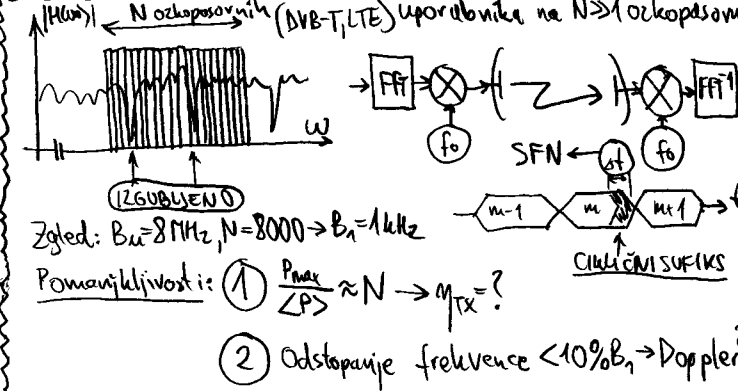
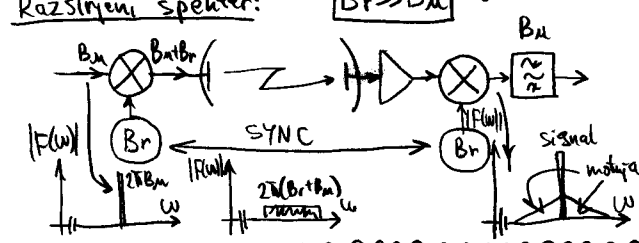
$d_m = d_u \sqrt[4]{\frac{P_s}{P_m}} \approx 5.6 \cdot d_u$
 $d_m \approx 5 d_u$
 Celice:



Zglej GSM: $B_u = 200\text{kHz} \rightarrow$ adaptivno sito
 NEPOREDNI
 IR sito \rightarrow stabilnost?

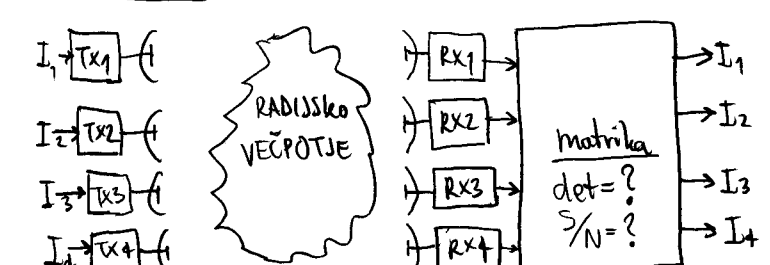


Razširjeni spekter:



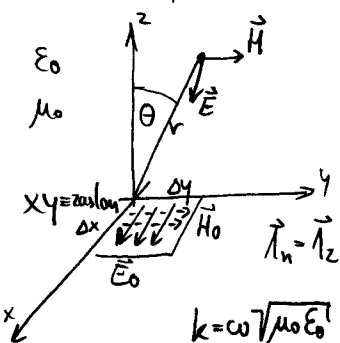
MIMO: (Multiple-In-Multiple-Out)

Zglej 4x4:



MIMO 2x2 = POLARIZACIJSKI MUX (WiFi, LTE)

EM Huygens-ov izvor 24.10.2014



$$\vec{E}_0 = \vec{n}_x E_0$$

$$\vec{H}_0 = \vec{n}_y \frac{E_0}{Z_0}$$

$$\vec{K} = \vec{n}_x \times \vec{H}_0 = -\vec{n}_z \frac{E_0}{Z_0}$$

$$\vec{n}_n = \vec{n}_z$$

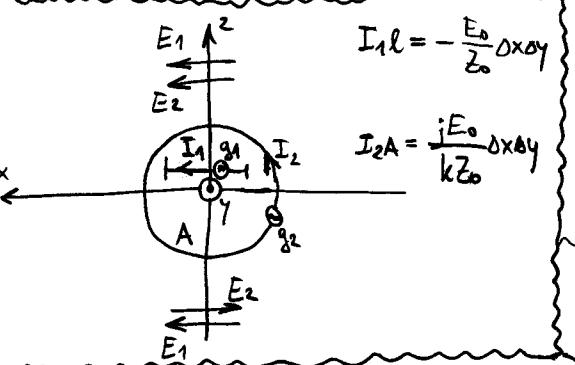
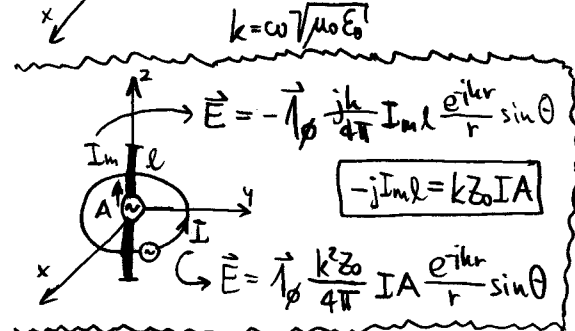
$$\vec{K}_m = \vec{E}_0 \times \vec{n}_n = -\vec{n}_y E_0$$

$$\vec{E}_1 = \vec{n}_{\theta x} \frac{jkz_0}{4\pi} I \Delta x \frac{e^{i(kr - \omega t)}}{r} \sin \theta_x = -\vec{n}_{\theta x} \frac{jkz_0}{4\pi} |\vec{K}| \Delta x \frac{e^{i(kr - \omega t)}}{r} \sin \theta_x$$

$$\vec{E}_1 = -\vec{n}_{\theta x} \frac{jk}{4\pi} E_0 \Delta x \Delta y \frac{e^{i(kr - \omega t)}}{r} \sin \theta_x$$

$$\vec{E}_2 = -\vec{n}_{\phi y} \frac{jk}{4\pi} I_m \Delta y \frac{e^{i(kr - \omega t)}}{r} \sin \theta_y = \vec{n}_{\phi y} \frac{jk}{4\pi} |\vec{K}_m| \Delta x \Delta y \frac{e^{i(kr - \omega t)}}{r} \sin \theta_y$$

$$\vec{E}_2 = \vec{n}_{\phi y} \frac{jk}{4\pi} E_0 \Delta x \Delta y \frac{e^{i(kr - \omega t)}}{r} \sin \theta_y$$



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{jk}{4\pi} E_0 \Delta x \Delta y \frac{e^{i(kr - \omega t)}}{r} [-\vec{n}_{\theta x} \sin \theta_x + \vec{n}_{\phi y} \sin \theta_y]$$

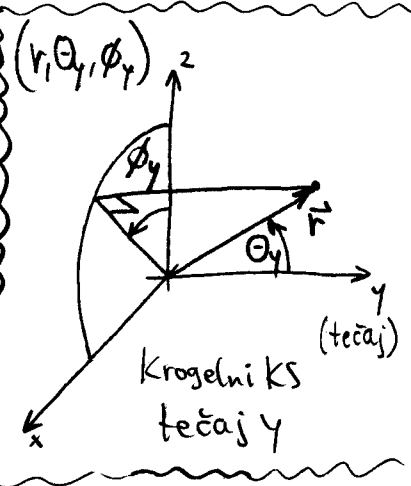
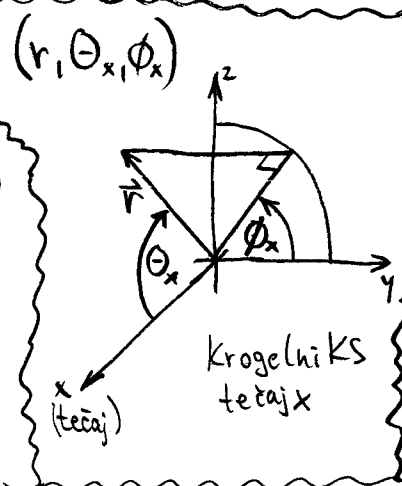
$$-\vec{n}_{\theta x} \sin \theta_x + \vec{n}_{\phi y} \sin \theta_y = (\cos \theta + 1) [\vec{n}_{\theta} \cos \phi - \vec{n}_{\phi} \sin \phi]$$

$$\vec{E} = (\vec{n}_{\theta} \cos \phi - \vec{n}_{\phi} \sin \phi) \frac{jk}{4\pi} E_0 \Delta x \Delta y \frac{e^{i(kr - \omega t)}}{r} (\cos \theta + 1)$$

$$\vec{E} = (\vec{n}_{\theta} \cos \phi - \vec{n}_{\phi} \sin \phi) \frac{j}{2\lambda} E_0 \Delta x \Delta y \frac{e^{i(kr - \omega t)}}{r} (\cos \theta + 1)$$

endni vektor!
POLARRACISA

F(theta, phi)



$$\vec{n}_{\theta} = \vec{n}_x \cos \theta \cos \phi + \vec{n}_y \cos \theta \sin \phi - \vec{n}_z \sin \theta$$

$$\vec{n}_{\phi} = -\vec{n}_x \sin \phi + \vec{n}_y \cos \phi$$

$$\cos \theta = \frac{z}{r} \quad \sin \theta = \frac{\sqrt{x^2 + y^2}}{r}$$

$$\cos \phi = \frac{x}{\sqrt{x^2 + y^2}} \quad \sin \phi = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\vec{n}_{\theta_x} = \vec{n}_y \cos \theta_x \cos \phi_x + \vec{n}_z \cos \theta_x \sin \phi_x - \vec{n}_x \sin \theta_x = \vec{n}_y \frac{xy}{r\sqrt{y^2+z^2}} + \vec{n}_z \frac{xz}{r\sqrt{y^2+z^2}} - \vec{n}_x \frac{\sqrt{y^2+z^2}}{r} \quad \sin \theta_x = \frac{\sqrt{y^2+z^2}}{r}$$

$$-\vec{n}_{\theta_x} \sin \theta_x = -\vec{n}_y \frac{xy}{r^2} - \vec{n}_z \frac{xz}{r^2} + \vec{n}_x \frac{y^2+z^2}{r^2} = \vec{n}_x (1 - \sin^2 \theta \cos^2 \phi) - \vec{n}_y \sin^2 \theta \cos \phi \sin \phi - \vec{n}_z \sin \theta \cos \theta \cos \phi$$

$$\vec{n}_{\phi_y} = -\vec{n}_z \sin \phi_y + \vec{n}_x \cos \phi_y = -\vec{n}_z \frac{x}{\sqrt{z^2+x^2}} + \vec{n}_x \frac{z}{\sqrt{z^2+x^2}} \quad \sin \theta_y = \frac{\sqrt{z^2+x^2}}{r}$$

$$\vec{n}_{\phi_y} \sin \theta_y = -\vec{n}_z \frac{x}{r} + \vec{n}_x \frac{z}{r} = \vec{n}_x \cos \theta - \vec{n}_z \sin \theta \cos \phi$$

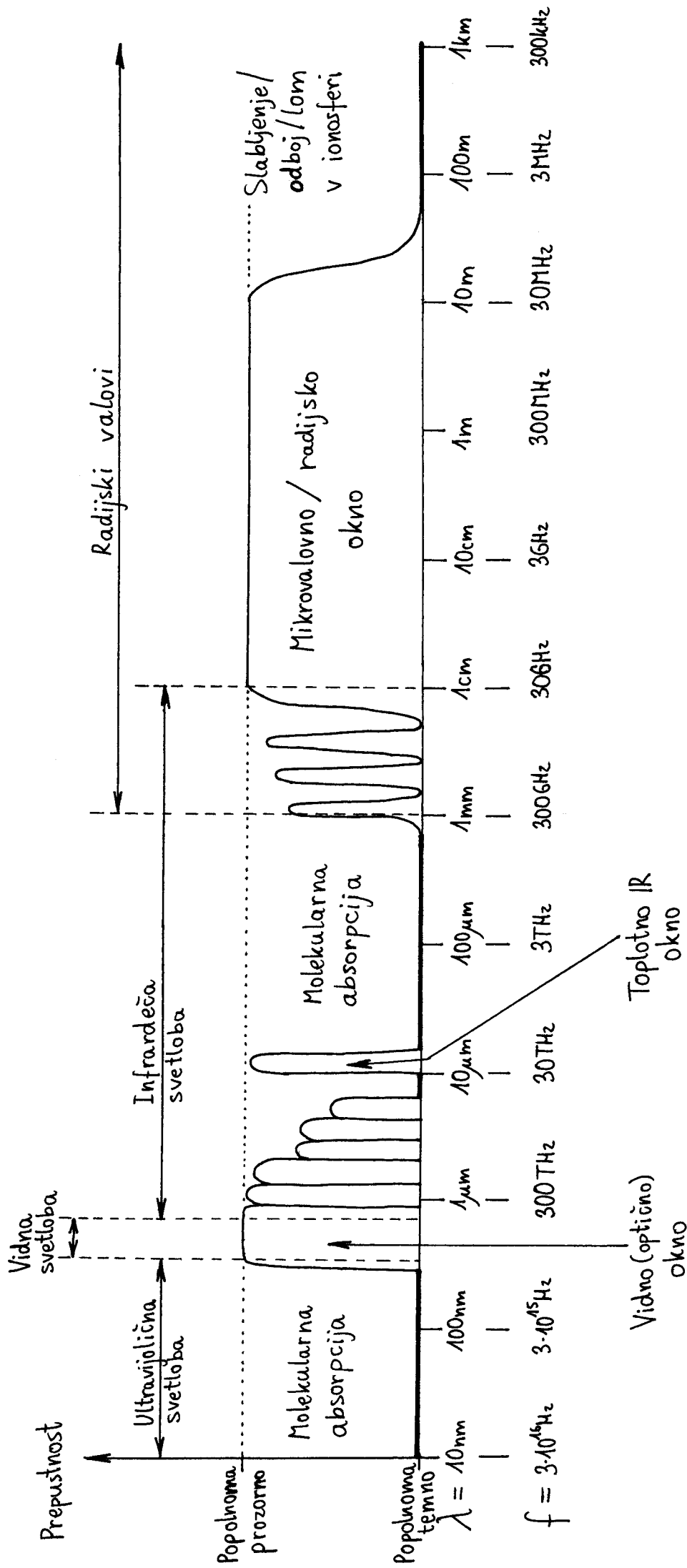
$$-\vec{n}_{\theta_x} \sin \theta_x + \vec{n}_{\phi_y} \sin \theta_y = \vec{n}_x (1 + \cos \theta - \sin^2 \theta \cos^2 \phi) - \vec{n}_y \sin^2 \theta \cos \phi \sin \phi - \vec{n}_z (1 + \cos \theta) \sin \theta \cos \phi =$$

$$= \vec{n}_x (1 + \cos \theta + (\cos^2 \theta - 1) \cos^2 \phi) + \vec{n}_y (\cos^2 \theta - 1) \cos \phi \sin \phi - \vec{n}_z (1 + \cos \theta) \sin \theta \cos \phi =$$

$$= (\cos \theta + 1) [\vec{n}_x (1 + \cos \theta \cos^2 \phi - \cos^2 \phi) + \vec{n}_y (\cos \theta \cos \phi \sin \phi - \cos \phi \sin \phi) - \vec{n}_z \sin \theta \cos \phi] =$$

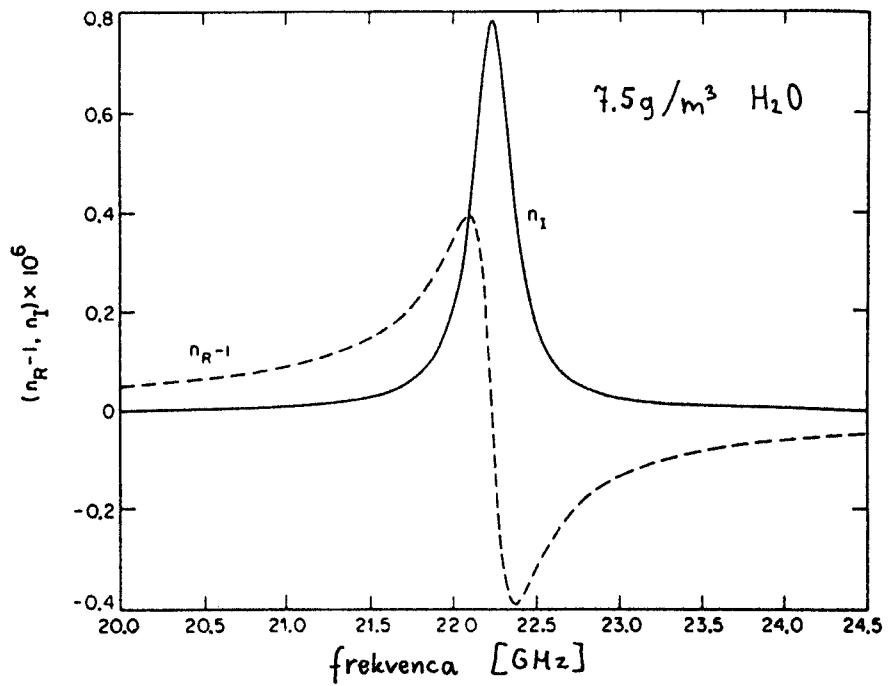
$$= (\cos \theta + 1) [\vec{n}_x (\cos \theta \cos^2 \phi + \sin^2 \phi) + \vec{n}_y (\cos \theta \cos \phi \sin \phi - \cos \phi \sin \phi) - \vec{n}_z \sin \theta \cos \phi] =$$

$$= (\cos \theta + 1) [\vec{n}_{\theta} \cos \phi - \vec{n}_{\phi} \sin \phi]$$



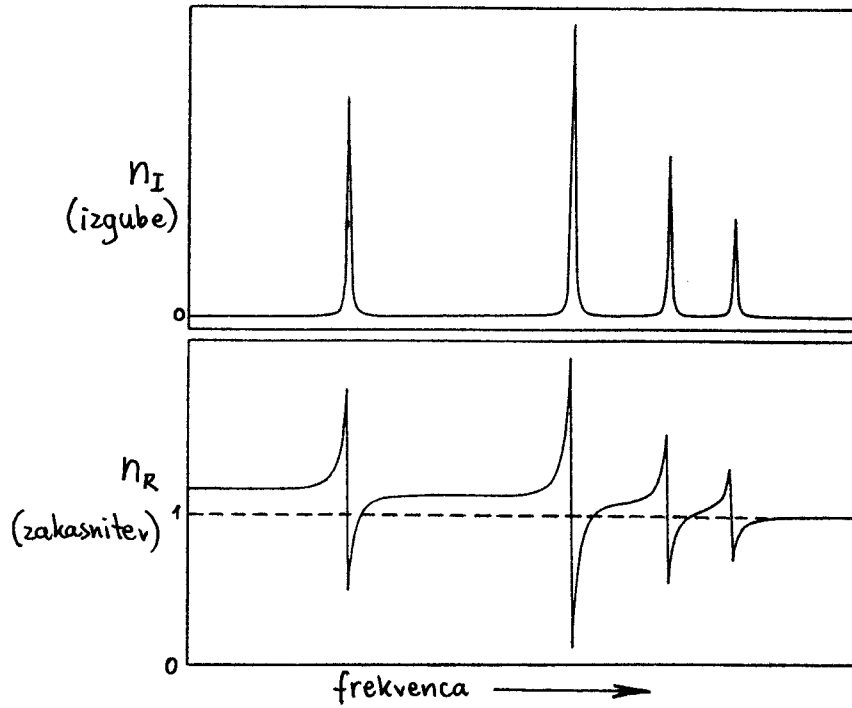
Prepustnost zemeljskega ozračja za elektromagnetno valovanje

Kompleksni
lomni
količnik
 $n = n_R + jn_I$

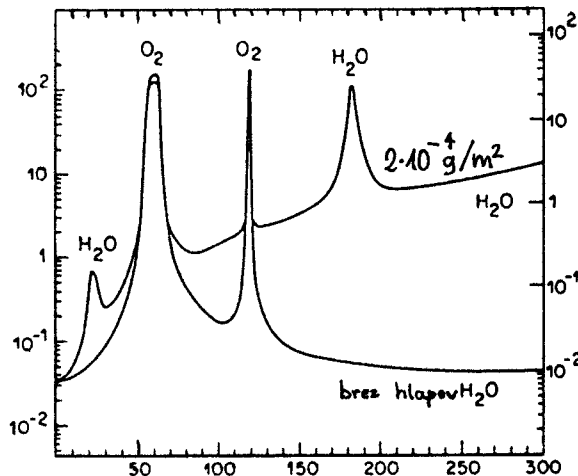


Kompleksni
lomni
količnik

$n = n_R + jn_I$



Zenitno
slabjenje
[dB]



Zenitno
slabjenje
[N_p]

Mikrovalovna molekularna absorpcija v zemeljskem ozračju

$$\lambda = \frac{\lambda_0}{n}$$

Lomni količnik v troposferi:

$$n = 1 + \Delta n e^{-\frac{h}{h_0}}$$

R poiščemo iz podobnih trikotnikov:

$$\lambda = \alpha R \quad \alpha \equiv \text{konstanta}$$

$$\frac{d\lambda}{dh} = \alpha \frac{dR}{dh} = \alpha$$

$$\frac{d\lambda}{dh} = \frac{d}{dh} \left(\frac{\lambda_0}{n} \right) = \frac{\lambda_0 \Delta n}{h_0 n^2} e^{-\frac{h}{h_0}}$$

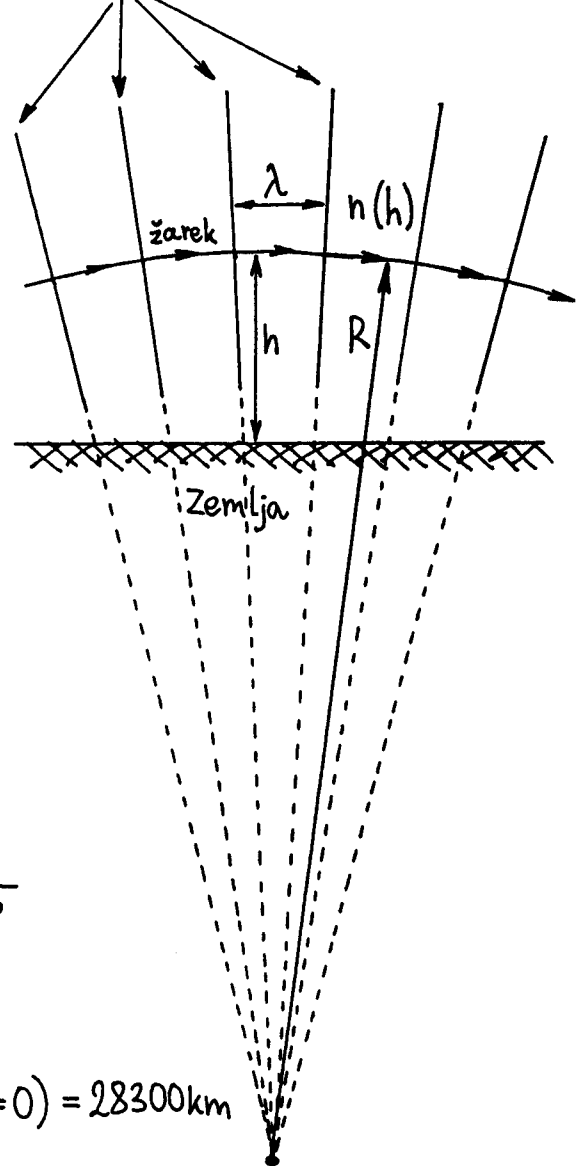
$$R = \frac{\lambda}{\alpha} = \frac{h_0 n^2}{\Delta n} e^{\frac{h}{h_0}} \approx \frac{h_0}{\Delta n} e^{\frac{h}{h_0}}$$

Suha troposfera:

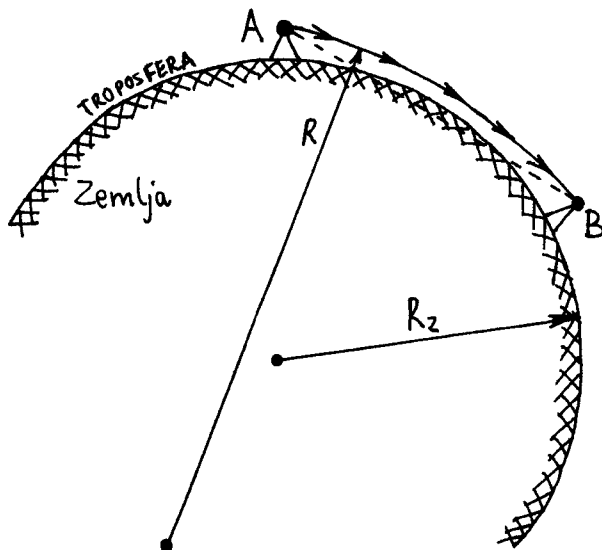
$$h_0 = 8.5 \text{ km}; \quad \Delta n = 0.0003 \longrightarrow R(h=0) = 28300 \text{ km}$$

$$\text{Vlažna troposfera: } R(h=0) \approx 25000 \text{ km}$$

VALOVNE FRONTE



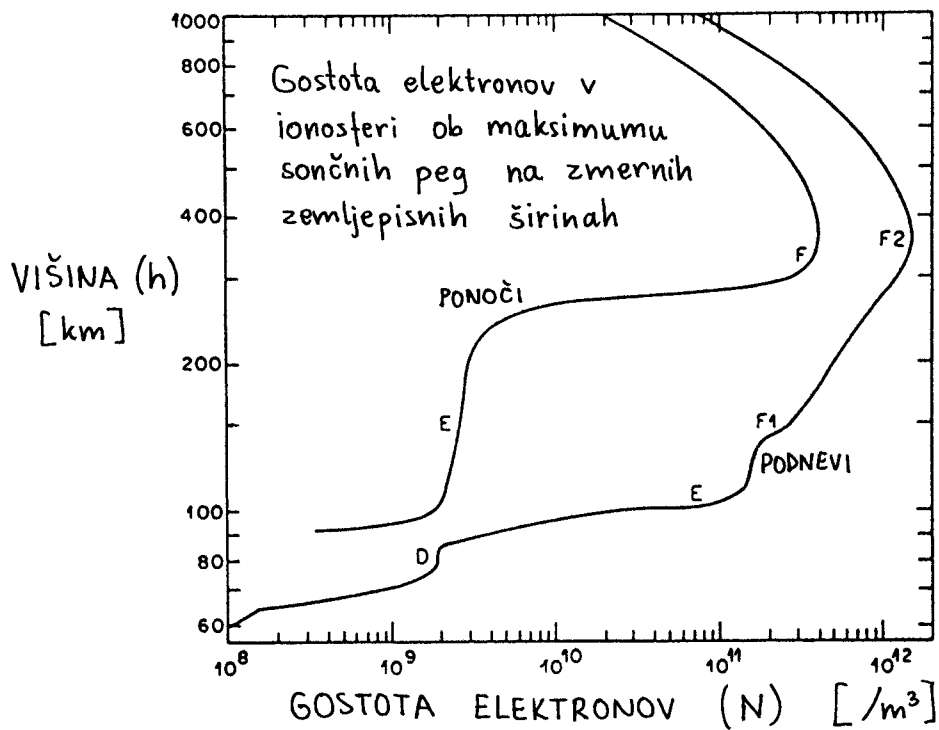
Lom radijskih valov v troposferi



$$\frac{1}{R_e} = \frac{1}{R_2} - \frac{1}{R}$$

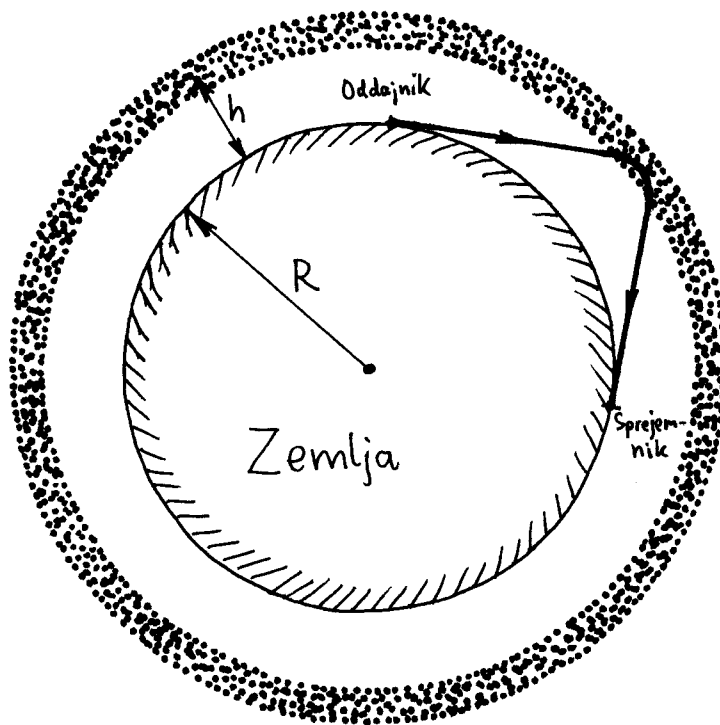
$$R_e \approx 8600 \text{ km} \approx \frac{4}{3} R_2$$

Efektivni polmer zemeljske površine



Lomni količnik: $n = \sqrt{1 - \left(\frac{f_p}{f}\right)^2}$

Frekvenca plazme: $f_p = \frac{1}{2\pi} \sqrt{\frac{Nq_e^2}{\epsilon_0 m_e}} = \sqrt{80.8 \frac{\text{m}^3}{\text{s}^2} N} = \begin{cases} \text{max} \\ \sim 12 \text{ MHz} \\ \text{PODNEVI} \\ \text{-----} \\ \text{max} \\ \sim 4 \text{ MHz} \\ \text{PONOČI} \end{cases}$



Zaradi loma ob poševnem vpadu valovanja:
 $MUF > f_p$

$$MUF \approx f_p \sqrt{\frac{R}{2h}}$$

$$MUF \approx 3 f_p$$

$$MUF \approx \begin{cases} 36 \text{ MHz} \text{ PODNEVI} \\ \text{-----} \\ 12 \text{ MHz} \text{ PONOČI} \end{cases}$$

Zelo visoka disperzija (snovna, rodovna) \rightarrow zmogljivost $\sim 100 \text{ bit/s}$

Radijska zveza preko loma/odboja v ionosferi