On Wire-Grid Representation of Solid Metallic Surfaces

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***Abstract*— This short paper deals with the wire-grid representation of metallic surfaces in numerical electromagnetic modeling. We discuss in particular the adequacy of the well-known and widely used Equal Area Rule to calculate the radii of wire-grid models. We show that the Equal Area Rule is accurate as long as the wire-grid consists of a simple rectangular mesh. For more complex body-fitted meshes, using of other polygons such as triangles, the Equal Area Rule appears to be less accurate in reproducing the electromagnetic field scattered by metallic bodies. The conclusions of the paper are supported by numerical simulations performed using a parallel version of NEC and experimental data obtained on a vehicle illuminated by an EMP simulator.**

I. Introduction

THE use of a wire grid model to approximate a continuous conducting surface was introduced by Richmond in 1966 [1]. By defining expressions for the scattered field of a wire segment, a point-matching solution was found for the scattering of a wire-grid structure by solving a system of linear equations [1]. The paper reports good agreement on simulation results of structures as simple as a conducting plate and as complex as a segmented sphere.

The wire-grid method has been adopted and the fast progress of digital computers has contributed to the evolution and development of even more complicated arbitrarily shaped models. The growth in complexity of the evaluated problems has permitted the observation of certain limitations derived from the fact that a wire-grid is, in fact, a highly simplified representation of reality. It has been observed in particular that the far-field results obtained with a wire-grid representation of a perfectly conducting closed surface are very reliable [2]. On the contrary, the wire-grid has been considered by some authors as a poor model of a closed surface when it comes to interaction calculations (currents and charge densities induced on the surface of a structure) [2], a rather reasonable conclusion, considering the fact that the wire-grid is an equivalent model of the solid surface.

A wire-grid approximation of a solid conducting surface introduces a number of new variables that affect the accuracy of the solution. For instance, the grid spacing must be carefully selected. This is further complicated by the fact that this particular parameter has an impact on the computation time and resource requirements [3]. In addition, the segments representing the solid structure must each be defined in terms of its length, width and position in space. Although the maximum segment length can be readily specified as a function of the frequency, it has been observed by many authors [2], [4], [5], [6], [7] that numerical simulation results are very sensitive to wires radii. As of the writing of this paper, the calculation of this parameter could still be characterized as an art form or guesswork.

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The wire-grid method has evolved and several numerical formulations based on segmented wires, patches or cells are available today for the solution of the electromagnetic scattering problem. One of these numerical solutions is based on the Method of Moments (MoM), for which the most popular incarnation in the frequency domain is the Numerical Electromagnetics Code (NEC) [8].

The "equal area rule" (EAR), also known as the "same surface area" and in some cases as the "twice surface area", has been for years a rule of thumb for the calculation of segment radius in wire-grid modeling using NEC (e.g. [4], [5], [6], [7]). The rule states that the surface area of the wires parallel to one linear polarization is made equal to the surface area of the solid surface being modeled. Ludwig [4] defines the issue as being "clearly complex" and even though his paper adds new information to the problem by running several variations of a canonical problem (an infinite circular cylinder) it does not provide the final answer to the wire radius question. The author does conclude, however, that "the results certainly enhance confidence in the same surface area wire size rule of thumb".

The problem of the modeling of an infinite cylinder was revisited by Paknys in 1991 [5]. The author arrives at the conclusion that the equal area rule gives the best accuracy for the E-field for this particular problem. However, the author also observes that the EAR does not always work and attempts to explain why a unique criterion has not yet been found.

In 1991, Trueman et al [6] summarized a series of rules for wire-grid simulation and produced a group of wire-grid modeling guidelines. They also considered a nonrectangular grid for which they derived a general expression allowing the calculation of the segment radius as a function of the two adjacent mesh surfaces to which it belongs.

The aim of the present paper is to analyze the degree of accuracy achieved by the EAR for uniform rectangular and body-fitted wire-grids.

The EAR, as it is known today, is described in Section II. Also in that section a particular form of the rule of thumb for squared meshes is presented as well as the generalized formula developed later for arbitrary meshes. In Section III, we give some numerical examples of the application of the EAR and we present some particular cases for which the generalization fails to give satisfactory results.

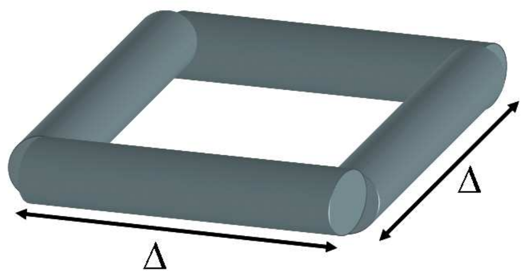


Fig. 1. Four single wires representing a solid square patch

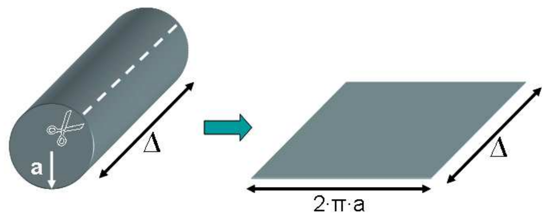


Fig. 2. Surface area of a wire

II. The Equal Area Rule

As mentioned in the introduction, the simulation of a solid surface by means of a wire-grid approximation requires the proper selection of a certain number of parameters. The segment length is determined by the frequency at which the model needs to be evaluated. An appropriate selection of the segment length results in a more computer efficient model. The use of too large a number of segments may produce unacceptably costly models in terms of memory and computing power. On the other hand, the use of a small number of segments may have an impact on the accuracy of the solution. The wire radius, on the contrary, does not affect in any way the computing power or memory requirements, but, as we shall see, may have a significant effect on the quality of the solution.

The rule of thumb for the selection of the wire radius which has been applied for more than a decade was obtained by empirical observation while testing several different radii on a canonical problem. As the optimal radius was found, it was observed that a numerical relationship appears to exist between the value of this radius and the area of the solid surface being modeled [4], [5], [6], [7]. Consider a square patch of side Δ. The simplest wire-grid representation of this structure would be the one formed by four single wires on the four sides of the square as seen in Figure 1. According to the EAR, the optimum wire radius for one single segment is the one obtained by calculating the surface area of the wire (Figure 2) and setting this area equal to the solid surface being modeled, in this case, the one already shown in Figure 1. As a result, the circumference of the cross section of the cylindrical conductor must be made equal to the segment length Δ and, therefore, the radius given by the EAR may be obtained as:

$$a = \frac{\Delta}{2 \cdot \pi} \tag{1}$$

This version of the EAR appears to have worked well for many problems over the years (e.g. [4], [5], [9]). On the other hand, the Method of Moments, on which NEC is based, allows the use of body-fitted meshes that nicely reproduce the geometry of the object. Clearly, in many cases, a square-mesh representation of a 3D structure will result in a rather rough model, from which we would expect less precision and, therefore, larger errors. Additionally, obtaining a square-mesh representation of an existing object is not always a simple task. In fact, complex structures may be represented by the CAD files that were created and used during design and construction. These CAD files often use triangular or even arbitrarily shaped meshes that better represent the real contours and small details of the geometry.

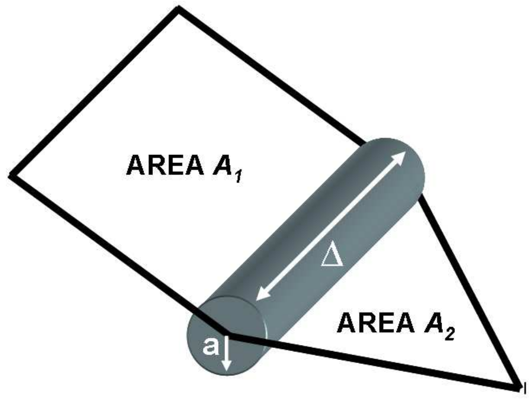


Fig. 3. Equal Area Rule for an arbitrarily shaped mesh

For arbitrarily shaped meshes, a general expression for the calculation of the wire radius has been presented in [6]. The formula takes into account the surface area of the two shapes adjacent to the segment (A_1 and A_2) for which the radius is required (Figure 3). The result is also dependent on the segment length Δ:

$$a = \frac{A_1 + A_2}{4 \cdot \pi \cdot \Delta} \tag{2}$$

For the particular case where both adjacent surfaces are square of side Δ, the two areas become Δ^2 and we obtain the expression for the EAR of a rectangular mesh as already presented in (1).

III. Numerical Examples using Parallel NEC

A model represented by a perfectly squared and homogeneous mesh guaranties that other rules in the construction of a NEC model are well respected. Indeed, a NEC model should avoid adjacent segments featuring large variations of segment length and radius. Some variations are allowed as long as they are smooth [8]. Clearly a perfectly square model will exhibit no changes at all in segment length and radius.

When a model is constructed using an arbitrarily shaped mesh, respecting the existing guidelines which include not having abrupt changes of length and radius in adjacent segments becomes an almost impossible task to accomplish for realistic body-fitted models. Obviously, some models can be constructed with complex meshes that exhibit homogenous segment lengths, but this only applies to particular figures and in no way does it apply to extracted CAD data for practical

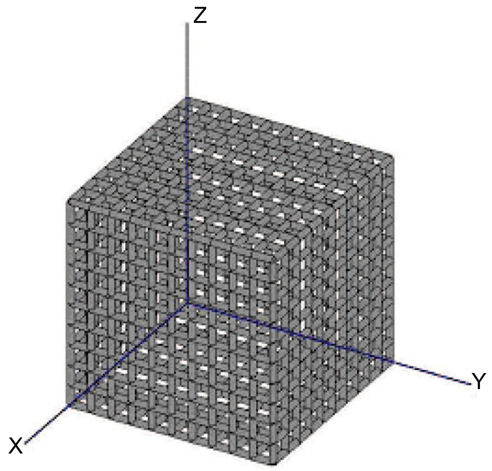


Fig. 4. Cube with rectangular Mesh

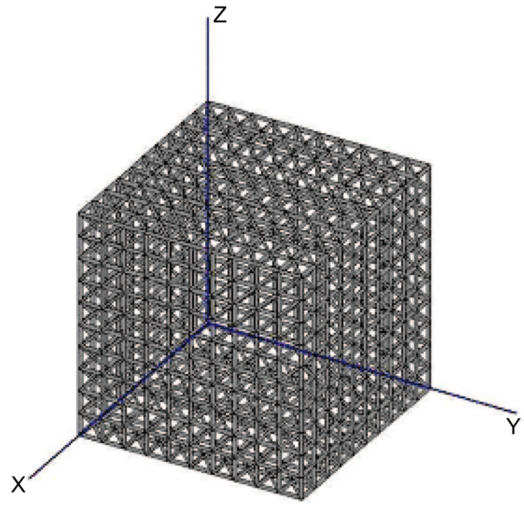


Fig. 5. Cube with triangular Mesh

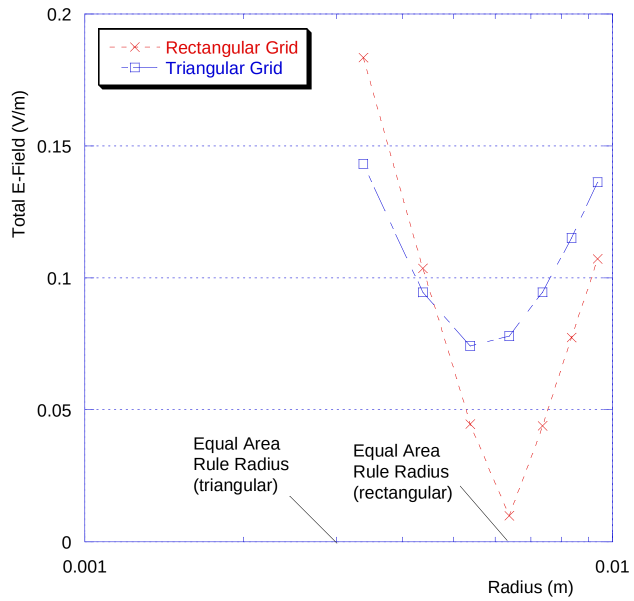
applications. Another problem derives from the difficulty in calculating the appropriate wire radii by applying the EAR general formula. This requires the determination of the surface area of the two shapes adjacent to the segment under consideration and, therefore, additional and sometimes complicated post processing of the mesh is necessary.

Let us examine in what follows a simple example of a closed metallic cube illuminated by a plane wave characterized by an E-field with an amplitude of 1 V/m and a frequency of 300 MHz (λ = 1 m). Two different polarizations where taken into account (1) vertical and (2) oblique polarization. The side of the cube is 40 cm long. The cube was meshed using a perfectly square and homogenous grid of 4 cm length (Figure 4). If the surfaces of the cubic Faraday cage are well represented by their wire grid homologues, illuminating the cube with a plane wave and then measuring the E field anywhere in the interior should give a result very close to zero. Several versions of the cube were created with different radii, including the one predicted by the EAR formula. We added diagonal segments cutting each square patch of the grid in half, creating a triangular mesh out of the same cube (Figure 5). At first, one would imagine that this should improve the accuracy of the wire-grid representation of the cube. All the radii were recalculated so that they would comply with the EAR general formula (Equation 2). Since all of the triangles of the resulting mesh are isosceles (i.e. two equal sides) and identical, the resulting model exhibited two different radii. Again, several versions of this cube were created by systematically changing the values of these two radii keeping the proportionality factor between them. We found basically no fundamental differences using other frequencies, in particular the cutoff frequency of the TE101 mode which is 530 MHz, and 1000 MHz (approximately twice the cutoff frequency of the TE101 mode). The E field calculated at the center of the square-mesh cube as a function of the wire radius is presented in Figure 6 for a vertical and oblique polarization of the incident field. Since the triangular model is characterized by two different radii (corresponding to the vertical/horizontal wires and to the diagonal ones, respectively), we presented its results in Figure 6 as a function of the radius of the vertical/horizontal segments only. The values for the radii predicted by the EAR formula (for both rectangular and triangular meshes) are also shown in that figure. As it can be seen, the prediction of the EAR for the rectangular case corresponds with the minimum of the total field evaluated at the center of the square meshed cube. This minimum was, as expected, very close to zero.

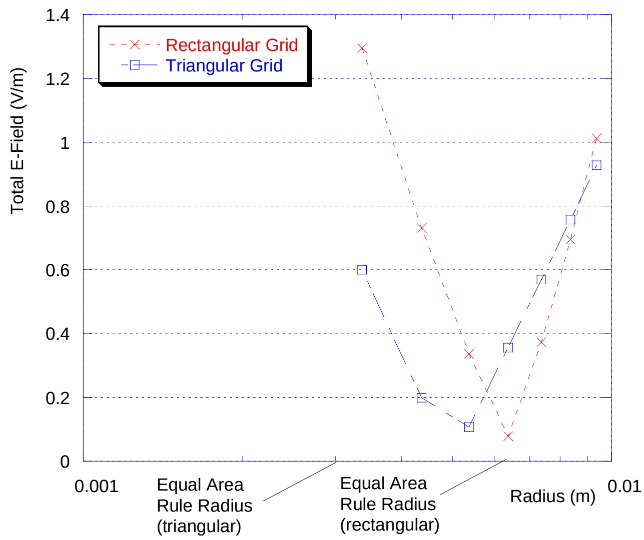
Surprisingly, the application of the radius predicted by the EAR for a square mesh to the vertical and horizontal segments in the triangular case appears to give nearly optimum results. Moreover, the radius predicted by the EAR for the triangular mesh is far from the optimum radius. Contrary to our expectations, the triangular model was far less effective than the simple square model (see Fig. 6(a)). Using an oblique polarization for the incident field (Fig. 6(b)), we obtain a similar behavior. In this case, the shielding effectiveness of both models results affected. However, the radius predicted by the general formula of the EAR for the triangular mesh does not correspond to the optimum value.

A possible interpretation of this rather unanticipated result could be the following. The idea behind the EAR is that we suppose that a metallic surface could be accurately represented by an equivalent wire grid model with proper values for segment lengths and radii. In other words, we substitute the original geometry (a closed metallic surface) by an equivalent one (a wire grid model). In this case, adding new elements to the grid making the holes smaller does not improve the performance of the whole surface because the size of the holes is not the only parameter involved. For this particular example, there might exist an optimum combination of parameters (segment lengths, radii) that would render the triangular version even better than the rectangular one, at the same given frequency. However, this combination of parameters includes segment radius as probably the most important value, and the existing EAR formula appears not able to properly predict it.

As a further example, consider the two models in Figure 7 representing a vehicle used in a recent electromagnetic compatibility study [10]. Model (a) is a very good approximation of the original CAD data. The triangular mesh faithfully re-

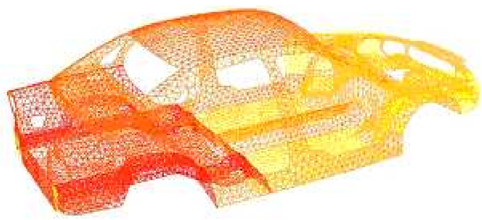


(a) Incident plane wave has vertical polarization.

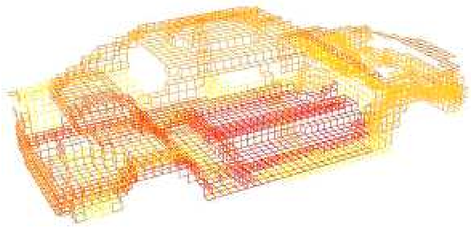


(b) Incident plane wave has oblique polarization.

Fig. 6. Simulated E-field as a function of the wire radius for a rectangular (squared) and a triangular meshes



(a) Triangular(bodyfitted) mesh (17000+ segments)



(b) Squared mesh (8000 segments)

Fig. 7. Two different meshing techniques for a NEC input file of a car

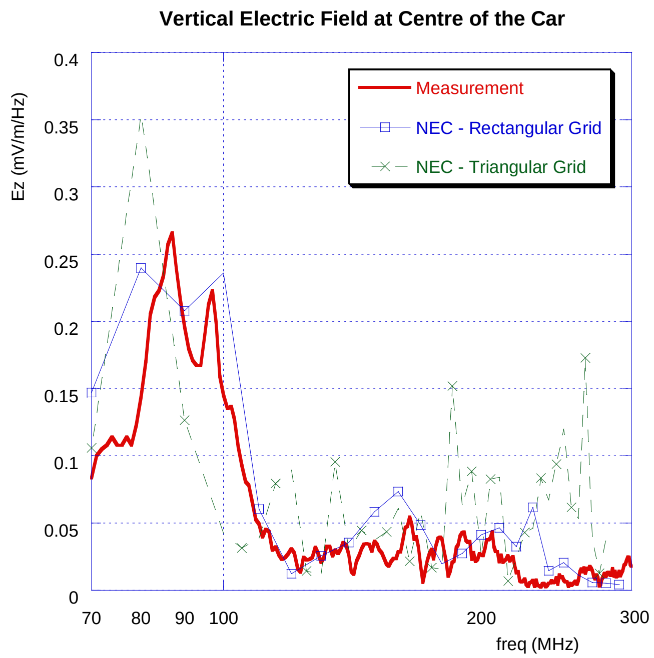


Fig. 8. Comparison of measurement and simulation for the two different meshes

produces the contours and details of the real model. However, this level of complexity requires some compromises. In fact, while some of the segments were 14 cm long, others were as smaller than 1 cm. The first attempts at running the body-fitted triangular model produced unsatisfactory results. For this reason, a routine was written to eliminate very small segments, in an effort to smooth out the differences in length. The modified model consisted of over 17.000 segments. Model (b), consisting of about 8.000 segments, is a less dense staircase approximation of the model. All of the segments have the same length and radius. It was found that the simplified version of the model, despite being a less faithful representation of the real car, exhibited far better results when compared to measurements. An example of these results can be seen in Figure 8. This figure shows the measured vertical component of the electric field penetrating inside the vehicle illuminated by an EMP simulator (for the details of the experiment, see [10], [11]). In the same figure, we also present the computed results using a parallel version of NEC [10] and obtained using the two meshes shown in Figs. 7(a) and 7(b). It can be seen from Figure 8 that the square-mesh model yields very good results, whereas the results obtained using the triangular mesh are not satisfactory over a wide frequency range.

IV. Summary and Conclusions

In this paper, we discussed the wire-grid representation of metallic surfaces in numerical electromagnetic modeling. Considering two types of geometries, namely (1) a simple cube, and (2) a complex structure representing a metallic car shell, we showed that the Equal Area Rule is accurate as long as the wire-grid consists of a simple rectangular mesh. For more complex body-fitted meshes, such as triangular ones, the Equal Area Rule appears to be less accurate in reproducing the electromagnetic field scattered by metallic bodies. Work is in progress to derive more accurate criteria to specify the parameters of the wire-grid model for complex geometries.

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