1/f noise

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Repetition: Noise

$$d = R \cdot s + n$$

White Noise $\rightarrow$ Gaussian noise

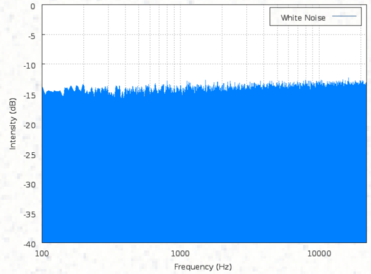
$\rightarrow$ signal independent

$$n = \frac{1}{|2\pi N|^{1/2}} e^{-\frac{1}{2} n^{+} N^{-1} n}$$

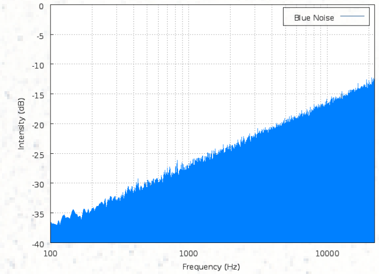
But: There's frequency dependent noise!

Colours of noise

- White

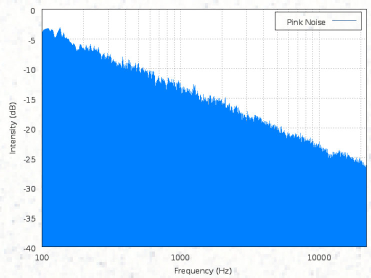


- Blue



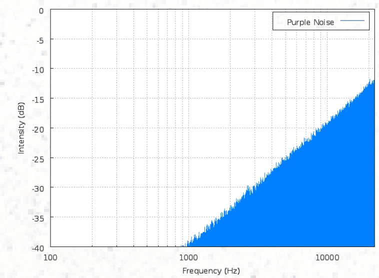
f

- Pink



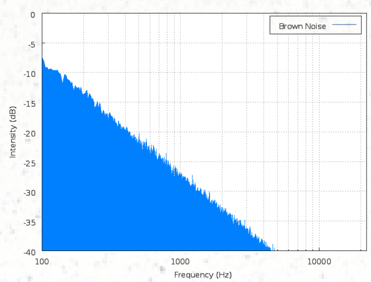
$$\frac{1}{f}$$

- Violet



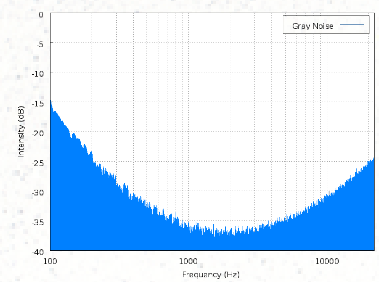
f^2

- Brown



$$\frac{1}{f^2}$$

- Grey



log(Noise power density) is plotted vs. log(f)

[1] Pictures created with Audacity

Occurrence of noise

- Acoustics
- Visual
- Electronic
- Vibrational

Kinds of electronic noise

- Thermal (Johnson-Nyquist) noise
- Shot noise
- Burst noise
- Avalanche noise
- Flicker noise

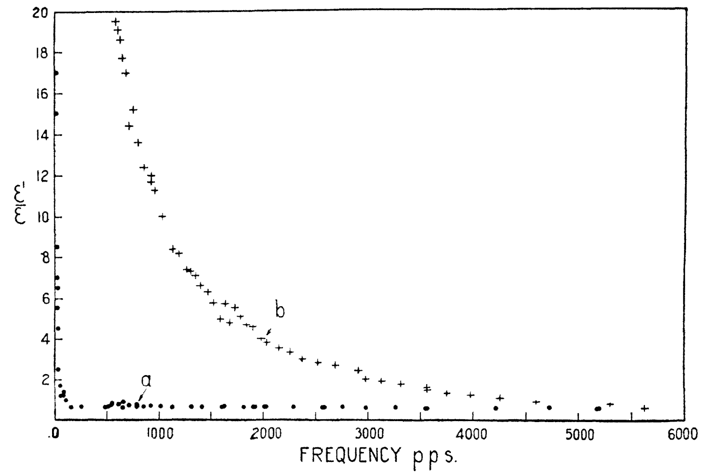
Measuring noise

1. Measure U/I
2. FFT
3. Calculate power density spectrum

→ Integrate over many measurements

History of 1/f noise

- 1918 W. Schottky predicted the occurance of frequency-independent white noise
- 1925 J. B. Johnson successfully measured it, but discovers unexpected “flicker noise” at low frequency



[2] W.Schottky Ann. d. Phys. **57** (1918) 157
[3] J.B. Johnson Phys. Rev. **26** (1925) 71

Flicker Noise

- $\frac{1}{f^{\alpha}}$ behaviour
- α range: 0.5 – 1.5
- Extends: Several frequency decades!

First explanation: W. Schottky 1926

Ansatz: Superposition of relaxation processes

$$N(t) = N_0 e^{-\lambda t} \quad t \geq 0$$

$$F(\omega) = \int_0^\infty N(t) e^{-i\omega t} = \frac{N_0}{\lambda + i\omega}$$

Now: Train of such pulses

$$N(t, t_k) = N_0 e^{-\lambda(t - t_k)} \quad t \geq t_k$$

$$F(\omega) = \int_0^\infty \sum_k N(t, t_k) e^{-i\omega t} = \frac{N_0}{\lambda + i\omega} \sum_k e^{i\omega t_k}$$

[4] W. Schottky, Phys. Rev. **28** (1926) 74

Spectrum

$$S(\omega) = \lim_{t \to \infty} \frac{1}{T} \langle |F(\omega)|^2 \rangle = \frac{N_0^2 n}{\lambda^2 + \omega^2}$$

n: average pulse rate

$\rightarrow \dfrac{1}{f^2}$ dependence

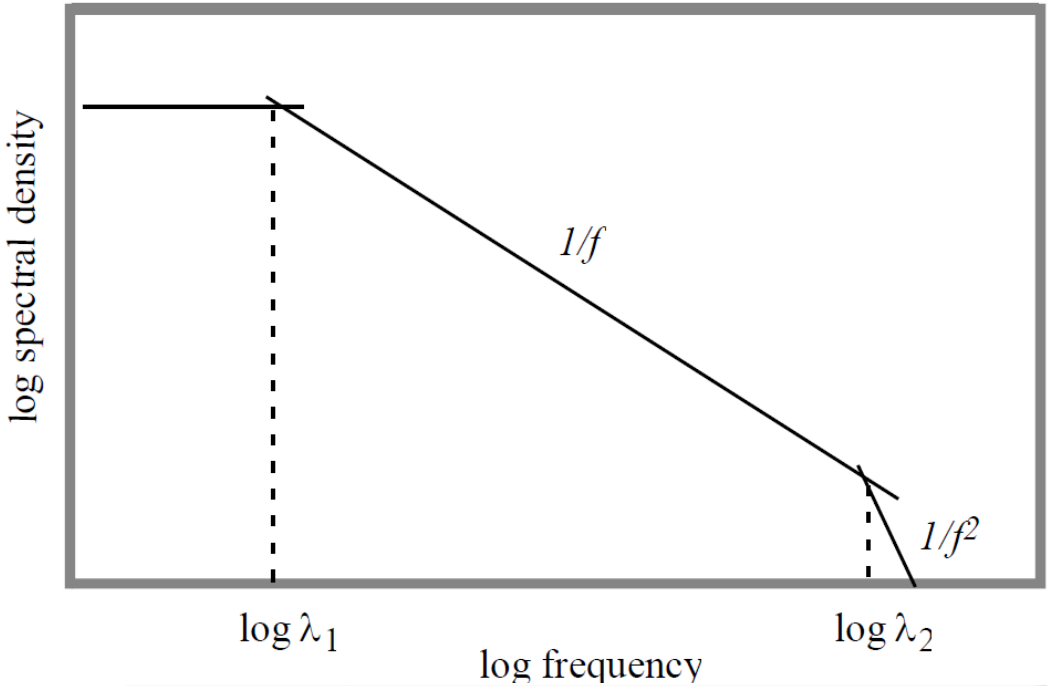
1/f dependence

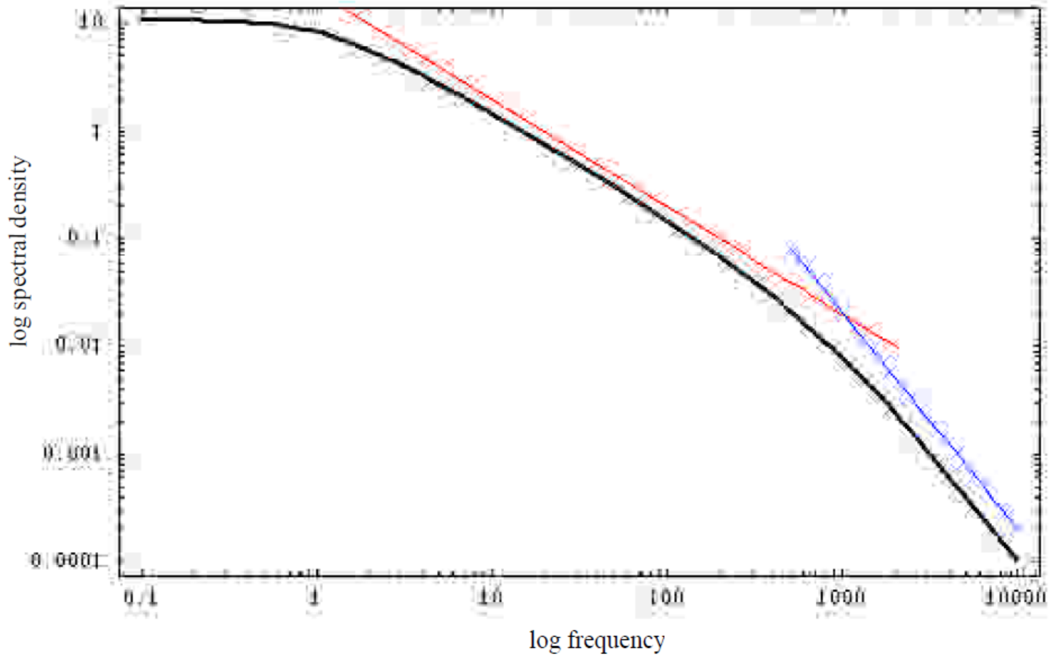
Uniform distributed relaxation rate between $\lambda_{1,} \lambda_2$

$$S(\omega) = \frac{1}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} \frac{N_0^2 n}{\lambda^2 + \omega^2} d\lambda$$

$$\approx \begin{cases} N_0^2 n & 0 < \omega \ll \lambda_{1,} \lambda_2 \\ \dfrac{N_0^2 n \pi}{2\omega(\lambda_1 - \lambda_2)} & \lambda_1 \ll \omega \ll \lambda_2 \\ \dfrac{N_0^2 n}{\omega^2} & \lambda_{1,} \lambda_2 \ll \omega \end{cases}$$

[5] E. Milotti arXiv:physics/0204033v1



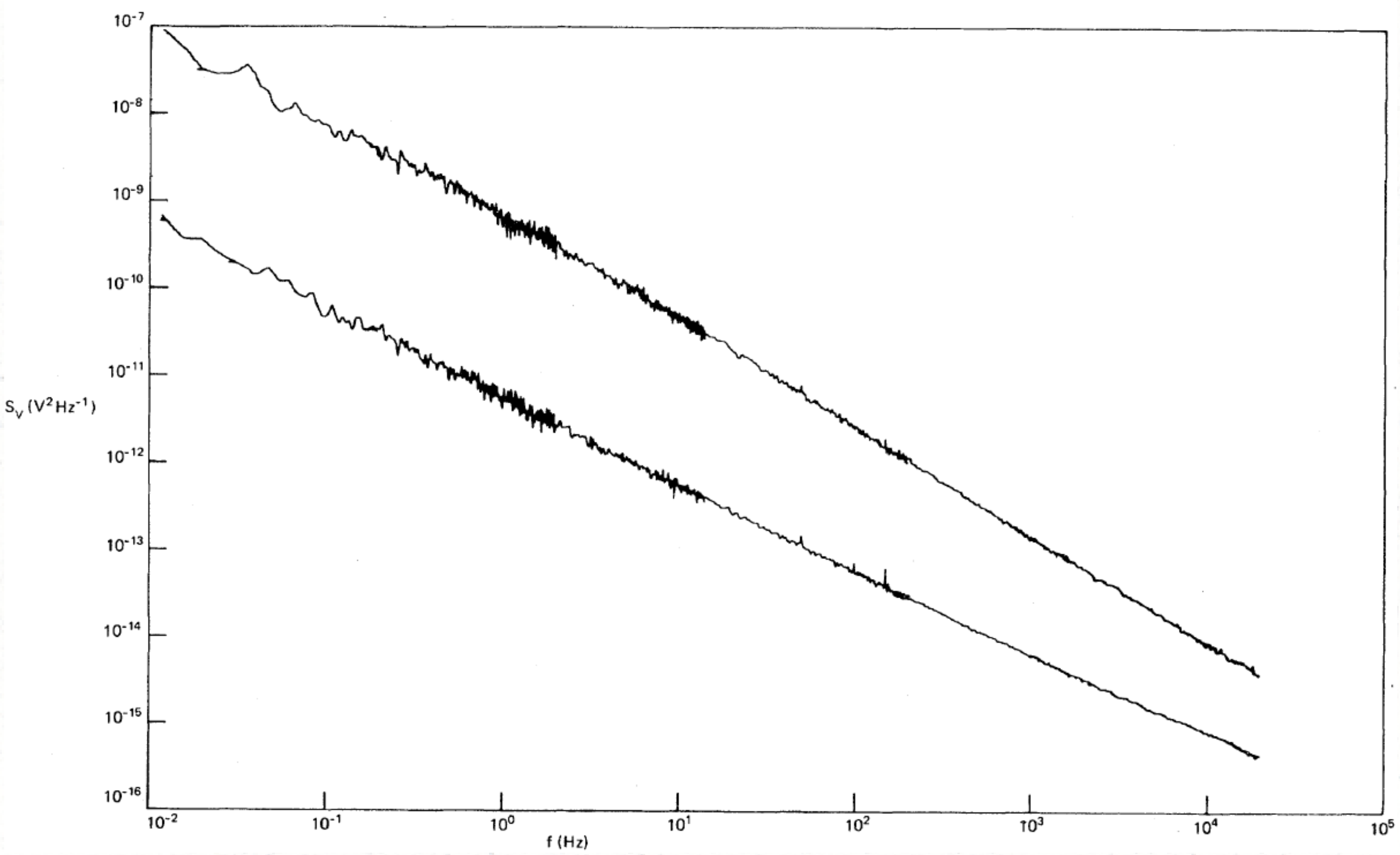


Non-uniform distribution

$$S(\omega) \sim \frac{1}{\omega^{1+\beta}}$$

→ Whole class of flicker noises with different exponents

Infinitely large fluctuations?



[6] B. Pellegrini, R.Saletti, P. Terreni and M. Prudenziati, Phys. Rev. B **27** (1983) 1233

Don't worry!

$$\int_{10^{-17}\,Hz}^{10^{43}\,Hz} \frac{df}{f} = \ln\left(10^{60}\right) \approx 138$$

Highest possible fluctuation can only be 138 times the total fluctuation between 1 Hz and 3 Hz

[7] I. Flinn Nature **219** (1968) 1356

Investigation of Brownian motion

$$\frac{dx}{dt} = Gaussian(t)$$

$$-i\omega X(\omega) = Gaussian(\omega)$$

$$S_x = \frac{\sigma^2}{2\pi\omega^2}$$

Brownian motion has a $\frac{1}{f^2}$ spectrum!

→ No description for $\frac{1}{f}$ noise

Diffusion processes

In principle:

- Possibility to derive flicker noise

- But: No physical meaning

[8] P. Dutta and P. M. Horn, Rev. Mod. Phys. **17** (1945) 323
[9] E. Milotti Phys. Rev. E **51** (1995) 3087

Bak Tang Wiesenfeld Model 1987

- Sandpile model
- Statistical approach: $\frac{1}{f^{\alpha}}$ dependence
- Numerical simulations: α near 1
- Confirmed by renormalization group

Experiment:

→ Sandpiles doesn't behave like theory

Reason: Calculation Error!

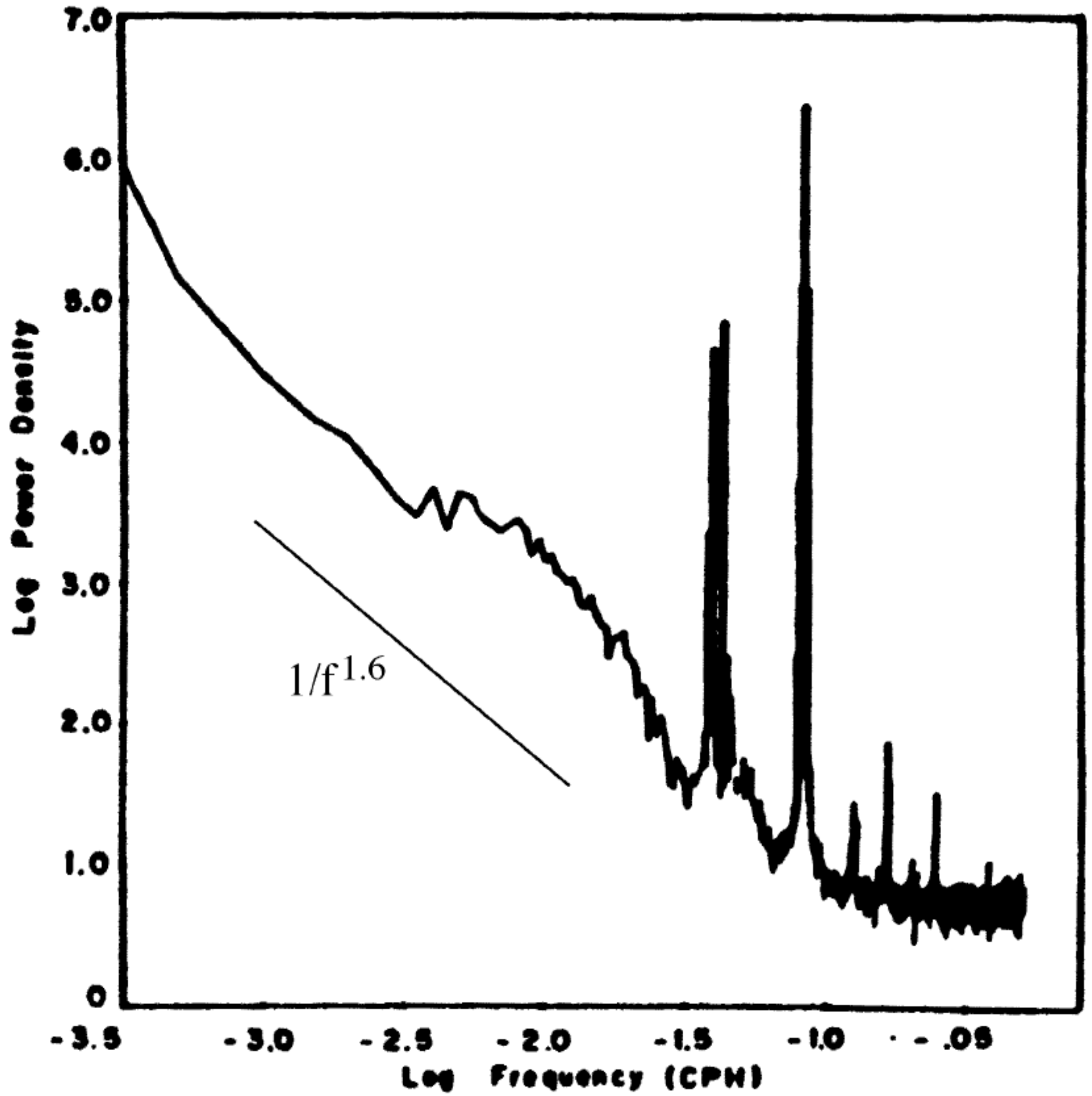
[10] P. Bak, C. Tang, K. Wiesenfeld, Phys. Rev. Lett. **59** (1987) 381
[11] H. M. Jaeger, C. Liu and S. R. Nagel, Phys. Rev. Lett. **62** (1989) 40
[12] H. J. Jensen, K. Christensen ans H. C. Fogedby, Phys. Rev. B **40** (1989) 7425

Theoretical Conclusion:

- Relaxation ansatz leads to wrong low and high frequency behaviour
- Diffusion ansatz yields $\frac{1}{f^2}$ noise
- Sandpile model not applicable

$\rightarrow$ No true explanation yet

Experiment: Sea Level at Bermuda



[13] C. Wunsch, Rev. Geophys. and Space Phys. **10** (1972) 1

Loudness fluctuation spectra of different radio stations and Bach

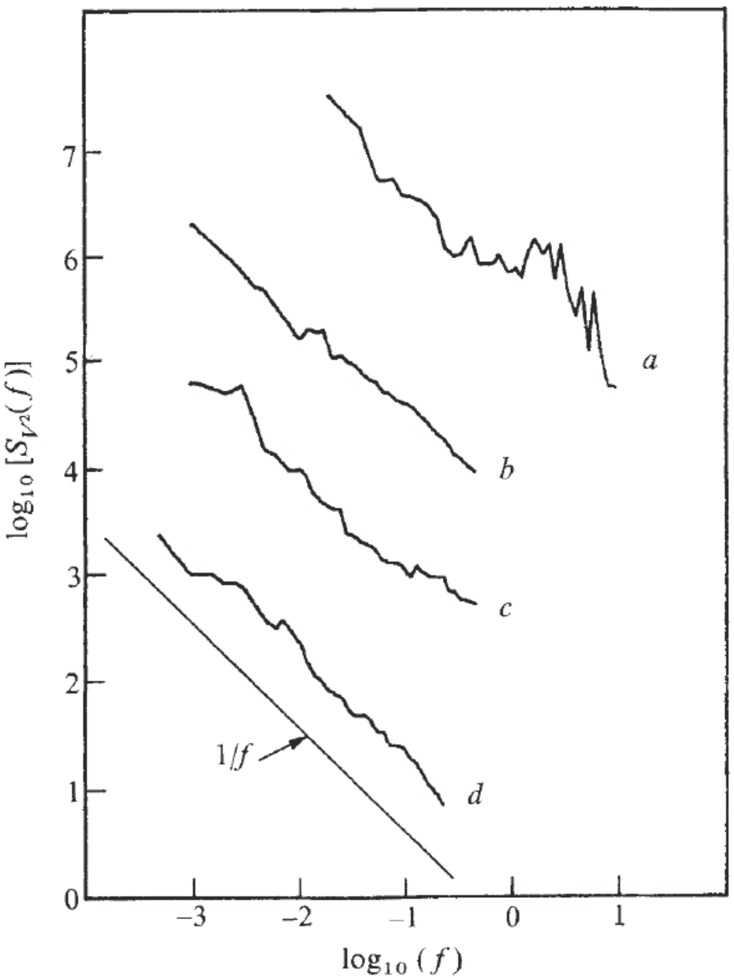
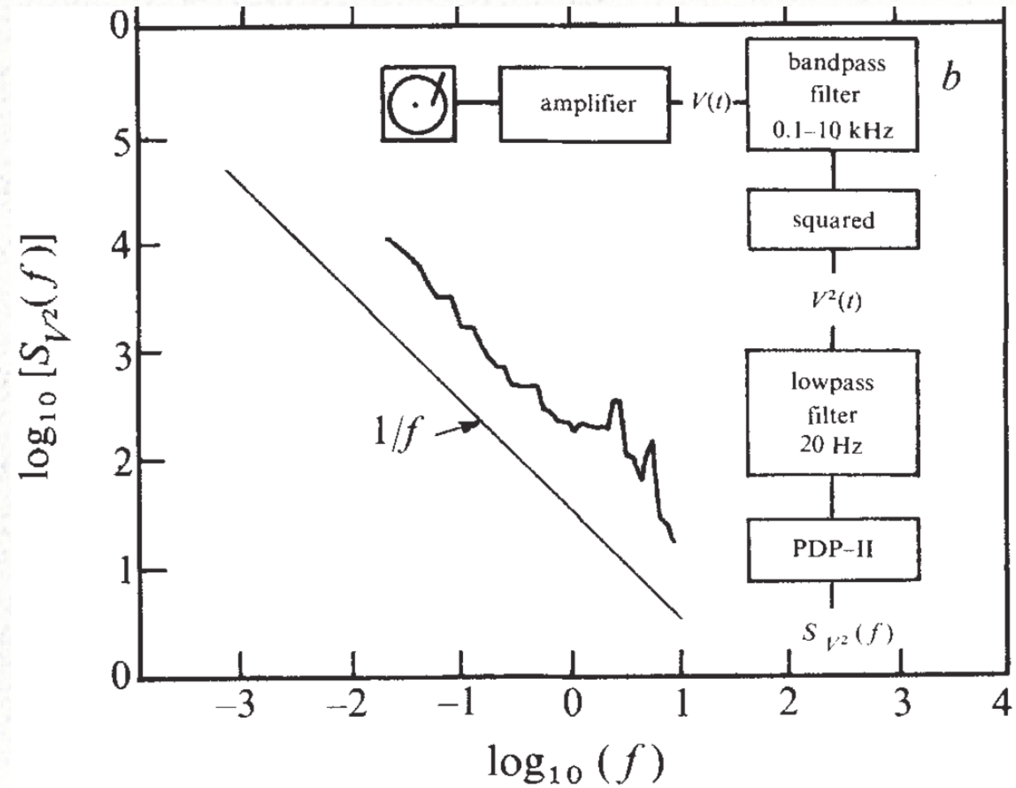
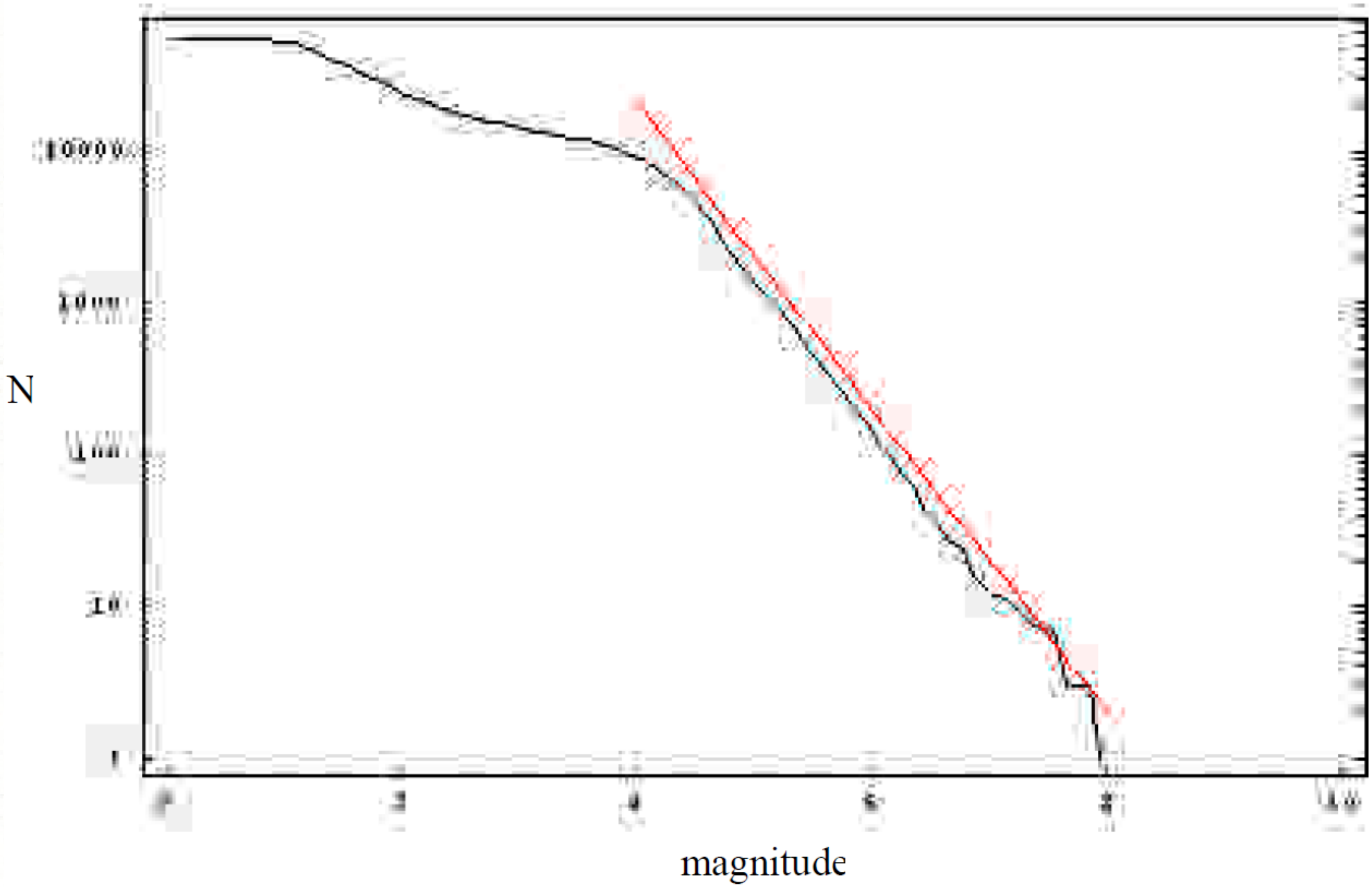


Fig. 2 Loudness fluctuation spectra, $S_{V^2}(f)$ against f for: *a*, Scott Joplin Piano Rags; *b*, classical radio station; *c*, rock station; *d*, news and talk station.



[14] R. F. Voss and J. Clarke, Nature **258** (1975) 317

Earthquakes 2000



[15] Data collected from Northern California Earthquake Data Center
http://quake.geo.berkeley.edu/

Ohmic contacts

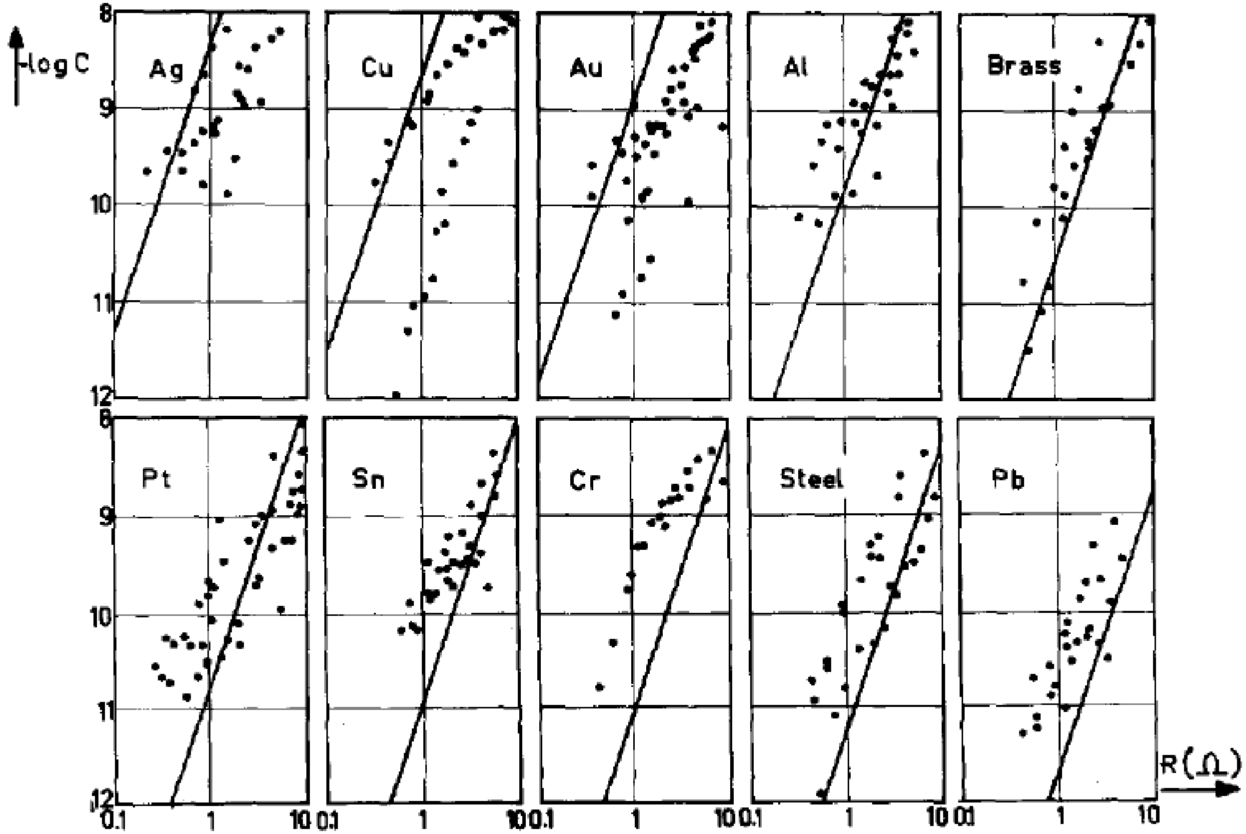
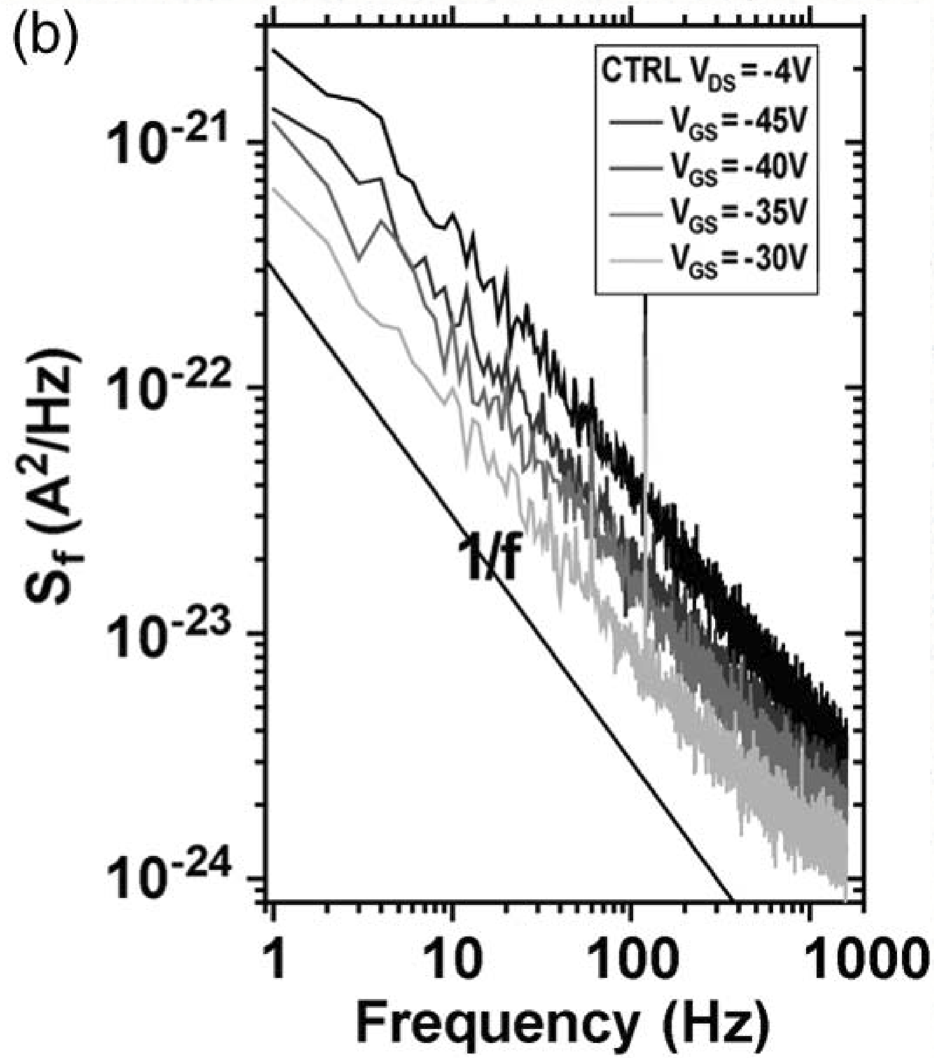


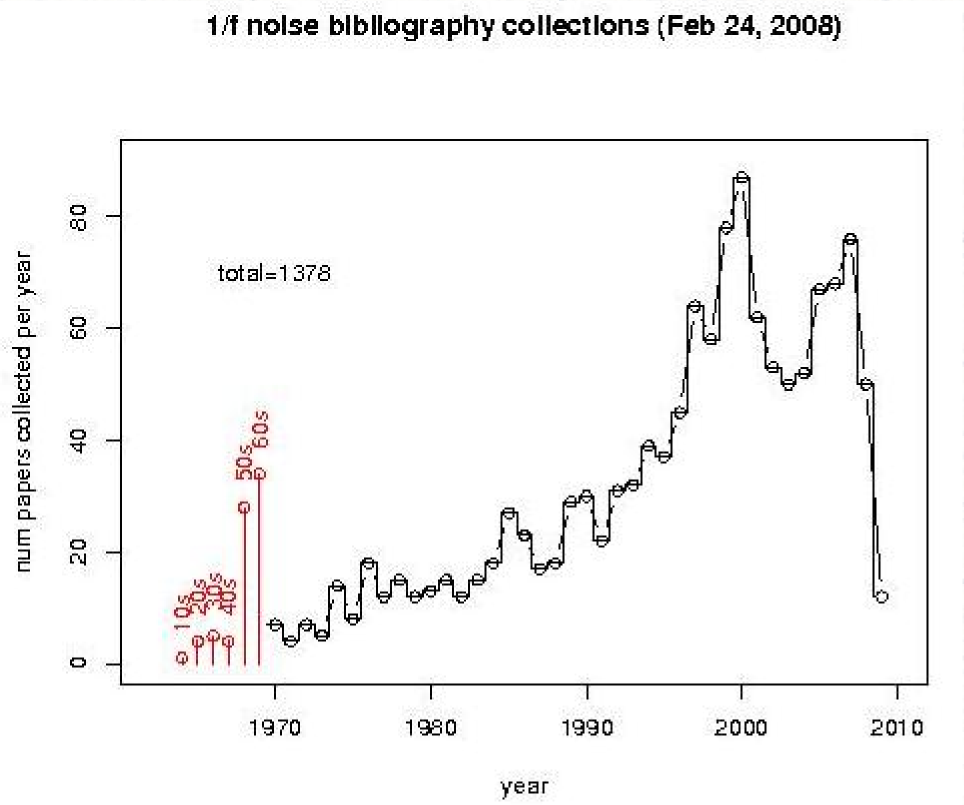
Fig. 2. 1/*f* Noise of point contacts of 10 metals. The solid lines represent values expected on the basis of eq. (3). From F.N. Hooge, Physica 60 (1972) 130.

TFTs



[17] IEEE Elec. Dev. Lett. **31** (2010) 1050

Last slide



[18] http://www.nslij-genetics.org/wli/1fnoise/index.html

Sources:

[1] Audacity 1.2.6 from http://audacity.sourceforge.net/?lang=de
[2] W. Schottky Ann. d. Phys. **57** (1918) 157
[3] J. B. Johnson Phys. Rev. **26** (1925) 71
[4] W. Schottky, Phys. Rev. **28** (1926) 74
[5] E. Milotti arXiv:physics/0204033v1
[6] B.Pellegrini, R.Saletti, P.Terreni and M.Prudenziati, Phys. Rev. B **27** (1983) 1233
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[11] H. M. Jaeger, C. Liu and S. R. Nagel, Phys. Rev. Lett. **62** (1989) 40
[12] H. J. Jensen, K. Christensen ans H. C. Fogedby, Phys. Rev. B **40** (1989) 7425
[13] C. Wunsch, Rev. Geophys. and Space Phys. **10** (1972) 1
[14] R. F. Voss and J. Clarke, Nature **258** (1975) 317
[15] Northern California Earthquake Data Center: http://quake.geo.berkeley.edu/
[16] F. N. Hooge Physica **60** (1972) 130
[17] IEEE Elec. Dev. Lett. **31** (2010) 1050
[18] http://www.nslij-genetics.org/wli/1fnoise/index.html