

# SpaceX in nosilne rakete vesoljskega tovora

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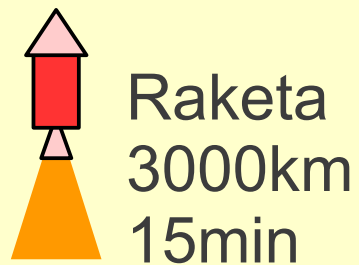
FE, Ljubljana, 16.5.2018

Seznam prosojnic predavanja:

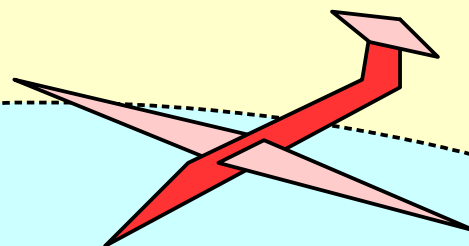
SpaceX in nosilne rakete vesoljskega tovora

- 1 - Visoka radijska postaja
- 2 - Naloga dveh teles
- 3 - Stožnice
- 4 - Eliptična tirnica
- 5 - Uporabne tirnice satelitov
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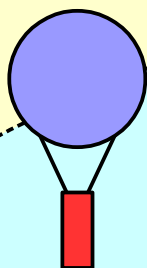
Vesolje



Letalo  
30km  
12h



Balon  
30km



Ozračje

Hrib  
3000m



Stolp  
100m



Kopno

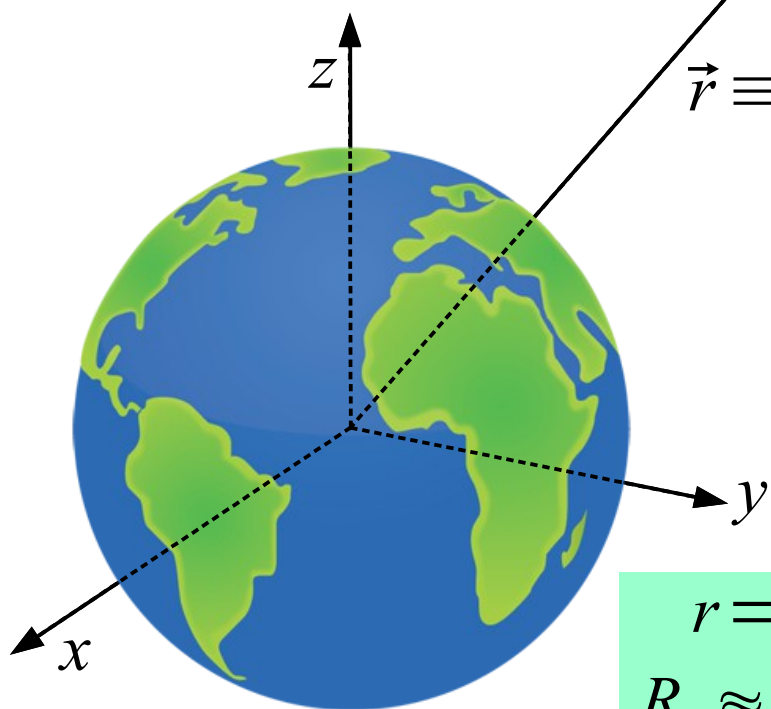
Morje

1 - Visoka radijska postaja

$$\vec{F} = -\vec{1}_r G \frac{M m}{r^2}$$

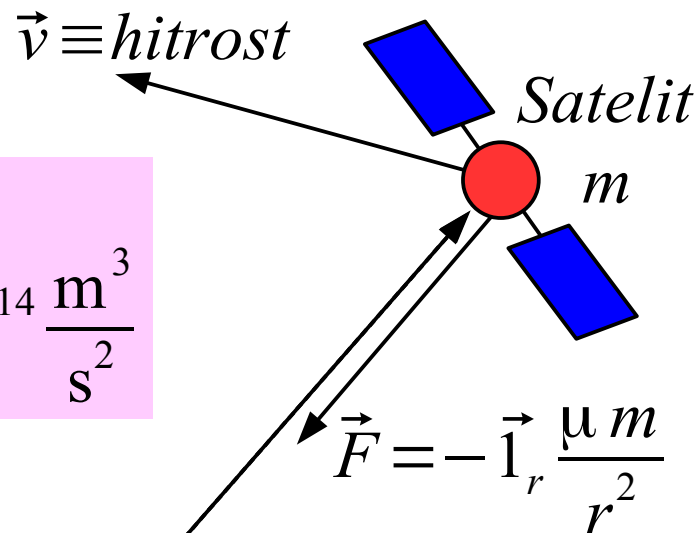
Zemlja

$$\mu = G M \approx 3.986 \cdot 10^{14} \frac{\text{m}^3}{\text{s}^2}$$



$$r = R_Z + h$$

$$R_Z \approx 6378 \text{ km}$$



Energija

$$W = \frac{1}{2} m v^2 - \frac{\mu m}{r} = \text{konst.}$$

Vrtilna količina

$$\vec{l} = \vec{r} \times m \vec{v} = \text{konst.}$$

Enačba gibanja

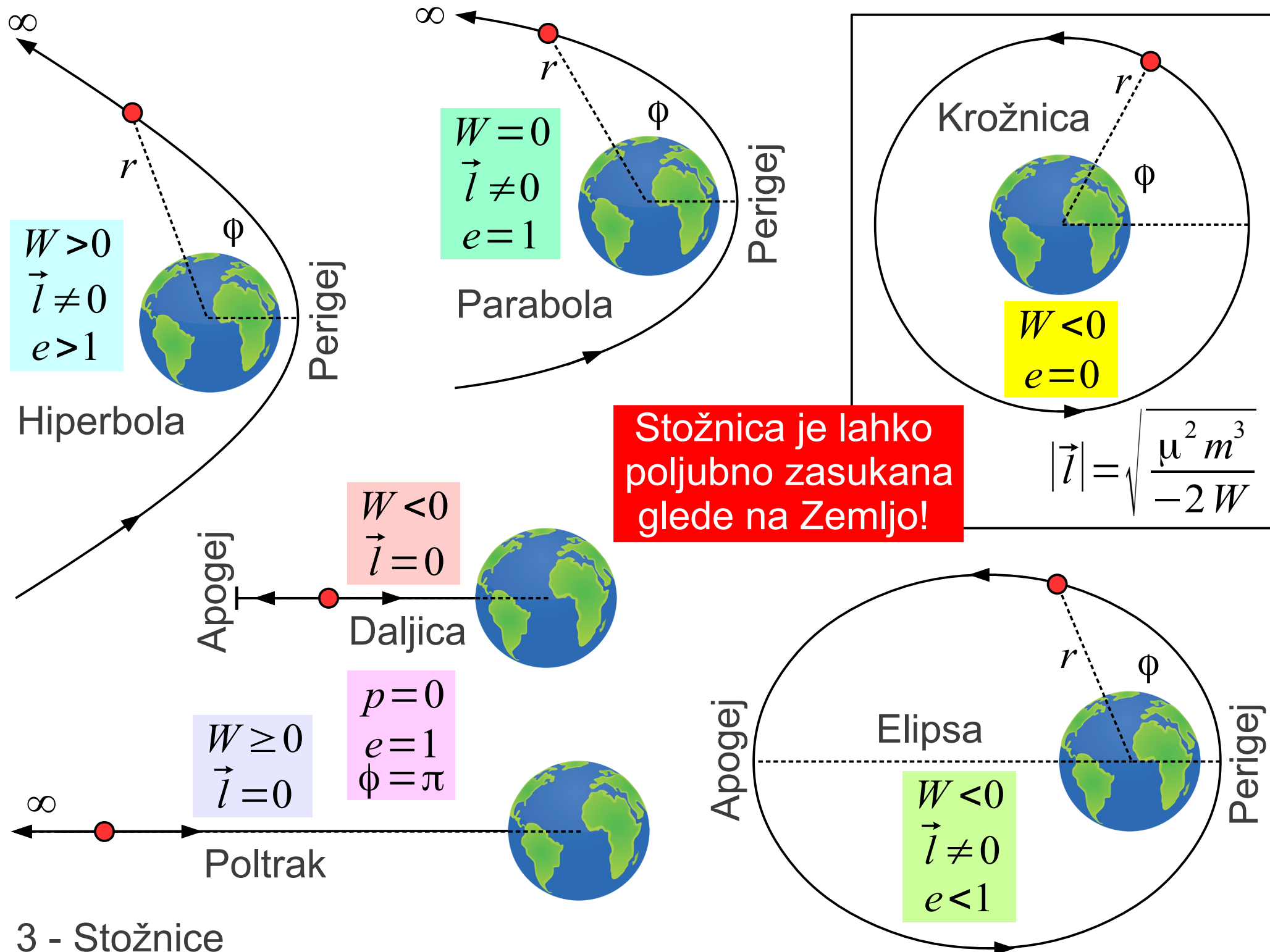
$$\vec{a} = \frac{d^2 \vec{r}}{dt^2} = \frac{\vec{F}}{m} = -\vec{1}_r \frac{\mu m}{r^2}$$

Enačba stožnice  $r = \frac{p}{1 + e \cos \phi}$

$$p = \frac{|\vec{l}|^2}{\mu m^2} \quad e = \sqrt{1 + \frac{2W|\vec{l}|^2}{\mu^2 m^3}}$$

$h < 200 \text{ km} \rightarrow$  trenje z ozračjem!

$h > 50000 \text{ km} \rightarrow$  težnost Sonca, Lune!



$$r_a = \frac{p}{1-e} = a(1+e)$$

$$h_a = r_a - R_Z$$

$$e = \frac{r_a - r_p}{r_a + r_p}$$

$e \equiv$  ekscentričnost

$$a = \frac{p}{1-e^2} \equiv \text{velika polos}$$

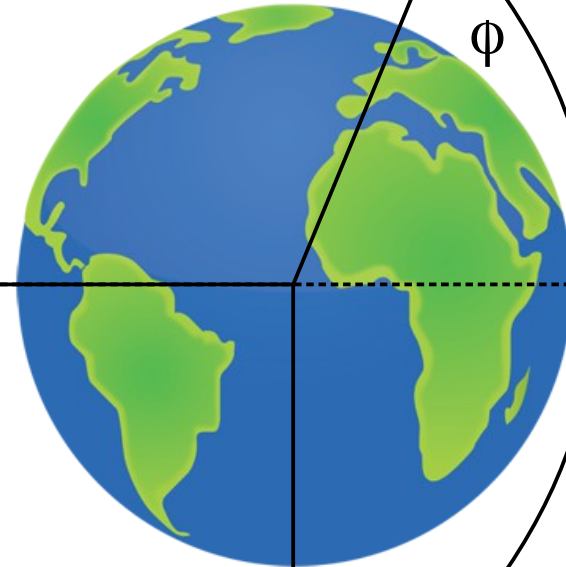
Apogej

$h_a$

$$f = ae \equiv \text{goriščnica}$$

$b \equiv$  mala polos

$f$



Perigej

$h_p$

Plovilo

$\vec{r}$

$\phi$

$$r_p = \frac{p}{1+e} = a(1-e)$$

$$h_p = r_p - R_Z$$

Elipsa

$$W < 0$$

$$\vec{l} \neq 0$$

$$0 \leq e < 1$$

$$b = \sqrt{a^2 - f^2}$$

$$p = a(1 - e^2)$$

$$a = \sqrt[3]{\mu \left( \frac{T}{2\pi} \right)^2}$$

$$\text{Perioda } T = 2\pi \sqrt{\frac{a^3}{\mu}}$$

4 - Eliptična tirnica

Elipsa je lahko poljubno zasukana glede na Zemljo!

$$\text{Hitrost } v = \sqrt{\mu \left( \frac{2}{r} - \frac{1}{a} \right)}$$

$$a = -\frac{\mu m}{2W}$$

$\Omega \equiv$  rektascenzija dvižnega vozla

$$\frac{d\Omega}{dt} = \frac{360^\circ}{1 \text{ leto}}$$

Molnija

$$h_a \approx 39500 \text{ km}$$

$$T \approx 12 \text{ h} \quad e \approx 0.72$$

$$\omega \approx 270^\circ$$

$$\sum \Delta v \approx 11 \text{ km/s}$$

Heliosinhrona

$$h \approx 850 \text{ km}$$

$$T \approx 1 \text{ h } 42'$$

$$e = 0$$

$$\sum \Delta v \approx 9 \text{ km/s}$$

$$i = 63.4^\circ$$

$$\frac{d\omega}{dt} = 0$$

$\omega \equiv$  argument perigeja

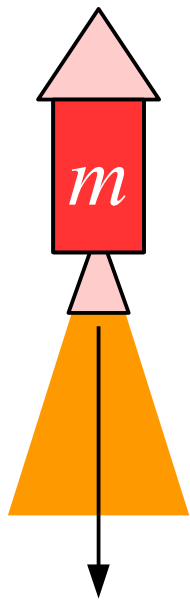
Risba ni v merilu!

$$h_p \approx 1000 \text{ km}$$

$$i = 0^\circ \quad e = 0$$

Geostacionarna

5 - Uporabne tirnice satelitov  $h \approx 35800 \text{ km} \quad T \approx 23 \text{ h } 56' \quad \sum \Delta v \approx 12 \text{ km/s}$



$$\sum \vec{F} = 0 = -\frac{d m}{d t} \vec{v}_i + \vec{a} m \rightarrow \vec{a} = \frac{\vec{v}_i}{m} \cdot \frac{d m}{d t}$$

$$\Delta \vec{v} = \int_{t_1}^{t_2} \vec{a} d t = \int_{m_1}^{m_2} \frac{\vec{v}_i}{m} d m = -\vec{v}_i \frac{m_1}{m_2} = -\vec{v}_i \ln \left( 1 + \frac{m_g}{m_t} \right)$$

$m_g \equiv$  masa goriva  $m_t \equiv$  masa tovora

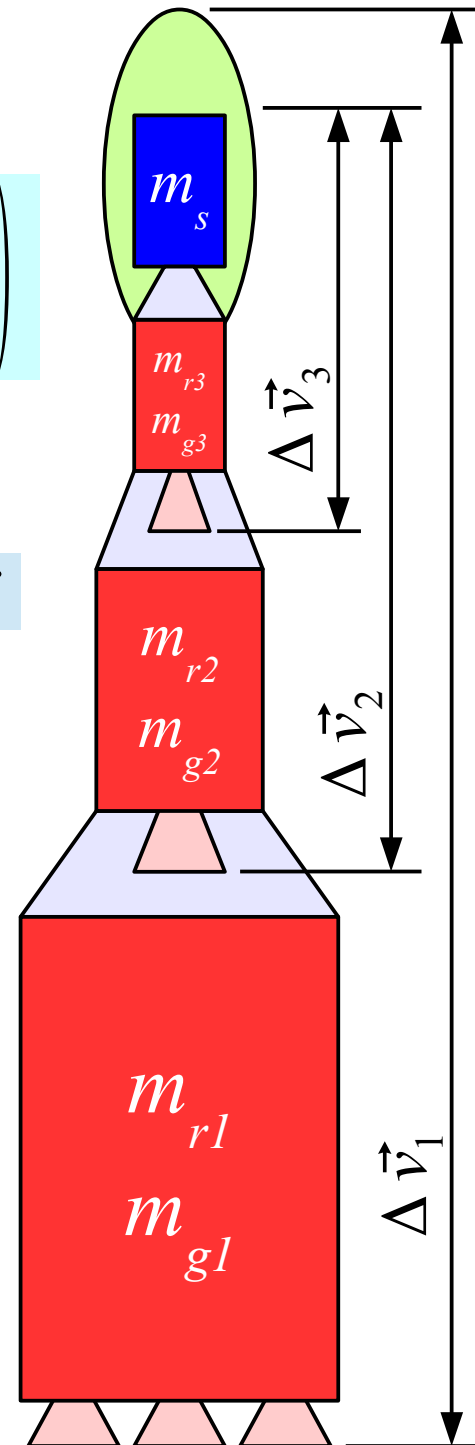
$\vec{v}_i \equiv$  hitrost izpuha

$\Delta \vec{v} \equiv$  sprememba hitrosti

Gorivo	$v_i$	$\Delta v$ $m_g = 9m_t$
Hladen $N_2$	0.7km/s	0.7km/s
$H_2O_2$	1.5km/s	3.5km/s
$N_2H_4$	2.2km/s	5.1km/s
$Al + NH_4ClO_4$	2.7km/s	6.2km/s
$N_2H_2(CH_3)_2 + N_2O_4$	3.0km/s	6.9km/s
Tekoča $H_2 + O_2$	4.5km/s	10.4km/s

Potrebno:  
 $\sum \Delta v = 8 \dots 12 \text{ km/s}$   
 Večstopenjska raketa!

$$m_g = m_t \left( e^{\frac{\Delta v}{v_i}} - 1 \right)$$

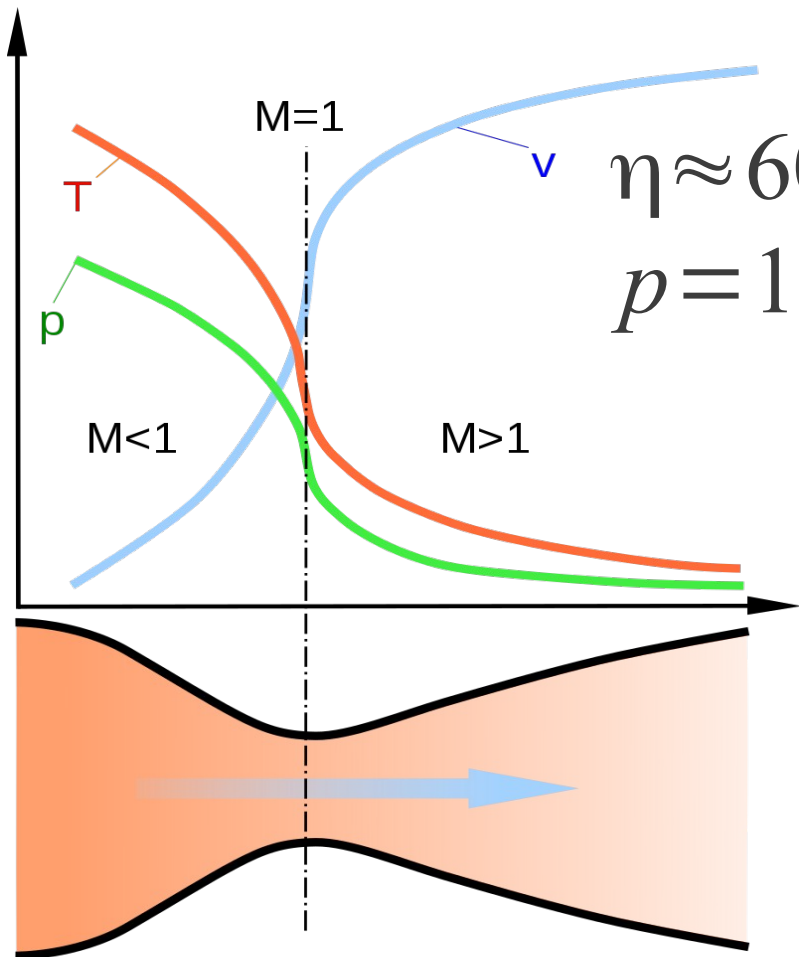


6 - Enačba gibanja rakete

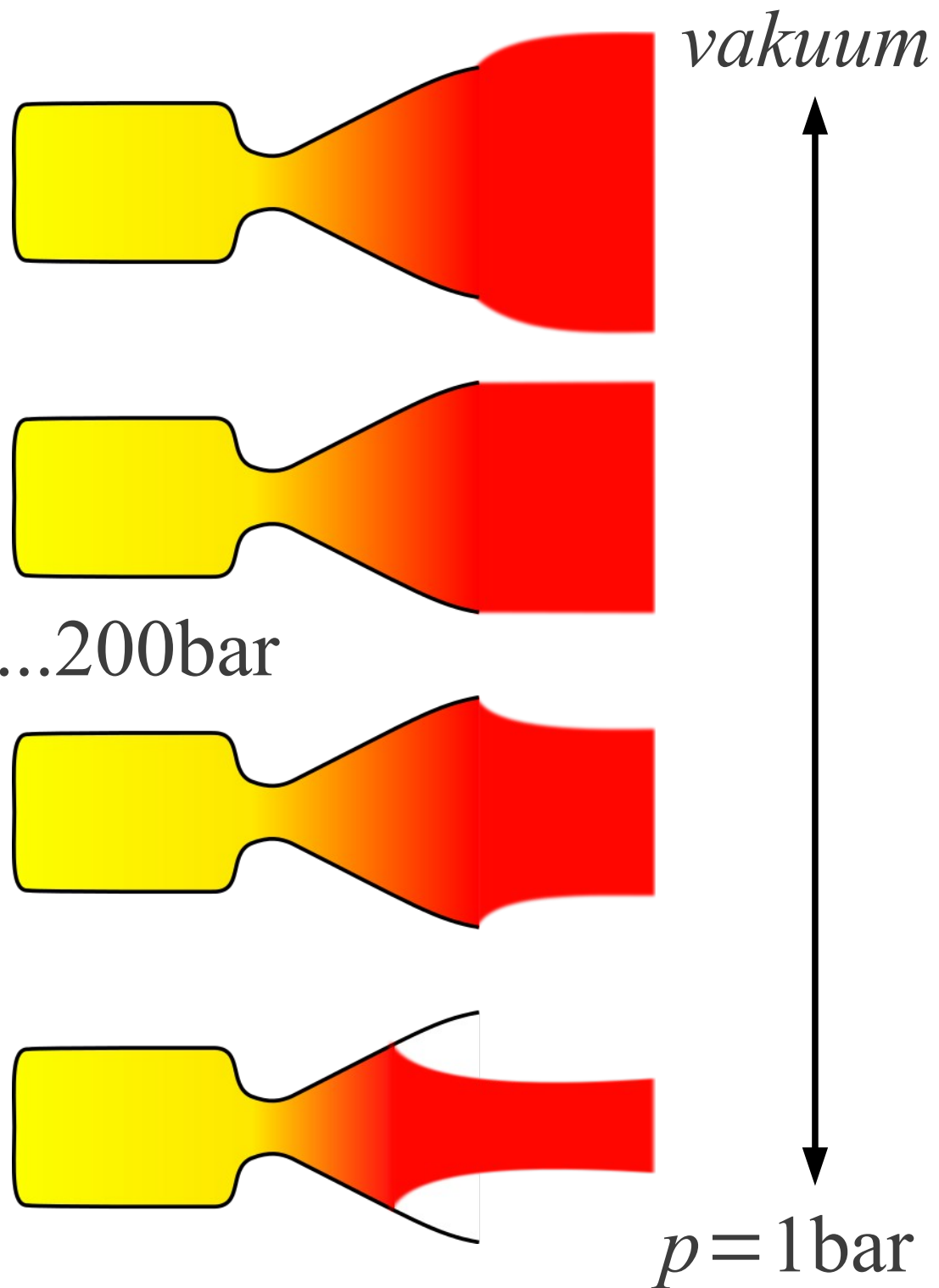


*Nicolas Léonard  
Sadi Carnot 1824*

$$\eta = \frac{W}{Q_V} \leq 1 - \frac{T_{HLADNO}}{T_{VROČE}}$$



7 - Raketni motor

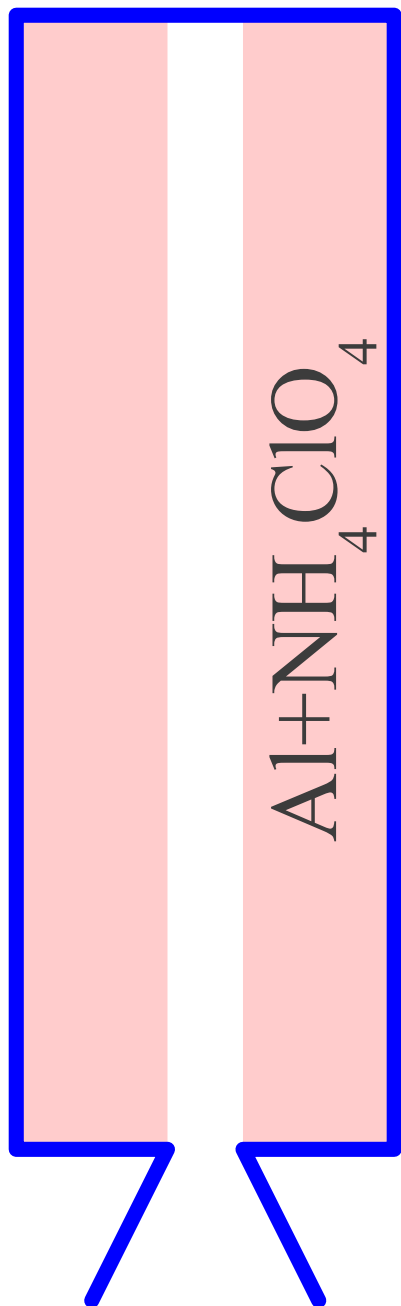


*Trdo gorivo*

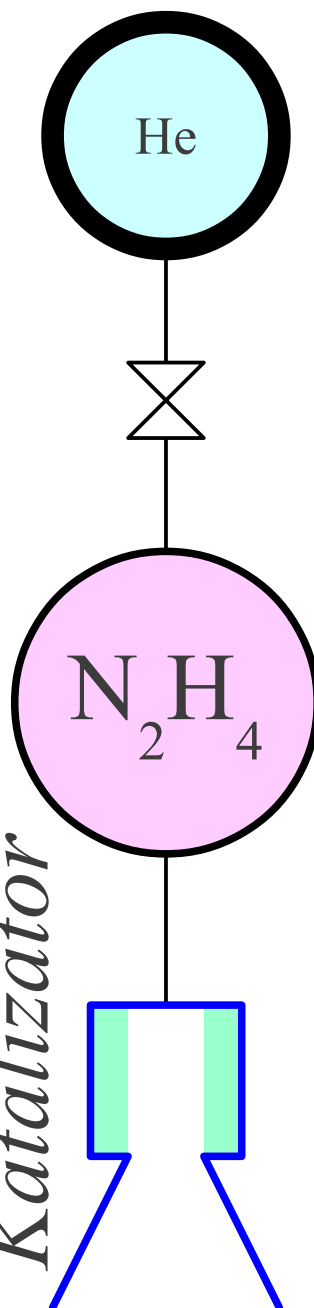
*Enokomponentni*

*Hipergoli*

*Tlačna posoda 70bar*



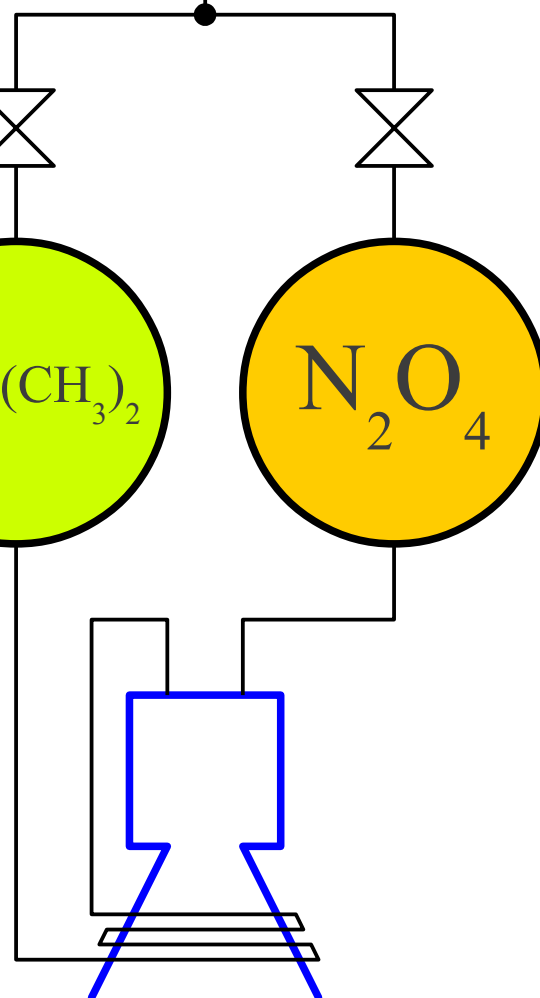
*Katalizator*

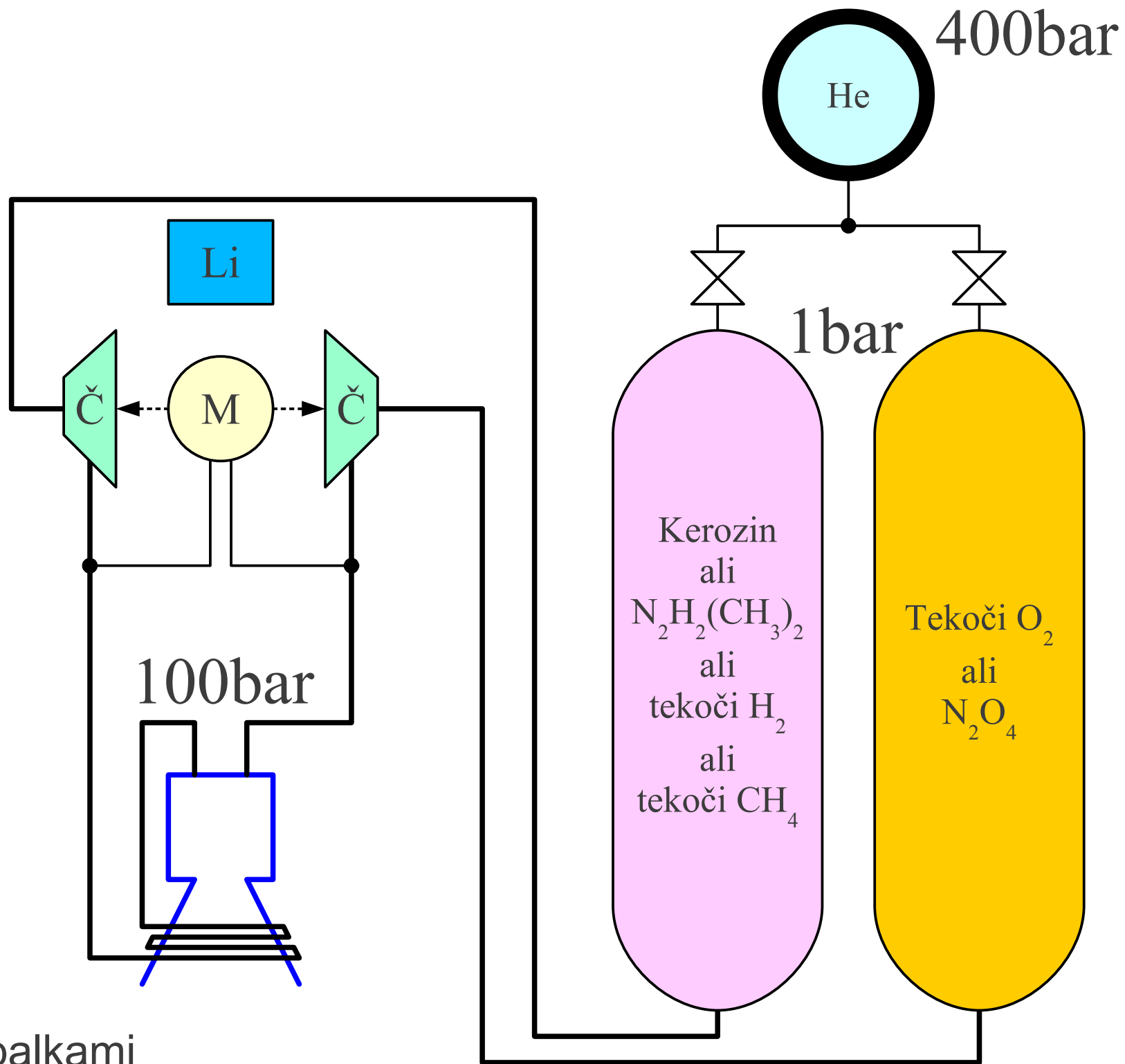


400bar

*Ventili*

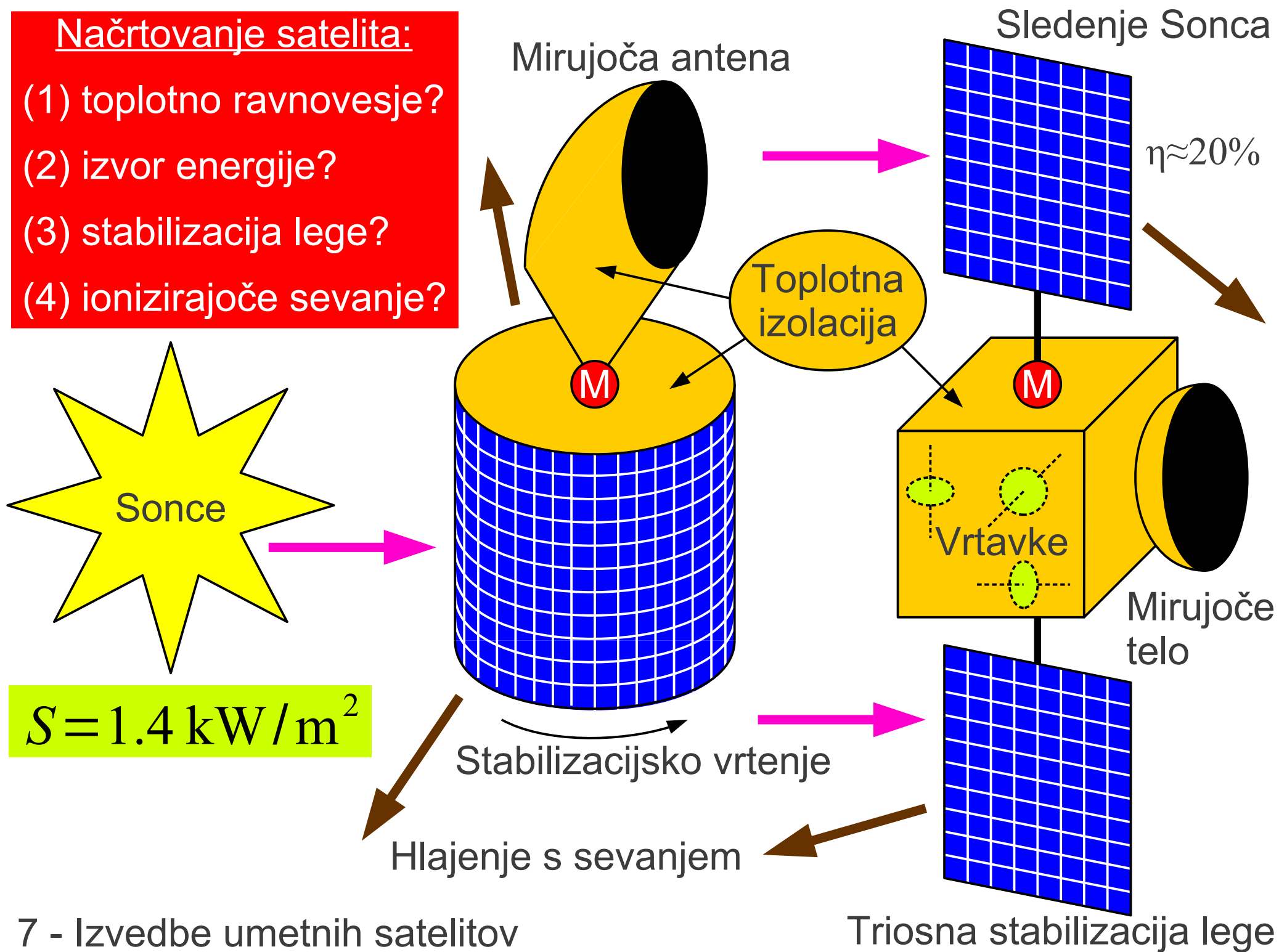
15bar





## Načrtovanje satelita:

- (1) toplotno ravnovesje?
- (2) izvor energije?
- (3) stabilizacija lege?
- (4) ionizirajoče sevanje?



$$P_{\text{e}} \leq 10\text{kW}$$

*Prepustnost ozračja*  
 $100\text{MHz} \leq f \leq 50\text{GHz}$

$$A \leq 10\text{m}^2$$

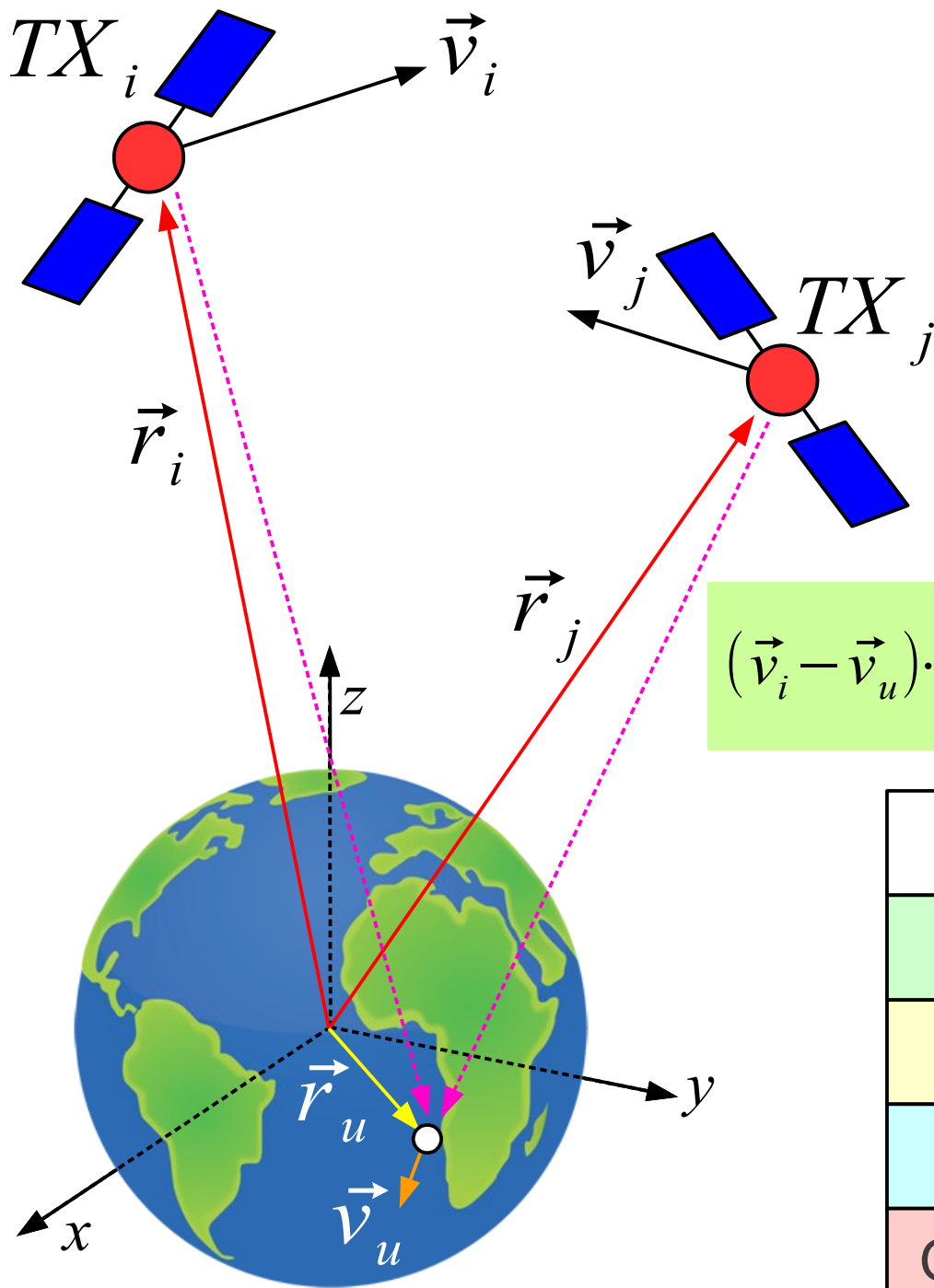
$$r \approx 38000\text{km}$$

*Zakasnitev*

$$\Delta t = \frac{2r}{c_0} \approx 0.25\text{s}$$

Omejena zmogljivost:

- (1) radiodifuzija
- (2) radionavigacija
- (3) opazovanje Zemlje



Zakasnitev modulacije:

$$|\vec{r}_i - \vec{r}_u| - |\vec{r}_j - \vec{r}_u| = c_0 \Delta t_{ij}$$

*Neznanke :  $\vec{r}_u, \vec{v}_u, t_u, f_u$*

*Potreben nabor enačb ?*

Doppler nosilca:

$$(\vec{v}_i - \vec{v}_u) \cdot \frac{(\vec{r}_i - \vec{r}_u)}{|\vec{r}_i - \vec{r}_u|} - (\vec{v}_j - \vec{v}_u) \cdot \frac{(\vec{r}_j - \vec{r}_u)}{|\vec{r}_j - \vec{r}_u|} = \frac{c_0}{f_0} \Delta f_{ij}$$

Sistem	Tirnica	Frekvence
Transit	1100km/90°	399.968MHz 149.988MHz
Cikada	970km/83°	~400MHz ~150MHz
GPS	20200km/55°	1575.42MHz 1227.6MHz
GLONASS	19100km/65°	~1600MHz ~1250MHz
BeiDou	21500km/55°	1561.098MHz 1207.140MHz