Noise in radio communications

Matjaž Vidmar

LSO, FE, Ljubljana, 5.2.2017

List of figures: Noise in radio communications

Fifth Solvay International Conference on Electrons and Photons (October 1927). The leading figures Albert Einstein and Niels Bohr disagreed:

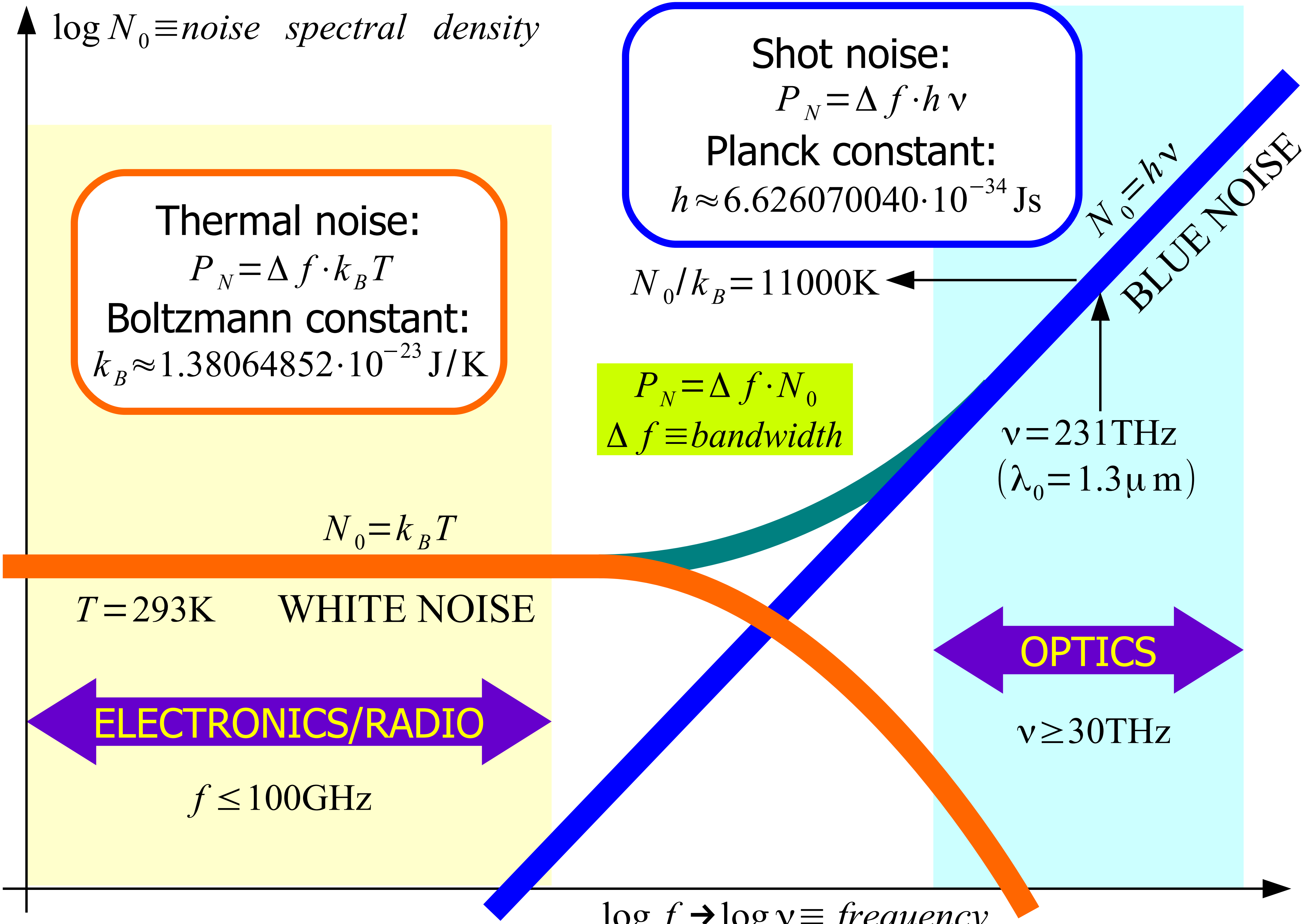
Albert Einstein: „God does not play dice!“

Niels Bohr: „Einstein, stop telling God what to do!“

In telecommunications random signals are called noise. Noise impairs the performance of any communication link.

Noise is a macroscopic description of quantum effects!

1 - The dispute between famous scientists



Thermal noise:

$P_N = \Delta f \cdot k_B T$

Boltzmann constant:

$k_B \approx 1.38064852 \cdot 10^{-23}\ \mathrm{J/K}$

Shot noise:

$P_N = \Delta f \cdot h \nu$

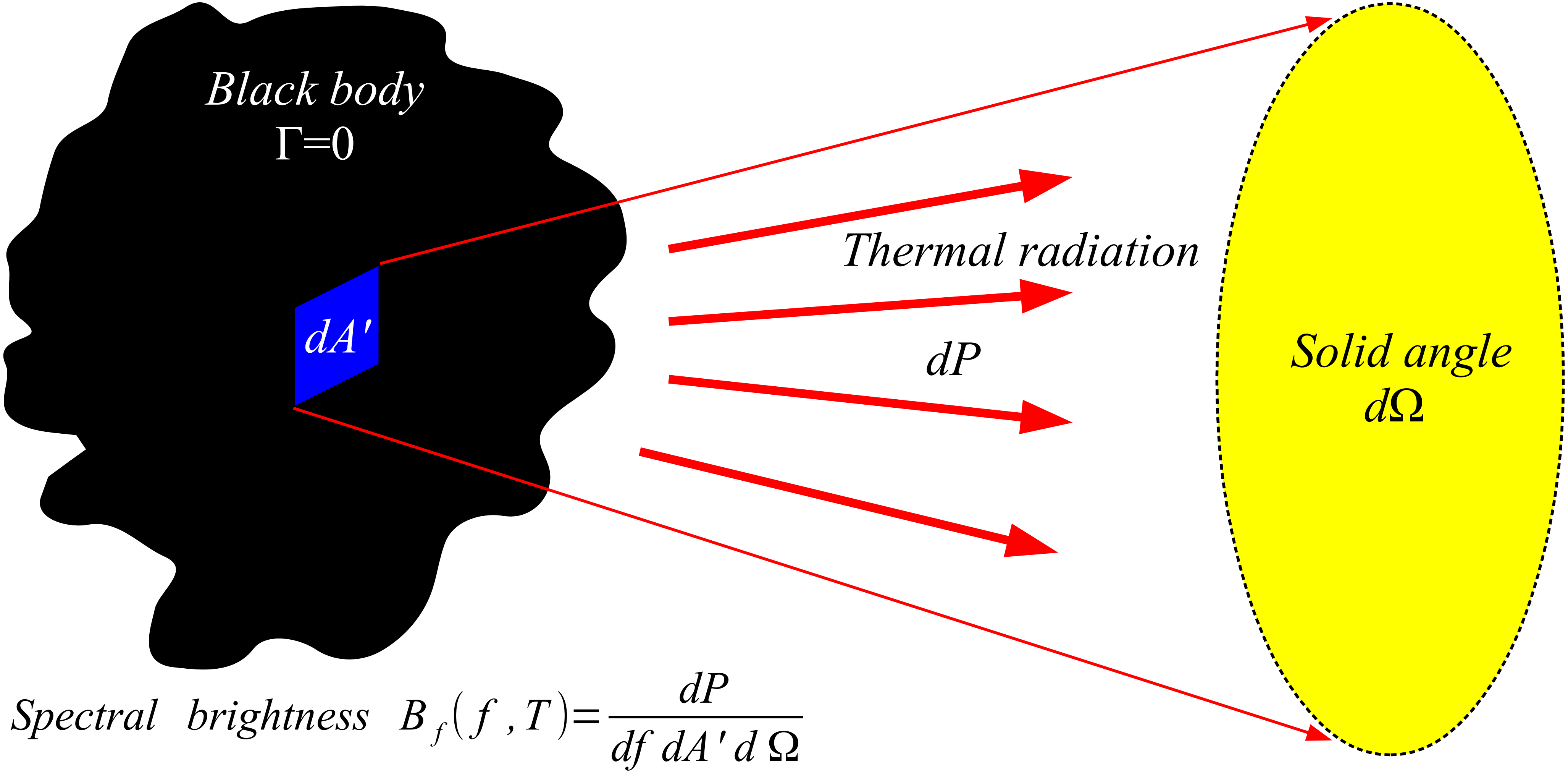
Planck constant:

$h \approx 6.626070040 \cdot 10^{-34}\ \mathrm{Js}$

$P_N = \Delta f \cdot N_0$

$\Delta f \equiv bandwidth$

2 - Noise spectral density



Spectral brightness $B_f(f,T)=\frac{dP}{df\,dA'\,d\Omega}$

Planck law $B_f(f,T)=\frac{2hf^3}{c_0^2}\cdot\frac{1}{\mathrm{e}^{\frac{hf}{k_BT}}-1}$

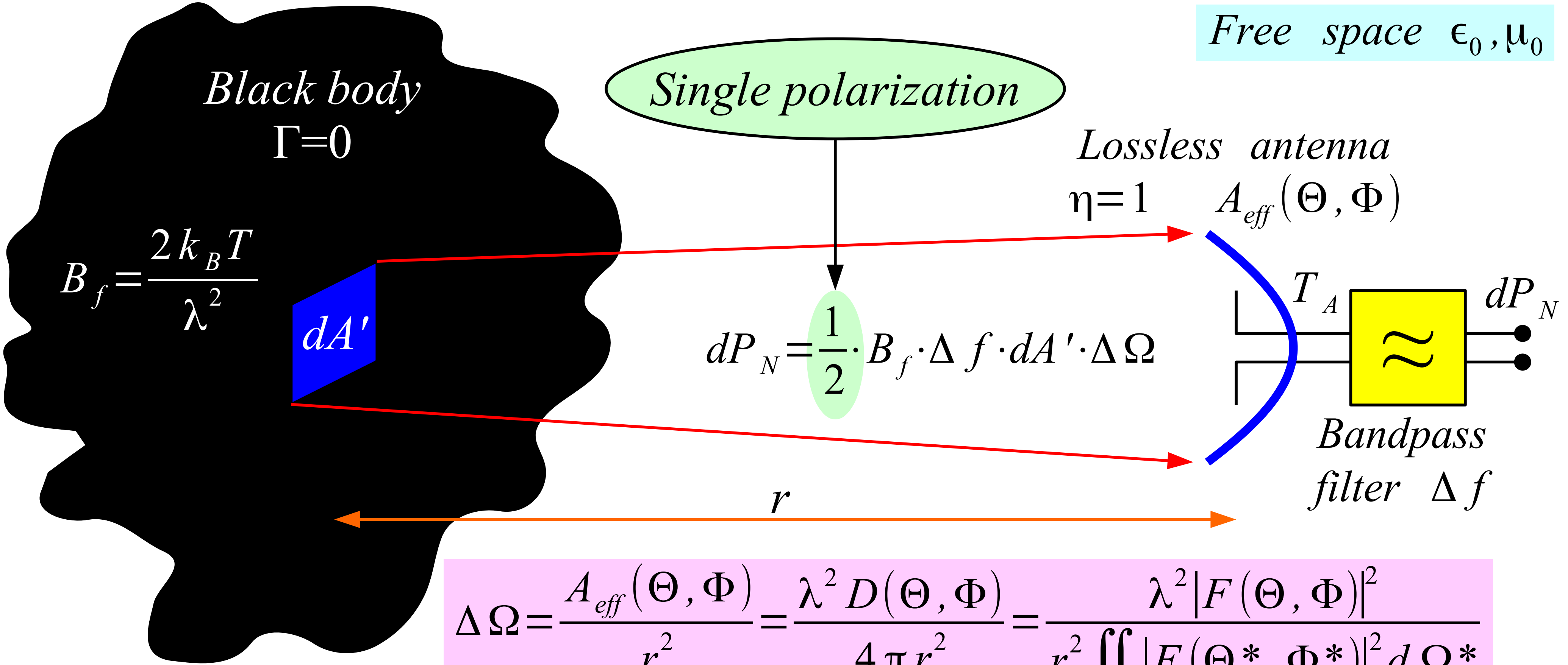
Free space ϵ_0,μ_0

$c_0=299792458\,\mathrm{m/s}\approx3\cdot10^8\,\mathrm{m/s}$

Radio $hf\ll k_BT$ → *Rayleigh–Jeans approximation* $B_f(f,T)\approx\frac{2k_BTf^2}{c_0^2}=\frac{2k_BT}{\lambda^2}$

3 – Black-body thermal radiation

4 – Received thermal-noise power



$$dP_N=\frac{1}{2}\cdot B_f\cdot\Delta f\cdot dA'\cdot\Delta\Omega$$

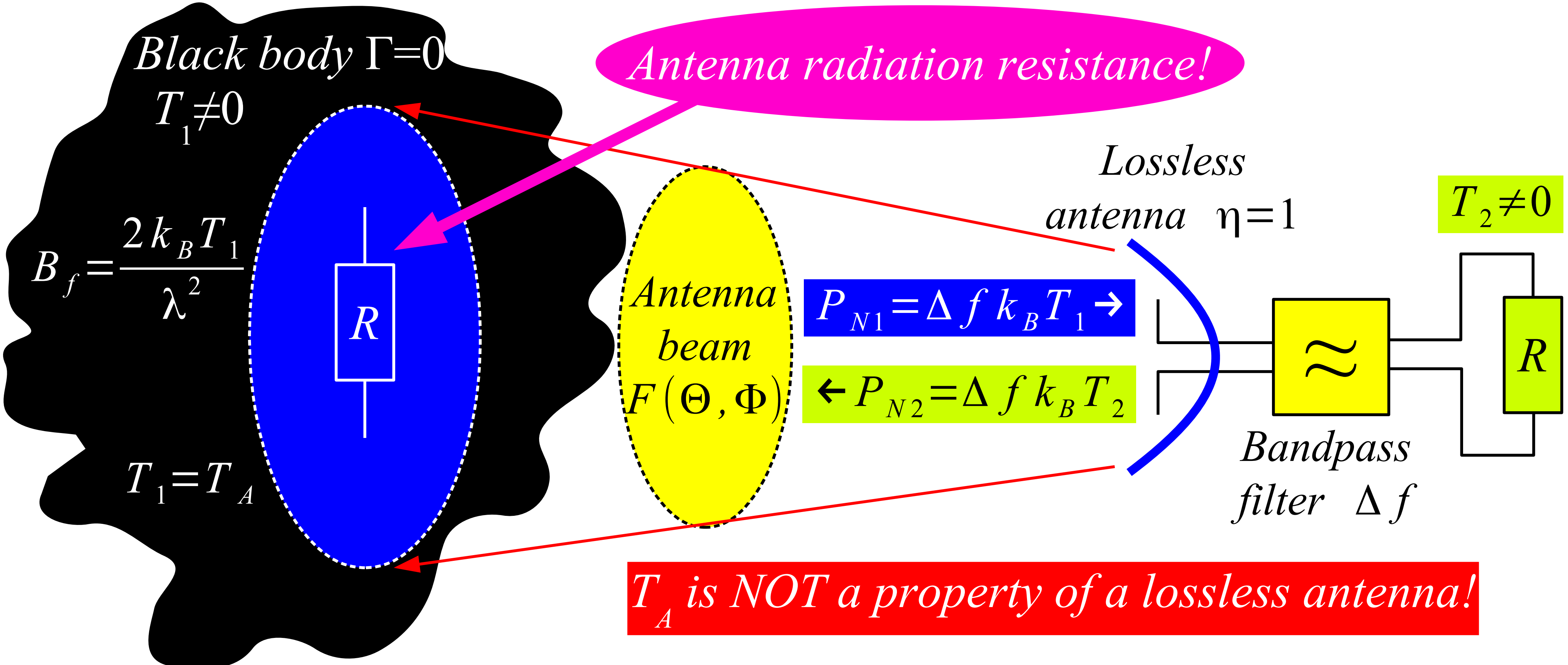
$$\Delta\Omega=\frac{A_{eff}(\Theta,\Phi)}{r^2}=\frac{\lambda^2 D(\Theta,\Phi)}{4\pi r^2}=\frac{\lambda^2|F(\Theta,\Phi)|^2}{r^2\iint\limits_{4\pi}|F(\Theta*,\Phi*)|^2 d\Omega*}$$

$$dA'=r^2 d\Omega$$

$$P_N=\iint\limits_{A'}\frac{1}{2}\cdot B_f\cdot\Delta f\cdot dA'\cdot\Delta\Omega=\iint\limits_{4\pi}\frac{1}{2}\cdot\frac{2k_BT(\Theta,\Phi)}{\lambda^2}\cdot\Delta f\cdot r^2 d\Omega\cdot\frac{\lambda^2|F(\Theta,\Phi)|^2}{r^2\iint\limits_{4\pi}|F(\Theta*,\Phi*)|^2 d\Omega*}$$

$$P_N=\Delta f\, k_B\frac{\iint\limits_{4\pi}T(\Theta,\Phi)|F(\Theta,\Phi)|^2 d\Omega}{\iint\limits_{4\pi}|F(\Theta,\Phi)|^2 d\Omega}=\Delta f\, k_B T_A$$

$$T_A=\frac{\iint\limits_{4\pi}T(\Theta,\Phi)|F(\Theta,\Phi)|^2 d\Omega}{\iint\limits_{4\pi}|F(\Theta,\Phi)|^2 d\Omega}$$



Black body $\Gamma=0$

$T_1 \neq 0$

$B_f = \frac{2k_B T_1}{\lambda^2}$

$T_1 = T_A$

R

Antenna radiation resistance!

Antenna beam $F(\Theta, \Phi)$

Lossless antenna $\eta=1$

$P_{N1} = \Delta f\, k_B T_1 \rightarrow$

$\leftarrow P_{N2} = \Delta f\, k_B T_2$

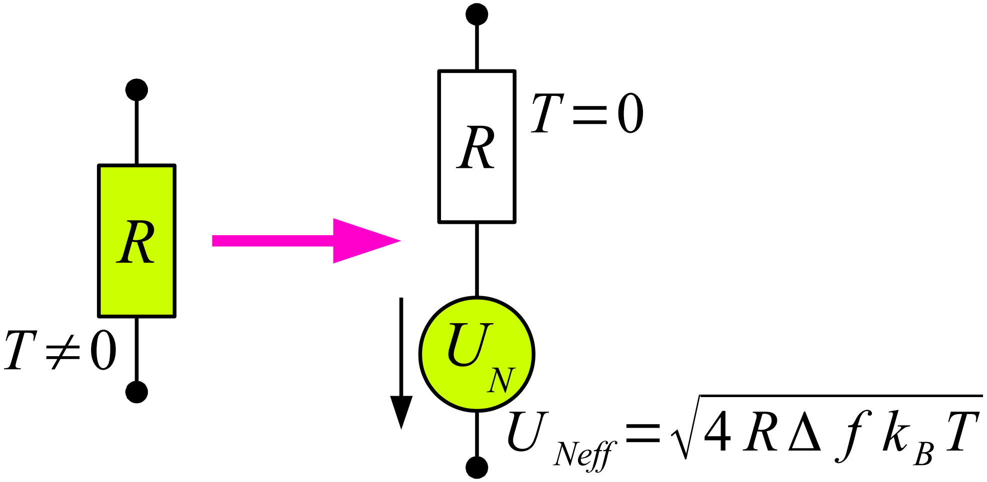
$\approx$

Bandpass filter Δf

$T_2 \neq 0$

R

T_A is NOT a property of a lossless antenna!



R

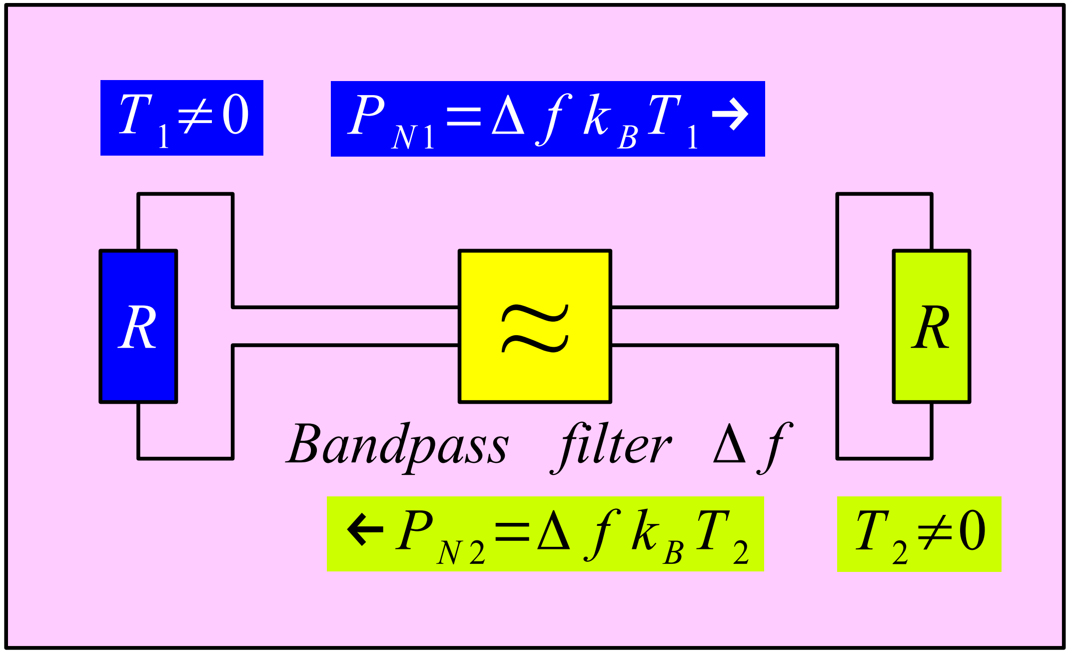
$T \neq 0$

R

$T=0$

U_N

$U_{Neff} = \sqrt{4 R \Delta f\, k_B T}$



$T_1 \neq 0$

$P_{N1} = \Delta f\, k_B T_1 \rightarrow$

R

$\approx$

R

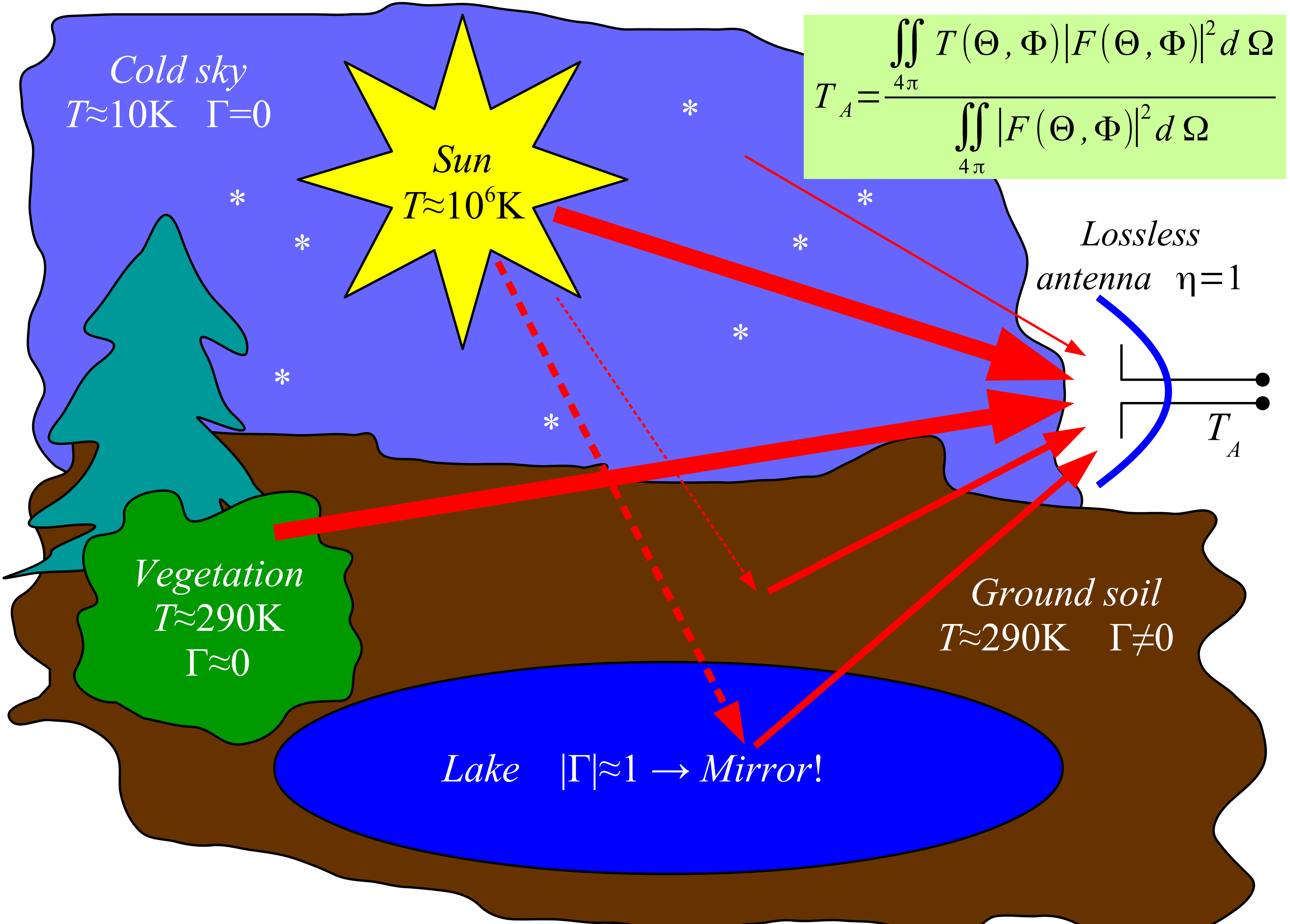
Bandpass filter Δf

$\leftarrow P_{N2} = \Delta f\, k_B T_2$

$T_2 \neq 0$

5 – Thermal equilibrium

6 – Natural noise sources



$$T_A = \frac{\iint_{4\pi} T(\Theta,\Phi)\,|F(\Theta,\Phi)|^2\, d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2\, d\Omega}$$

Cold sky
$T \approx 10\text{K}$ $\Gamma = 0$

Sun
$T \approx 10^6\text{K}$

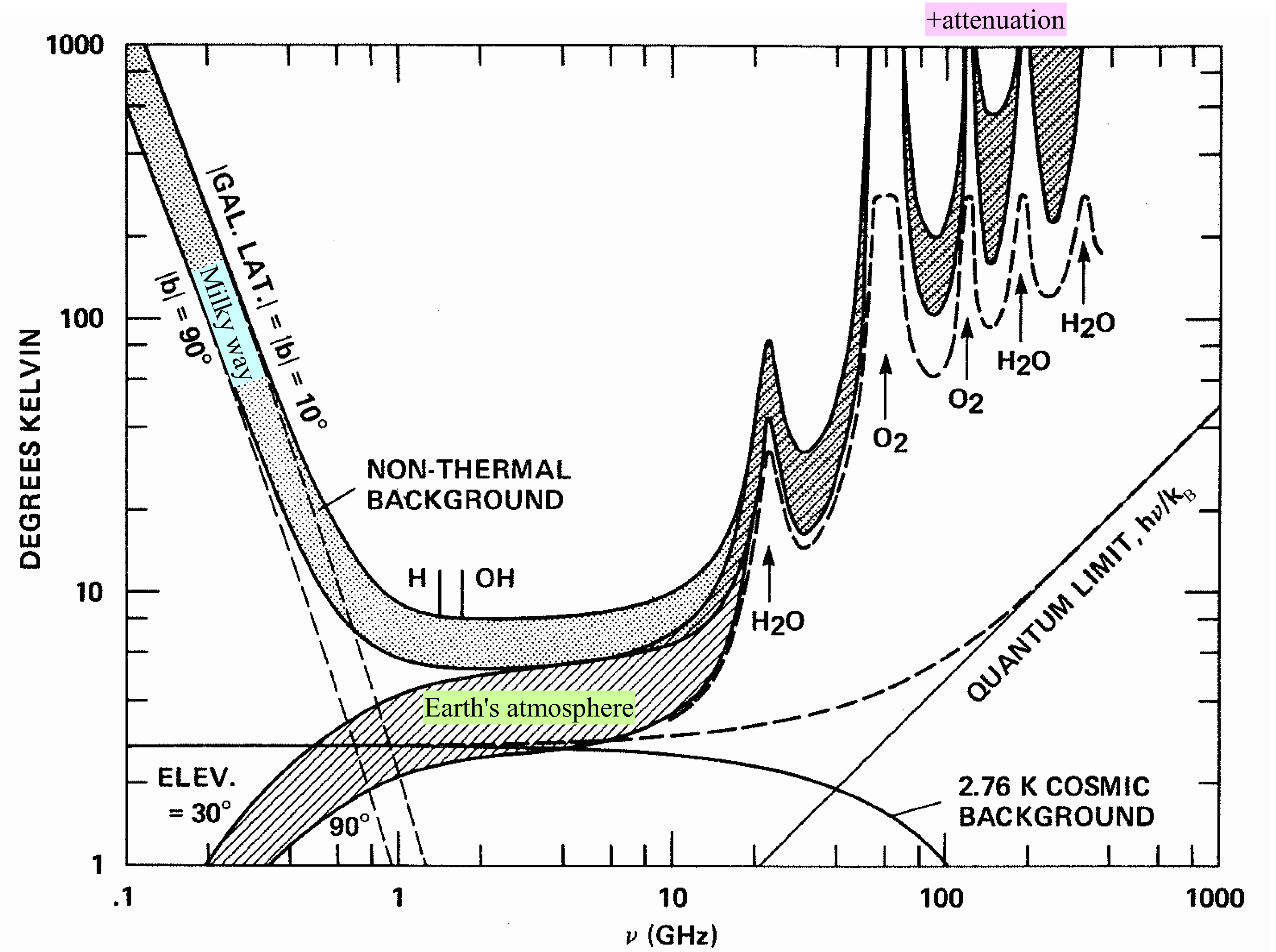
Lossless antenna $\eta = 1$

T_A

Vegetation
$T \approx 290\text{K}$
$\Gamma \approx 0$

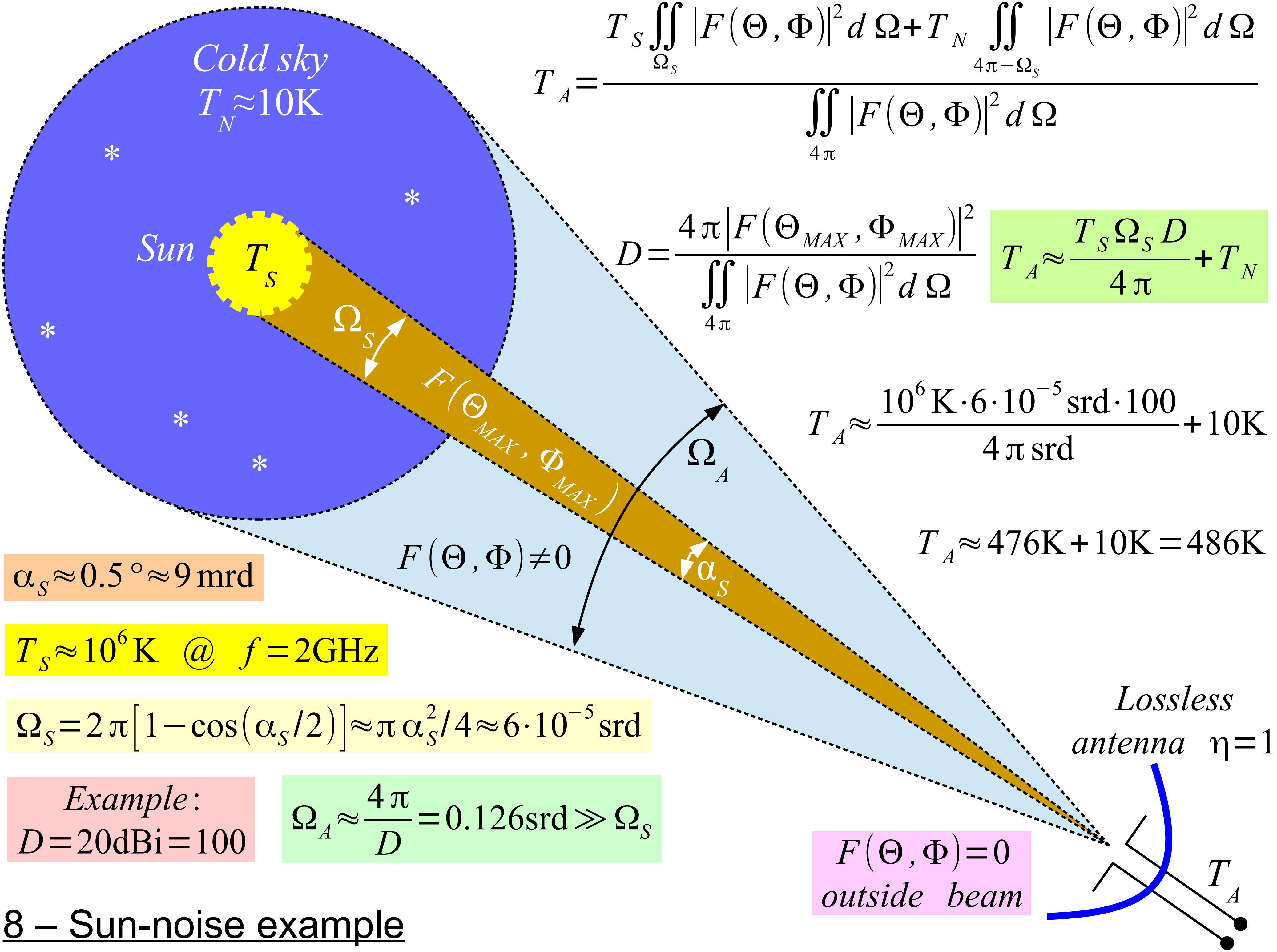
Ground soil
$T \approx 290\text{K}$ $\Gamma \neq 0$

Lake $|\Gamma| \approx 1 \rightarrow$ *Mirror!*



7 – Natural sky noise

8 – Sun-noise example



$$T_A = \frac{T_S \iint\limits_{\Omega_S} |F(\Theta,\Phi)|^2 d\Omega + T_N \iint\limits_{4\pi-\Omega_S} |F(\Theta,\Phi)|^2 d\Omega}{\iint\limits_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

$$D = \frac{4\pi |F(\Theta_{MAX},\Phi_{MAX})|^2}{\iint\limits_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

$$T_A \approx \frac{T_S \Omega_S D}{4\pi} + T_N$$

$$T_A \approx \frac{10^6\,\mathrm{K} \cdot 6 \cdot 10^{-5}\,\mathrm{srd} \cdot 100}{4\pi\,\mathrm{srd}} + 10\mathrm{K}$$

$$T_A \approx 476\mathrm{K} + 10\mathrm{K} = 486\mathrm{K}$$

$\alpha_S \approx 0.5° \approx 9\,\mathrm{mrd}$

$T_S \approx 10^6\,\mathrm{K}$ @ $f = 2\mathrm{GHz}$

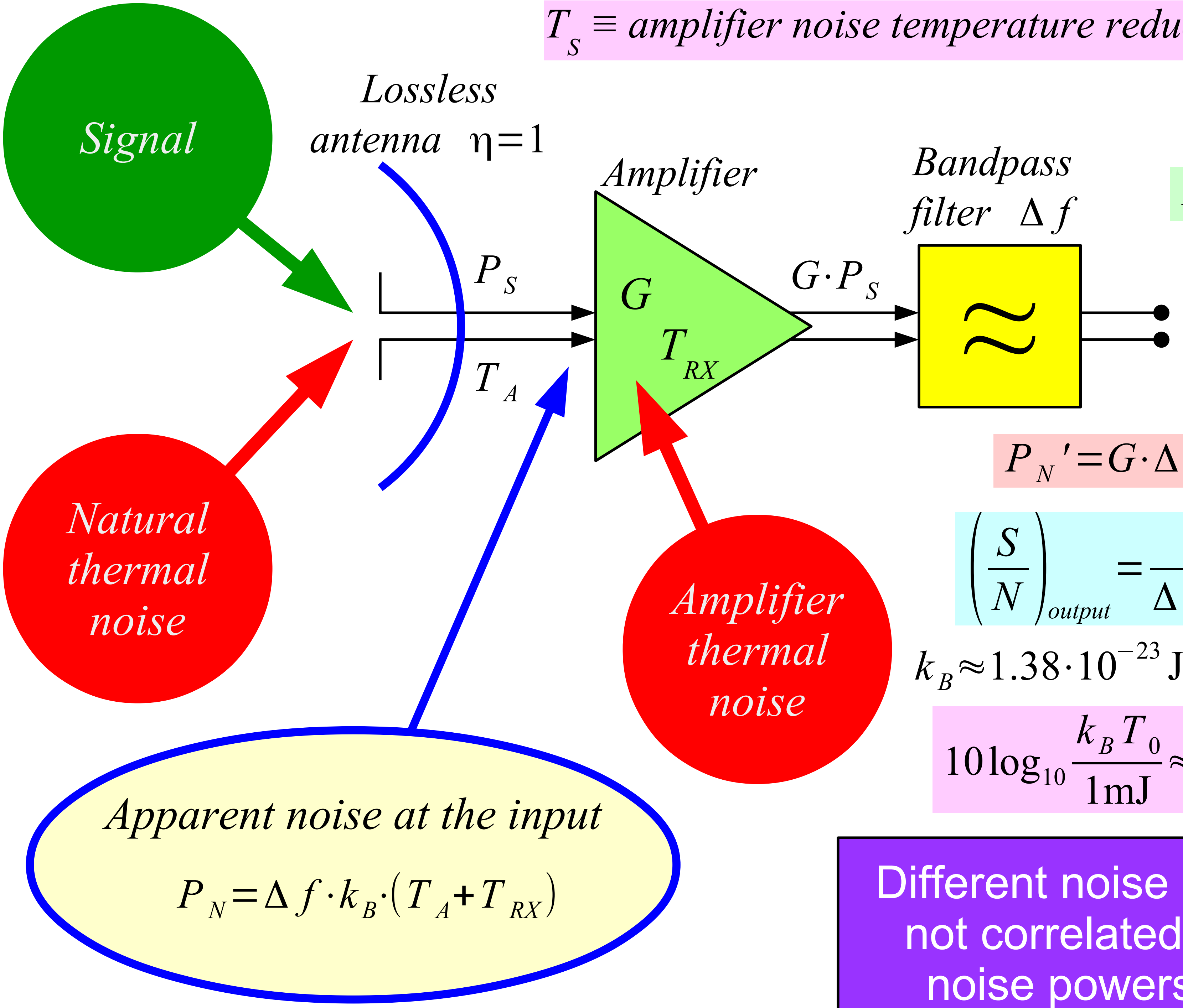
$\Omega_S = 2\pi[1 - \cos(\alpha_S/2)] \approx \pi\alpha_S^2/4 \approx 6 \cdot 10^{-5}\,\mathrm{srd}$

Example:
$D = 20\mathrm{dBi} = 100$

$$\Omega_A \approx \frac{4\pi}{D} = 0.126\mathrm{srd} \gg \Omega_S$$

9 – Receiver signal-to-noise ratio

$T_S \equiv$ *amplifier noise temperature reduced to the input!*



Apparent noise at the input

$$P_N = \Delta f \cdot k_B \cdot (T_A + T_{RX})$$

$$P_S' = G \cdot P_S$$

$$\left(\frac{S}{N}\right)_{output} = \frac{P_S'}{P_N'}$$

$$P_N' = G \cdot \Delta f \cdot k_B \cdot (T_A + T_{RX})$$

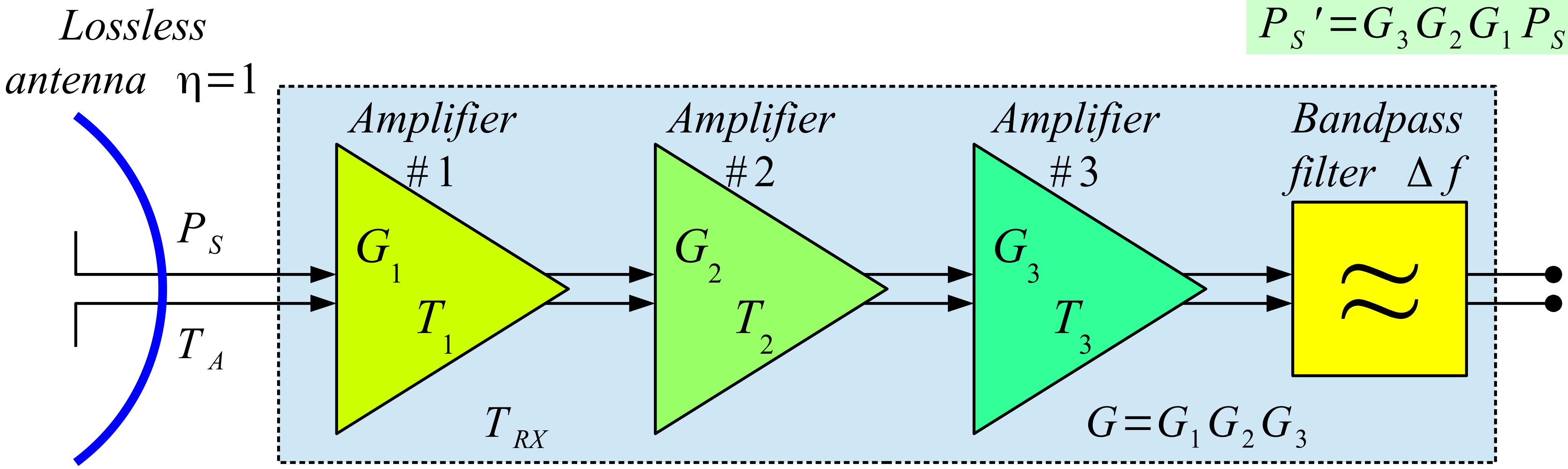
$$\left(\frac{S}{N}\right)_{output} = \frac{P_S}{\Delta f \cdot k_B \cdot (T_A + T_{RX})}$$

$$k_B \approx 1.38 \cdot 10^{-23}\ \mathrm{J/K} \qquad T_0 = 290\mathrm{K}$$

$$10\log_{10}\frac{k_B T_0}{1\mathrm{mJ}} \approx -174\ \mathrm{dBm/Hz}$$

Different noise sources are not correlated, therefore noise powers or noise temperatures are summed!

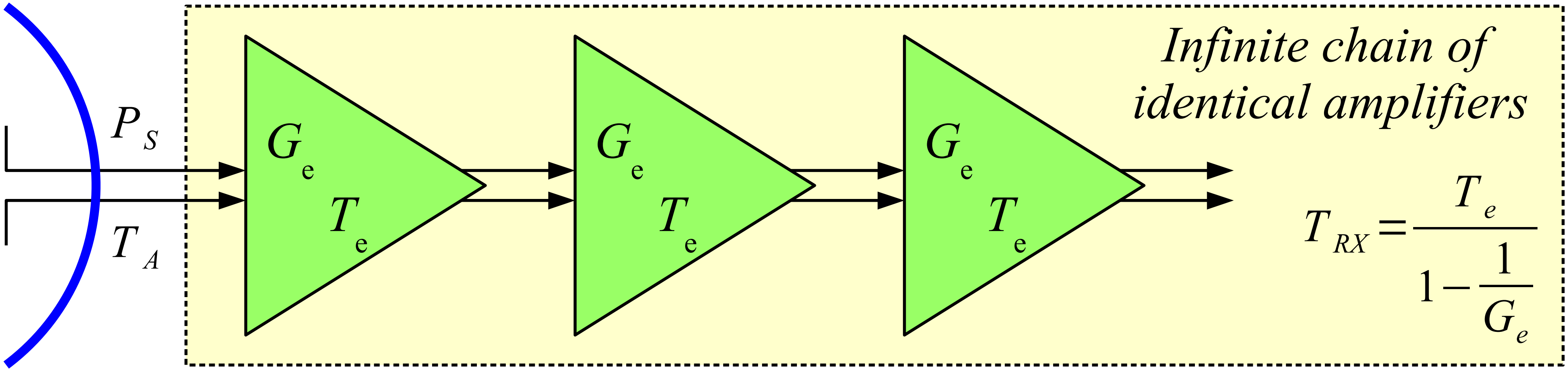
$$P_S' = G_3 G_2 G_1 P_S$$



$$G = G_1 G_2 G_3$$

$$P_N' = \Delta f\, k_B \left[G_3 G_2 G_1 (T_A + T_1) + G_3 G_2 T_2 + G_3 T_3 \right]$$

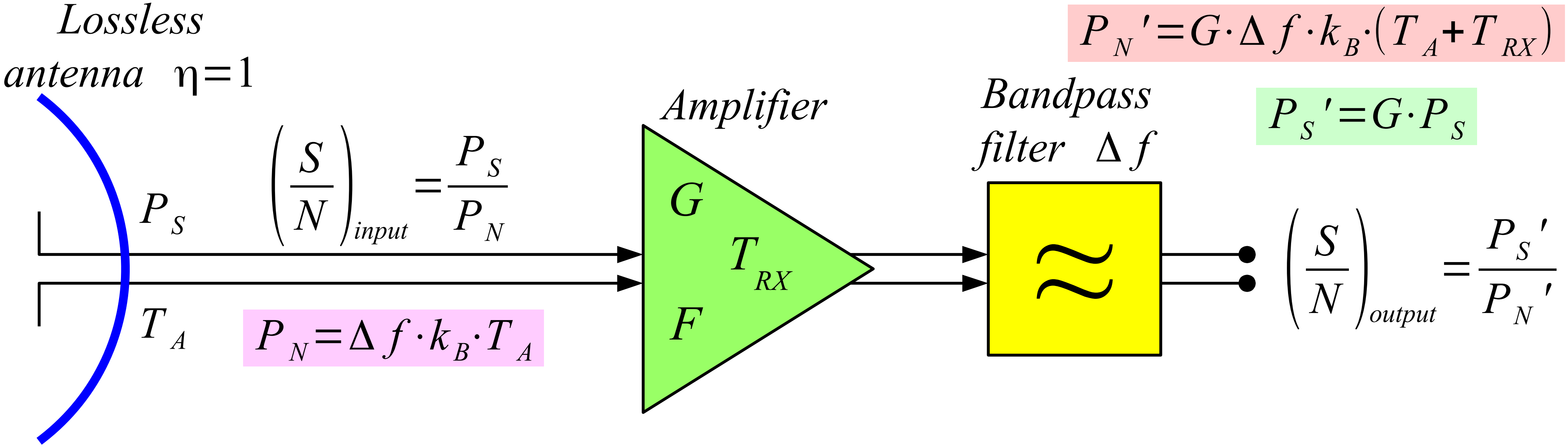
$$P_N' = G_3 G_2 G_1 \Delta f\, k_B (T_A + T_{RX}) \quad \rightarrow \quad T_{RX} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \dots$$



Infinite chain of identical amplifiers

$$T_{RX} = \frac{T_e}{1 - \frac{1}{G_e}}$$

10 – Chain noise temperature



$$\left(\frac{S}{N}\right)_{input} = \frac{P_S}{P_N}$$

$$P_N = \Delta f \cdot k_B \cdot T_A$$

$$P_N{}' = G \cdot \Delta f \cdot k_B \cdot \left(T_A + T_{RX}\right)$$

$$P_S{}' = G \cdot P_S$$

$$\left(\frac{S}{N}\right)_{output} = \frac{P_S{}'}{P_N{}'}$$

Nonsense definition of the noise figure:

$$F = \frac{\left(\frac{S}{N}\right)_{input}}{\left(\frac{S}{N}\right)_{output}} = \frac{\frac{P_S}{\Delta f\, k_B T_A}}{\frac{G P_S}{G \Delta f\, k_B \left(T_A + T_{RX}\right)}} = \frac{T_A + T_{RX}}{T_A} = 1 + \frac{T_{RX}}{T_A}$$

A property of an amplifier can not be a function of T_A!

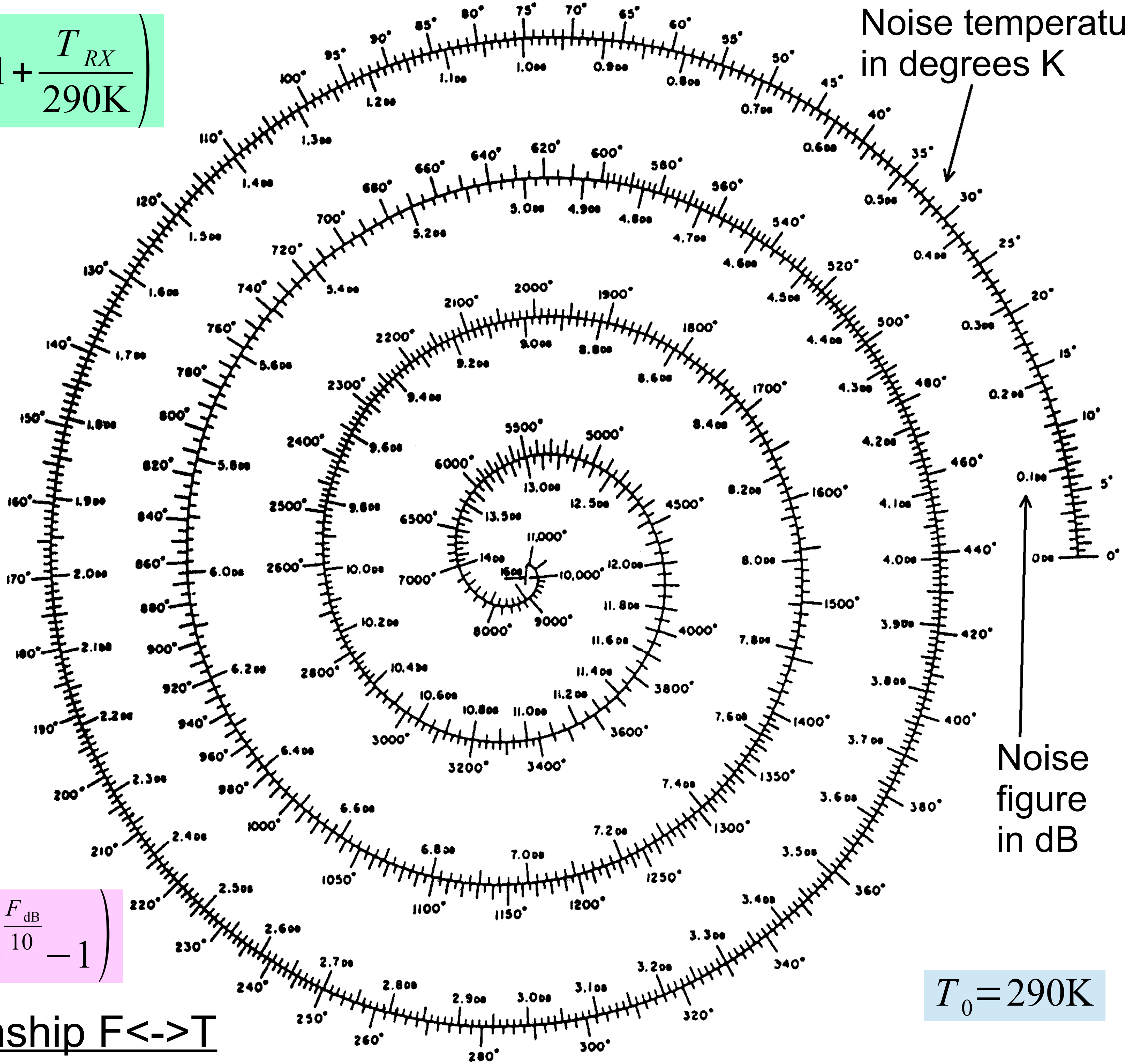
Sensible definition $F = 1 + \frac{T_{RX}}{T_0}$ @ $T_0 = 290\mathrm{K}$ ←→ $T_{RX} = T_0\left(F - 1\right)$

Logarithmic units $F_{\mathrm{dB}} = 10\log_{10} F = 10\log_{10}\left(1 + \frac{T_{RX}}{T_0}\right)$ ←→ $T_{RX} = T_0\left(10^{\frac{F_{\mathrm{dB}}}{10}} - 1\right)$

11 – Amplifier noise figure

$$F_{\mathrm{dB}} = 10\log_{10}\left(1 + \frac{T_{RX}}{290\mathrm{K}}\right)$$

Noise temperature in degrees K



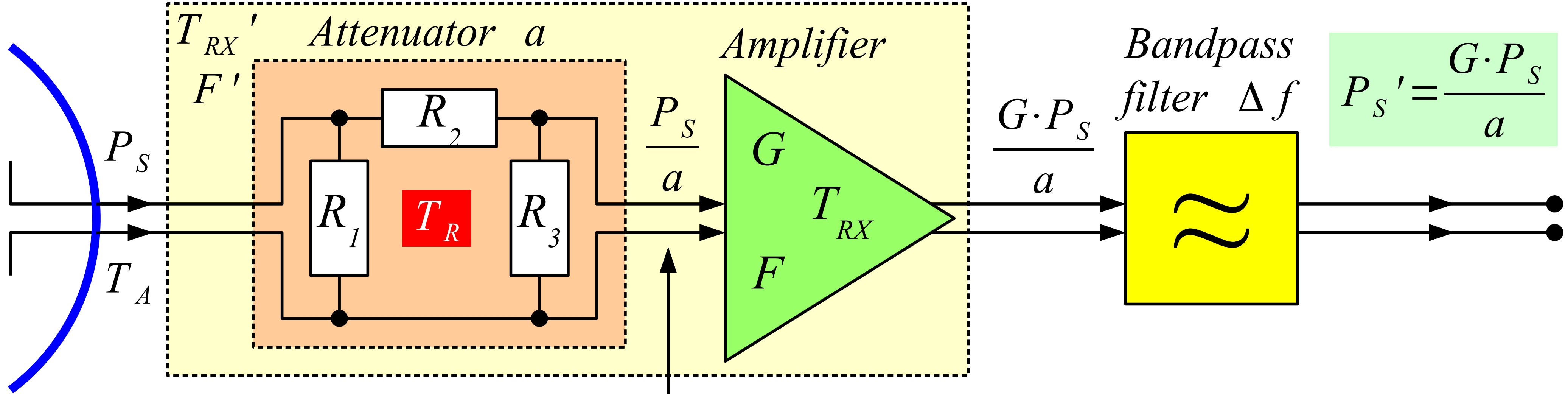
Noise figure in dB

$$T_{RX} = 290\mathrm{K}\left(10^{\frac{F_{\mathrm{dB}}}{10}} - 1\right)$$

$$T_0 = 290\mathrm{K}$$

12 – Relationship F<->T

13 – Attenuator noise



$$P_S'=\frac{G\cdot P_S}{a}$$

$$\frac{T_A}{a}+T_R\left(1-\frac{1}{a}\right)$$

$$P_N'=G\cdot\Delta f\cdot k_B\cdot\left[\frac{T_A}{a}+T_R\left(1-\frac{1}{a}\right)+T_{RX}\right]$$

$$T_{RX}'=T_R(a-1)+a\,T_{RX}$$

$$\left(\frac{S}{N}\right)_{output}=\frac{P_S'}{P_N'}=\frac{P_S}{\Delta f\cdot k_B\cdot\left[T_A+T_R(a-1)+a\,T_{RX}\right]}$$

$$F'=1+\frac{T_{RX}'}{T_0}=1+\frac{T_R}{T_0}(a-1)+a\frac{T_{RX}}{T_0}$$

Frequent case $T_R\approx T_0=290\mathrm{K}$

$$F'\approx a+a\frac{T_{RX}}{T_0}=a\left(1+\frac{T_{RX}}{T_0}\right)=a\cdot F$$

$$F_{\mathrm{dB}}'\approx a_{\mathrm{dB}}+F_{\mathrm{dB}}$$

Attenuator examples $T_R\approx T_0=290\mathrm{K}$

$F'\approx a\cdot F$ $\quad F_{\mathrm{dB}}'\approx a_{\mathrm{dB}}+F_{\mathrm{dB}}$

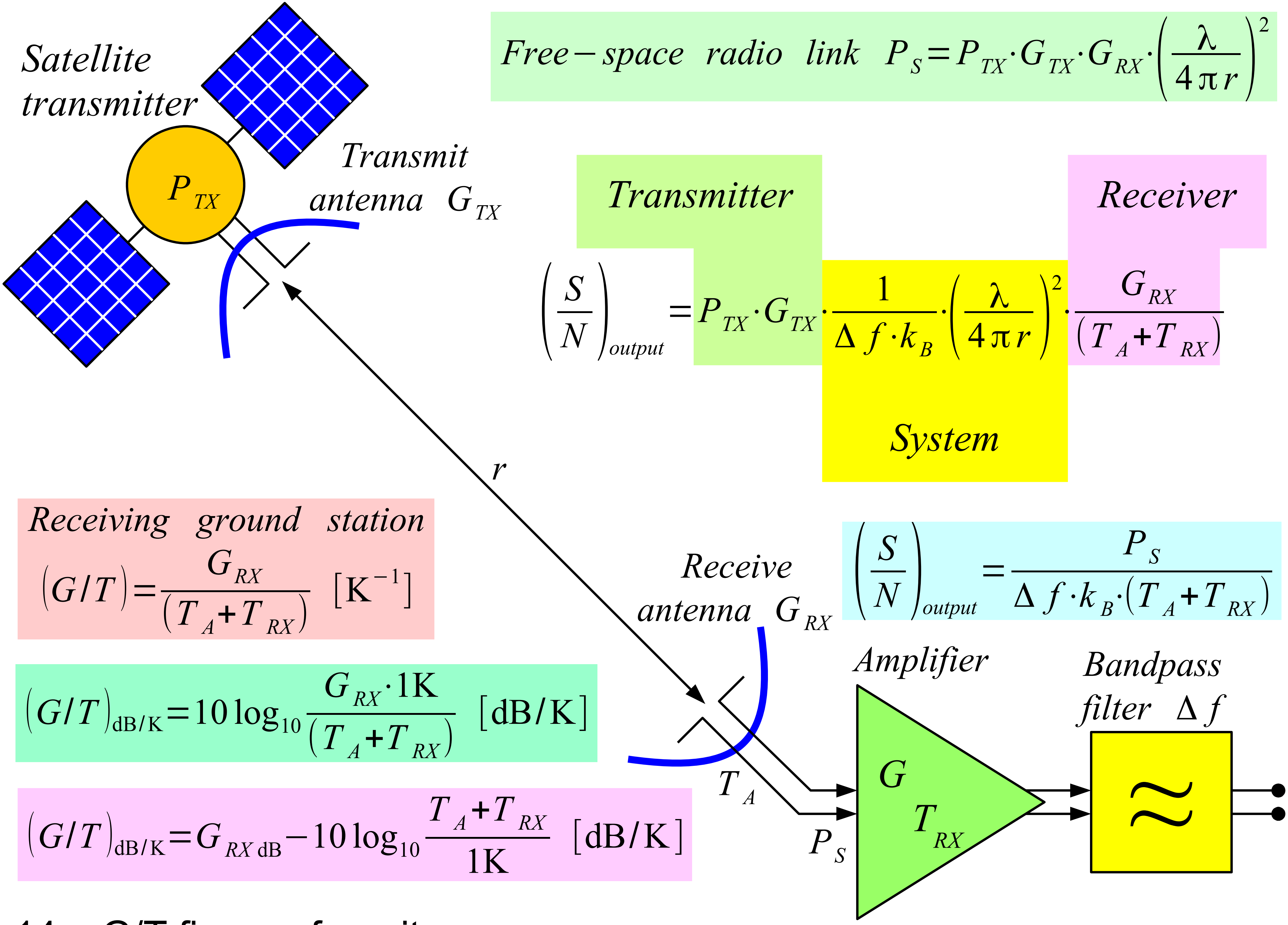
(1) *lossy antenna* $a_{\mathrm{dB}}=-10\log_{10}\eta$

(2) *lossy transmission line* a_{dB}

(3) *lossy bandpass filter* a_{dB}

(4) *passive−mixer loss* a_{dB}

14 – G/T figure of merit



Free-space radio link $P_S = P_{TX}\cdot G_{TX}\cdot G_{RX}\cdot\left(\frac{\lambda}{4\pi r}\right)^2$

Transmitter

Receiver

$$\left(\frac{S}{N}\right)_{output} = P_{TX}\cdot G_{TX}\cdot\frac{1}{\Delta f\cdot k_B}\cdot\left(\frac{\lambda}{4\pi r}\right)^2\cdot\frac{G_{RX}}{(T_A+T_{RX})}$$

System

$$\left(\frac{S}{N}\right)_{output} = \frac{P_S}{\Delta f\cdot k_B\cdot(T_A+T_{RX})}$$

Receiving ground station

$$(G/T) = \frac{G_{RX}}{(T_A+T_{RX})} \quad [\mathrm{K}^{-1}]$$

$$(G/T)_{\mathrm{dB/K}} = 10\log_{10}\frac{G_{RX}\cdot 1\mathrm{K}}{(T_A+T_{RX})} \quad [\mathrm{dB/K}]$$

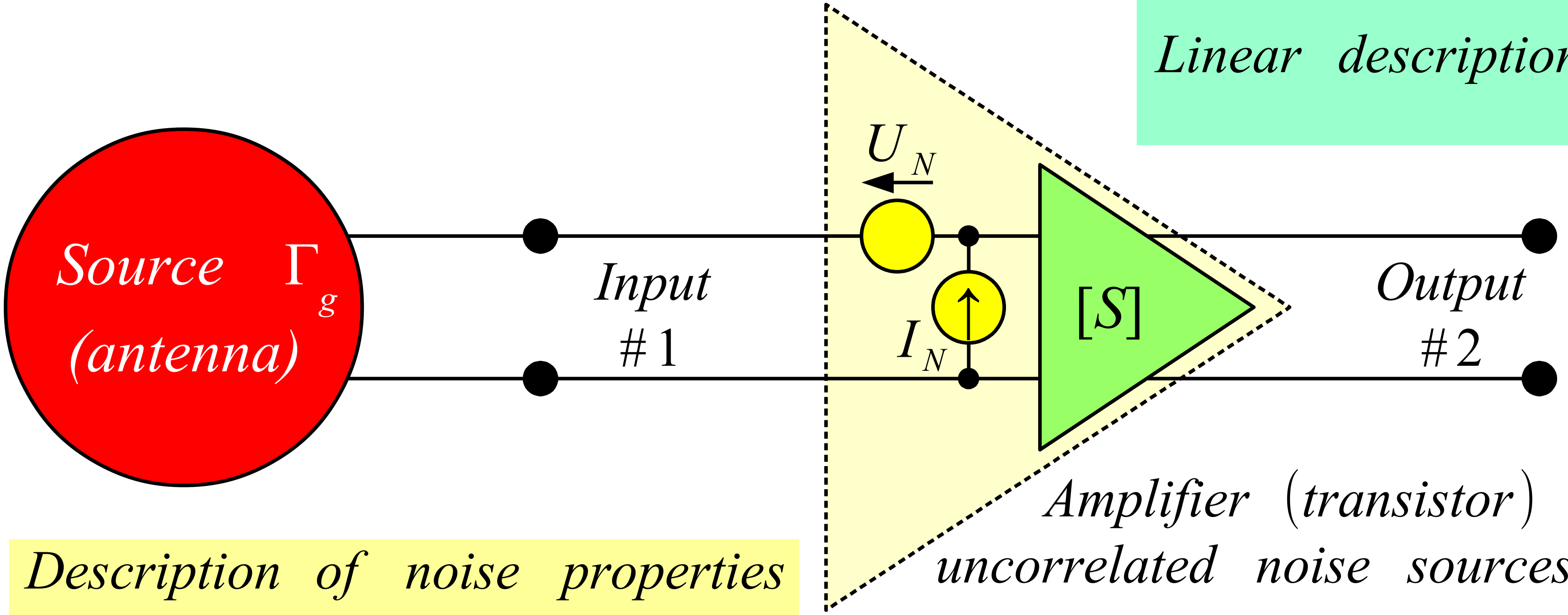
$$(G/T)_{\mathrm{dB/K}} = G_{RX\,\mathrm{dB}} - 10\log_{10}\frac{T_A+T_{RX}}{1\mathrm{K}} \quad [\mathrm{dB/K}]$$

Amplifier device	Gain G [dB]	Noise temperature T_{RX} [K]	Noise figure F_{dB} [dB]
Vacuum tube with control grid (triode, pentode)	10↔20	1600↔9000	8↔15
Vacuum tube with speed modulation (klystron, TWT)	20↔50	3000↔30000	10↔20
Parametric amplifier (room temperature)	10↔15	75↔300	1↔3
Si BJT, JFET or MOSFET (room temperature)	10↔20	75↔300	1↔3
GaAs FET or HEMT (room temperature)	10↔15	20↔120	0.3↔1.5
GaAs FET ali HEMT (liquid-nitrogen 77K)	10↔15	7↔35	0.1↔0.5
Si or GaAs MMIC amplifier	10↔25	170↔1600	2↔8
Operational amplifier	40↔100	10^4↔10^9	16↔66

15 – Noise of active components

16 – Noise parameters

Linear description $[S]=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$



Amplifier (transistor) with uncorrelated noise sources U_N, I_N

Description of noise properties
$U_N, I_N \rightarrow F_{MIN}, \Gamma_O, r_N$

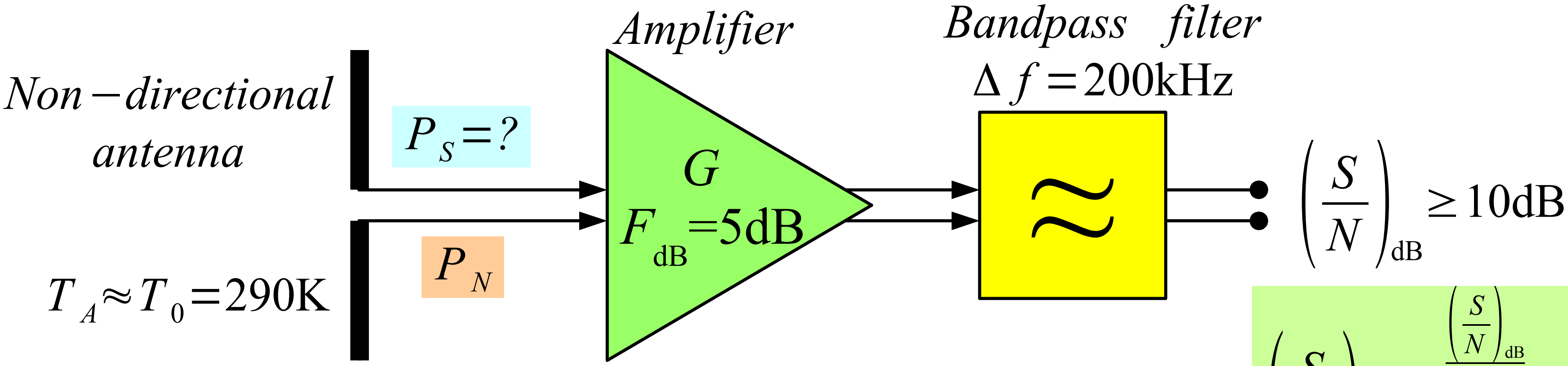
$$F = F_{MIN} + 4\frac{R_N}{Z_K}\cdot\frac{|\Gamma_g-\Gamma_O|^2}{\left(1-|\Gamma_g|^2\right)\cdot|1+\Gamma_O|^2} = F_{MIN} + 4r_N\cdot\frac{|\Gamma_g-\Gamma_O|^2}{\left(1-|\Gamma_g|^2\right)\cdot|1+\Gamma_O|^2}$$

$F_{MIN} \equiv$ lowest noise figure at $\Gamma_g=\Gamma_O$ in linear units (not in dB!)

$\Gamma_O \equiv$ optimum source reflectivity for F_{MIN} (unrelated to $[S]$!)

$r_N = \dfrac{R_N}{Z_K} \equiv$ normalized noise resistance (usually $Z_K = 50\,\Omega$)

Low-noise devices are usually NOT unconditionally stable amplifiers!



$$\left(\frac{S}{N}\right)=10^{\frac{\left(\frac{S}{N}\right)_{\mathrm{dB}}}{10}}\geq 10$$

$$T_{RX}=T_0\cdot\left(10^{\frac{F_{\mathrm{dB}}}{10}}-1\right)=290\mathrm{K}\cdot(3.162-1)=627\mathrm{K}$$

$$k_B\approx 1.38\cdot 10^{-23}\,\mathrm{J/K}$$

$$P_N=\Delta f\cdot k_B\cdot(T_A+T_{RX})=200\mathrm{kHz}\cdot 1.38\cdot 10^{-23}\,\mathrm{J/K}\cdot(290\mathrm{K}+627\mathrm{K})=2.531\cdot 10^{-15}\,\mathrm{W}$$

$$P_S=P_N\cdot\left(\frac{S}{N}\right)=2.531\cdot 10^{-15}\,\mathrm{W}\cdot 10=2.531\cdot 10^{-14}\,\mathrm{W}$$

$$P_{S\,\mathrm{dBm}}=10\log_{10}\frac{P_S}{1\mathrm{mW}}=-106\mathrm{dBm}$$

Simplified calculation exclusively in case $T_A\approx T_0=290\mathrm{K}$

$$P_{S\,\mathrm{dBm}}\approx(S/N)_{\mathrm{dB}}+(\Delta f)_{\mathrm{dB\cdot Hz}}+(k_B T_0)_{\mathrm{dBm/Hz}}+F_{\mathrm{dB}}$$

$$(k_B T_0)_{\mathrm{dBm/Hz}}=10\log_{10}\frac{k_B T_0}{1\mathrm{mJ}}\approx-174\,\mathrm{dBm/Hz}$$

$$(\Delta f)_{\mathrm{dB\cdot Hz}}=10\log_{10}\left(\frac{\Delta f}{1\mathrm{Hz}}\right)=53\mathrm{dB\cdot Hz}$$

$$P_{S\,\mathrm{dBm}}\approx 10\mathrm{dB}+53\mathrm{dB\cdot Hz}-174\mathrm{dBm/Hz}+5\mathrm{dB}=-106\mathrm{dBm}$$

17 – Example: the sensitivity of a GSM phone

18 – Change of S/N versus F

Two different receivers #1 and #2:

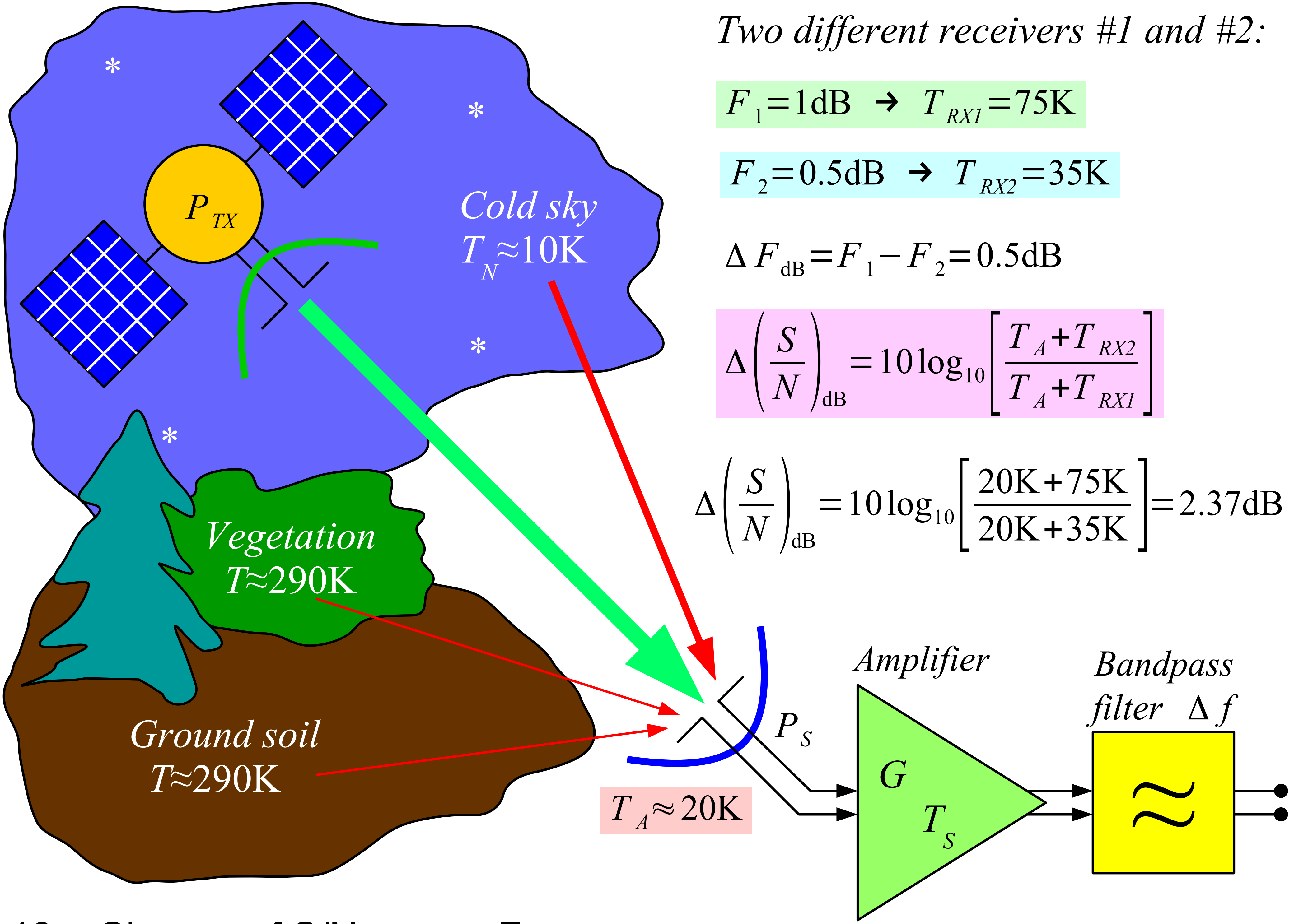
$F_1 = 1\text{dB} \rightarrow T_{RX1} = 75\text{K}$

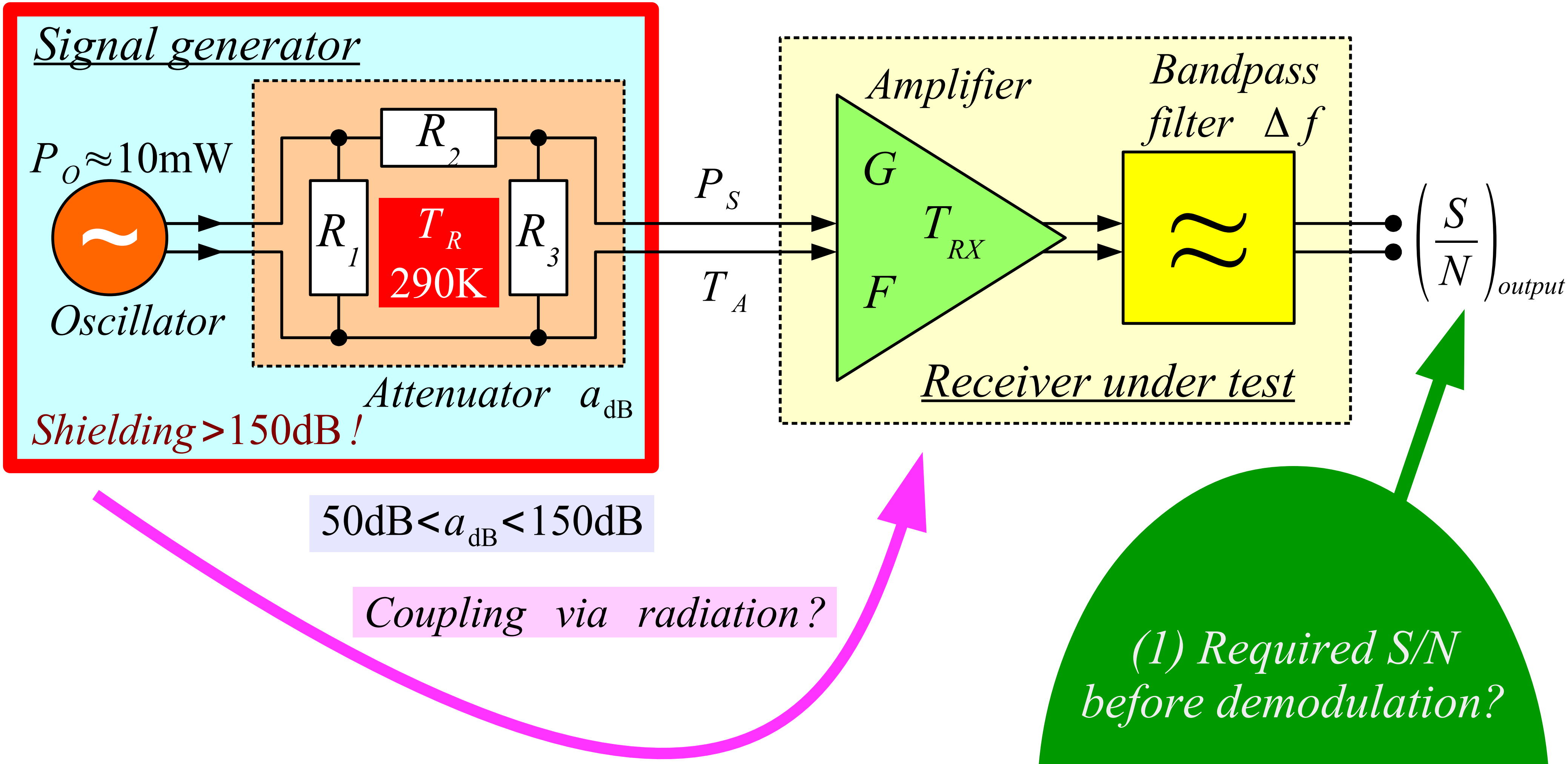
$F_2 = 0.5\text{dB} \rightarrow T_{RX2} = 35\text{K}$

$$\Delta F_{\text{dB}} = F_1 - F_2 = 0.5\text{dB}$$

$$\Delta\left(\frac{S}{N}\right)_{\text{dB}} = 10\log_{10}\left[\frac{T_A + T_{RX2}}{T_A + T_{RX1}}\right]$$

$$\Delta\left(\frac{S}{N}\right)_{\text{dB}} = 10\log_{10}\left[\frac{20\text{K} + 75\text{K}}{20\text{K} + 35\text{K}}\right] = 2.37\text{dB}$$





Shielding >150dB !

$50\text{dB} < a_{\text{dB}} < 150\text{dB}$

Coupling via radiation ?

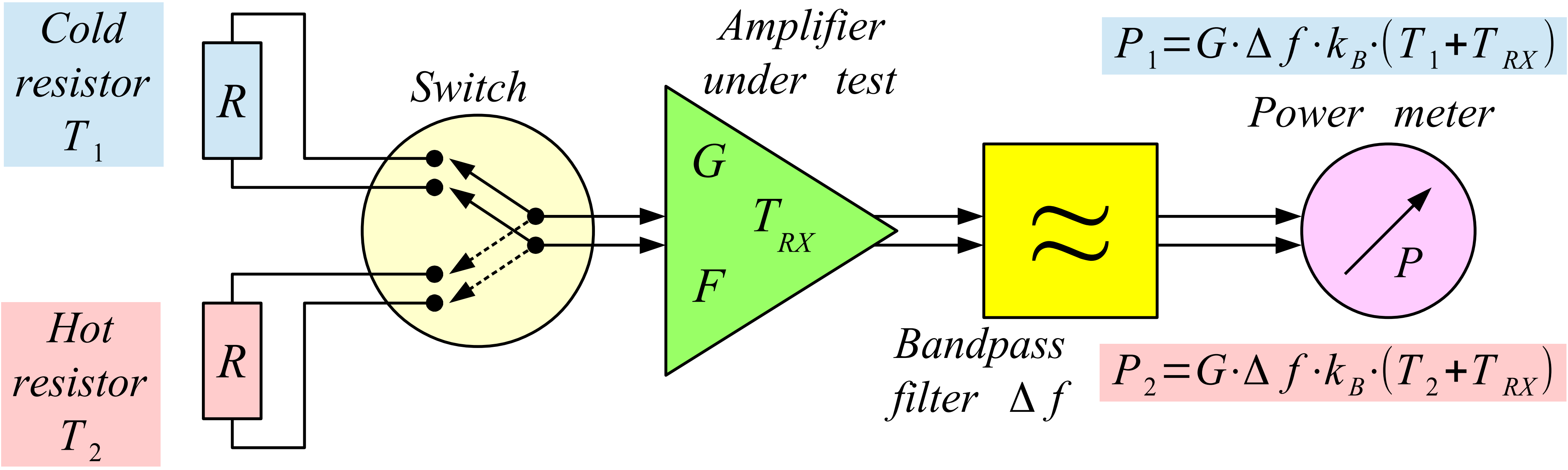
(1) Required S/N before demodulation?

(2) Required S/N after demodulation?

(3) Required BER?

Additional requirements for the test source (signal generator) for sensitivity measurements of radio receivers:
(1) Shielding>150dB
(2) $T_R = T_A = T_0 = 290\text{K}$

19 – Receiver-sensitivity measurement



$$P_1 = G \cdot \Delta f \cdot k_B \cdot (T_1 + T_{RX})$$

$$P_2 = G \cdot \Delta f \cdot k_B \cdot (T_2 + T_{RX})$$

The unknowns $G \cdot \Delta f \cdot k_B$ *cancel in the Y ratio!*

$$Y = \frac{P_2}{P_1} = \frac{T_2 + T_{RX}}{T_1 + T_{RX}}$$

$$T_{RX} = \frac{T_2 - Y \cdot T_1}{Y - 1}$$

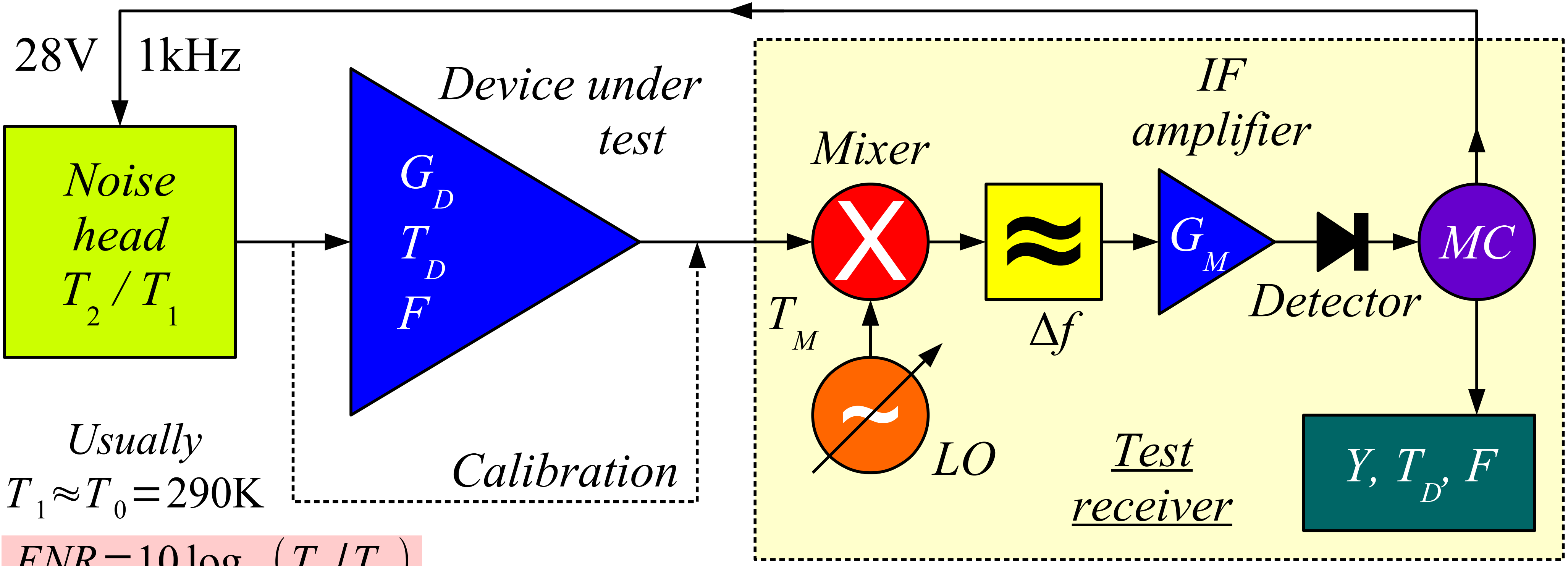
$$T_0 = 290\text{K}$$

$$F_{dB} = 10 \log_{10} \left[1 + \frac{T_2 - Y \cdot T_1}{(Y-1) \cdot T_0} \right]$$

Resistor type	Temperature
Antenna into cold sky	~20K
Liquid N_2 cooled R	~77K
Antenna into absorber	~290K
R at room temperature	~290K
Light-bulb filament as R	~2000K
Ionized gas as R	~10^4K
Avalanche breakdown	~10^6K

20 – Hot/cold method

21 – Noise-figure meter



Usually $T_1 \approx T_0 = 290\mathrm{K}$

$$ENR = 10\log_{10}\left(T_2/T_1\right)$$

$$G_M \Delta f \equiv uncertain\,!$$

Two measurements without calibration:

$$Y = \frac{P_2}{P_1} = \frac{T_2 + T_D + T_M/G_D}{T_1 + T_D + T_M/G_D}$$

$$T_D = \frac{T_2 - Y\cdot T_1}{Y-1} - \frac{T_M}{G_D} \leftarrow \ known\ \ G_D$$

$$F_{dB} = 10\log_{10}\left[1 + \frac{1}{T_0}\cdot\left(\frac{T_2 - Y\cdot T_1}{Y-1} - \frac{T_M}{G_D}\right)\right]$$

Four measurements with calibration:

$(1)\quad P_1 = G_M G_D \Delta f\, k_B\left(T_1 + T_D + T_M/G_D\right)$

$(2)\quad P_2 = G_M G_D \Delta f\, k_B\left(T_2 + T_D + T_M/G_D\right)$

$(3)\quad P_3 = G_M \Delta f\, k_B\left(T_1 + T_M\right)$

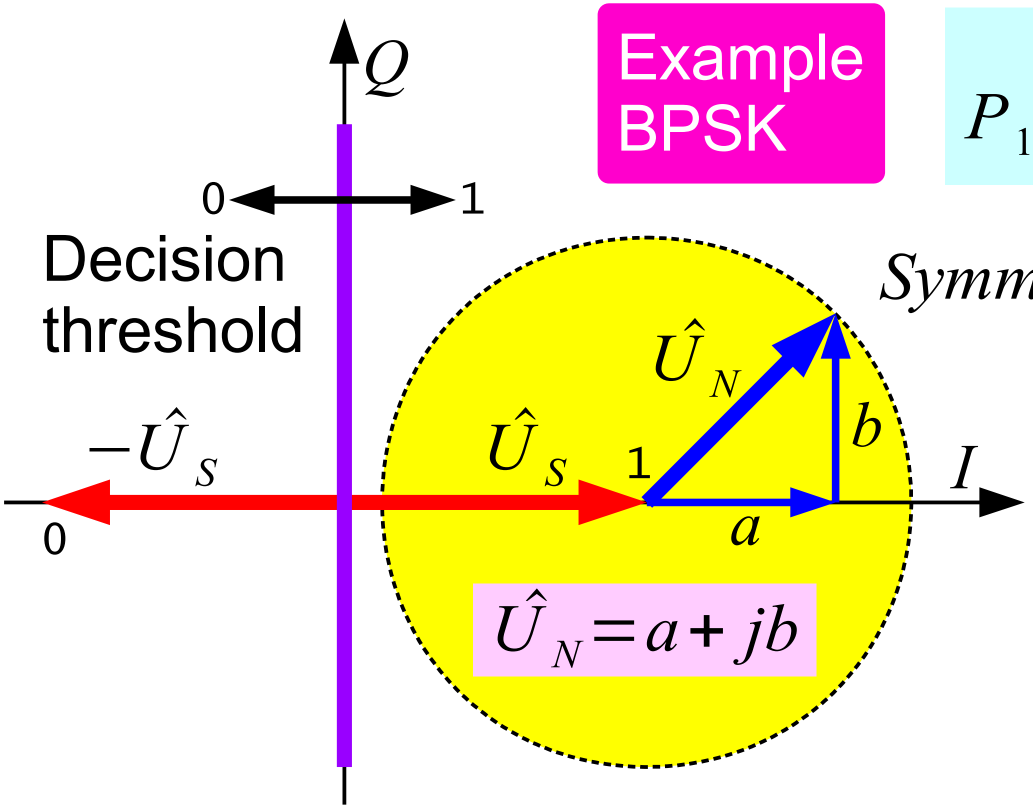
$(4)\quad P_4 = G_M \Delta f\, k_B\left(T_2 + T_M\right)$

Solve 4 equations for 4 unknowns:

$T_D,\ G_D,\ T_M$ and $\left(G_M \Delta f\, k_B\right)$

22 – Bit-Error Rate (BER) calculation

Example BPSK



$$\hat U_N = a + jb$$

$$P_{1\to0}=\int_{-\infty}^{-|\hat U_S|} p(a)\,da$$

$$P_{0\to1}=\int_{|\hat U_S|}^{\infty} p(a)\,da$$

Symmetric threshold: $P_{1\to0}=P_{0\to1}=BER$

$$BER=\int_{|\hat U_S|}^{\infty}\frac{1}{\sqrt{\pi\langle|\hat U_N|^2\rangle}}\,\mathrm{e}^{-\frac{a^2}{\langle|\hat U_N|^2\rangle}}\,da$$

$$\operatorname{erfc}(x)=\frac{2}{\sqrt{\pi}}\int_{x}^{\infty}\mathrm{e}^{-u^2}\,du$$

Gaussian distribution of probability density of the in-phase a and quadrature jb noise components

$$p(a)=\frac{1}{\sigma\sqrt{2\pi}}\,\mathrm{e}^{-\frac{a^2}{2\sigma^2}}$$

$$p(b)=\frac{1}{\sigma\sqrt{2\pi}}\,\mathrm{e}^{-\frac{b^2}{2\sigma^2}}$$

$$\langle|\hat U_N|^2\rangle=\langle a^2\rangle+\langle b^2\rangle=2\sigma^2$$

$$BER=\frac{1}{2}\operatorname{erfc}\left(\frac{|\hat U_S|}{\sqrt{\langle|\hat U_N|^2\rangle}}\right)$$

$$P_S=\alpha|\hat U_S|^2 \qquad P_N=\alpha\langle|\hat U_N|^2\rangle$$

$$\boxed{BER=\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{P_S}{P_N}}\right)}$$

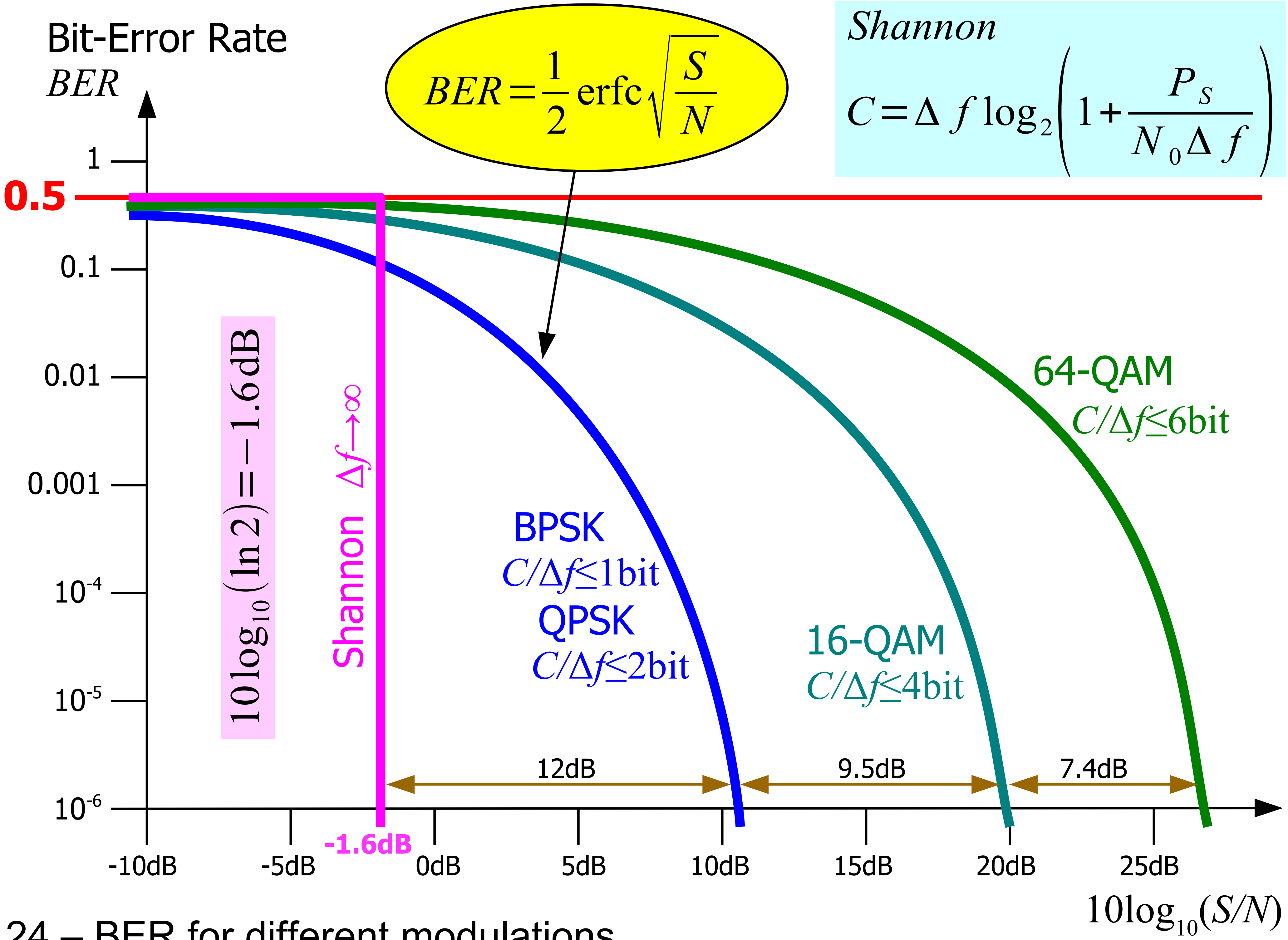
$(S/N)_{dB}$	BER
-5dB	23.6%
-4dB	18.6%
-3dB	15.9%
-2dB	13.1%
-1dB	10.4%
-0dB	7.9%
1dB	5.7%
2dB	3.8%
3dB	2.3%
4dB	1.3%
5dB	0.6%
6dB	0.24%
7dB	$7.7 \bullet 10^{-4}$
$(S/N)_{dB}$	BER

$(S/N)_{dB}$	BER
8dB	$1.9 \bullet 10^{-4}$
9dB	$3.4 \bullet 10^{-5}$
10dB	$3.9 \bullet 10^{-6}$
11dB	$2.6 \bullet 10^{-7}$
12dB	$9 \bullet 10^{-9}$
13dB	$1.3 \bullet 10^{-10}$
14dB	$6.8 \bullet 10^{-13}$
15dB	$9.2 \bullet 10^{-16}$
16dB	$2.3 \bullet 10^{-19}$
17dB	$6.8 \bullet 10^{-24}$
18dB	$1.4 \bullet 10^{-29}$
19dB	10^{-36}
20dB	10^{-45}
$(S/N)_{dB}$	BER

BER	$(S/N)_{dB}$
30%	-8.6dB
10%	-0.8dB
3%	2.5dB
1%	4.3dB
0.3%	5.8dB
0.1%	6.8dB
$3 \bullet 10^{-4}$	7.7dB
10^{-4}	8.4dB
$3 \bullet 10^{-5}$	9.1dB
10^{-5}	9.6dB
$3 \bullet 10^{-6}$	10.1dB
10^{-6}	10.5dB
$3 \bullet 10^{-7}$	11dB
BER	$(S/N)_{dB}$

BER	$(S/N)_{dB}$
10^{-7}	11.3dB
$3 \bullet 10^{-8}$	11.7dB
10^{-8}	12dB
$3 \bullet 10^{-9}$	12.3dB
10^{-9}	12.6dB
10^{-10}	13.1dB
10^{-11}	13.5dB
10^{-12}	13.9dB
10^{-13}	14.3dB
10^{-14}	14.7dB
10^{-15}	15dB
10^{-16}	15.3dB
10^{-17}	15.6dB
BER	$(S/N)_{dB}$

23 – BER <-> S/N table for BPSK

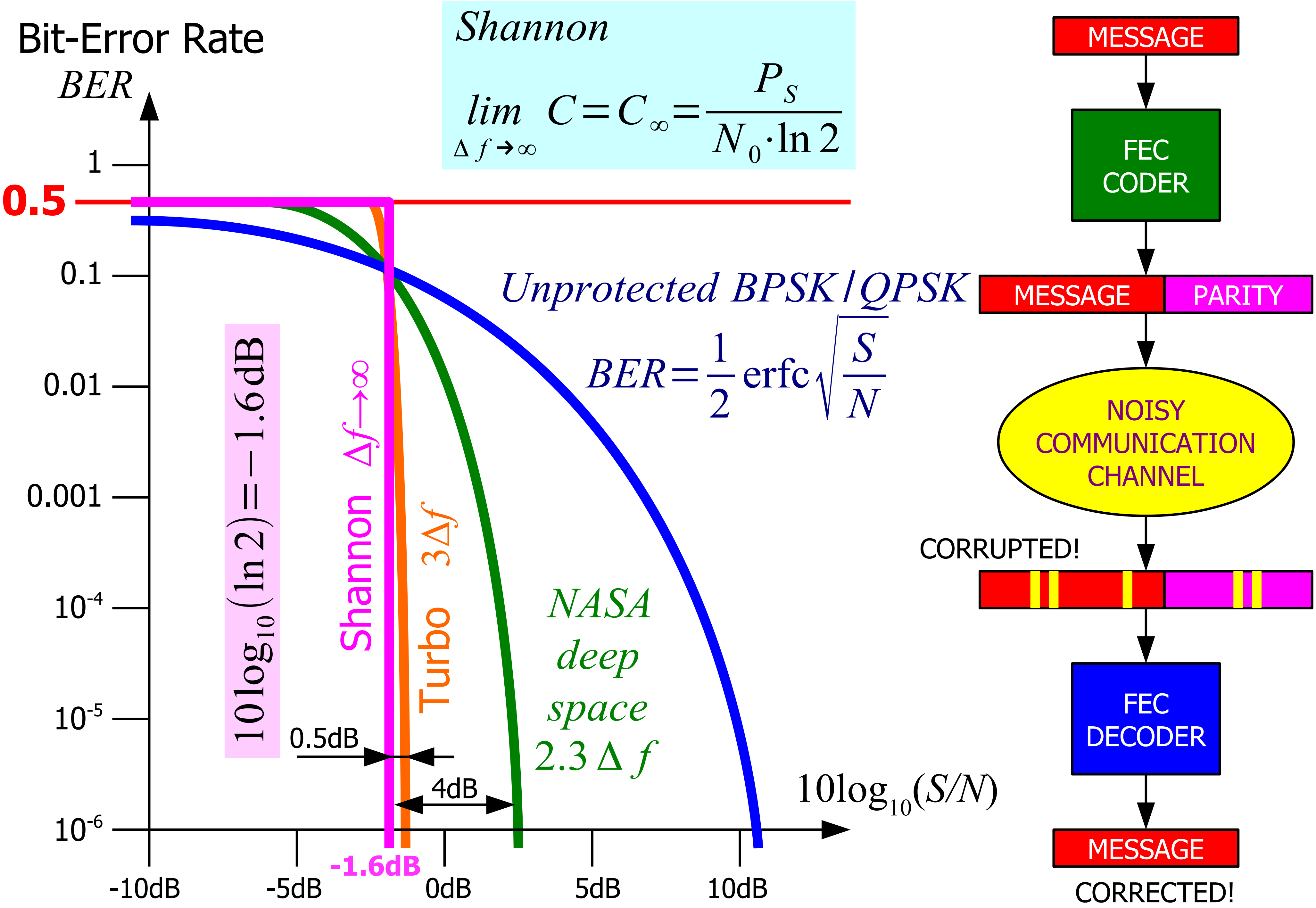


$$BER=\frac{1}{2}\operatorname{erfc}\sqrt{\frac{S}{N}}$$

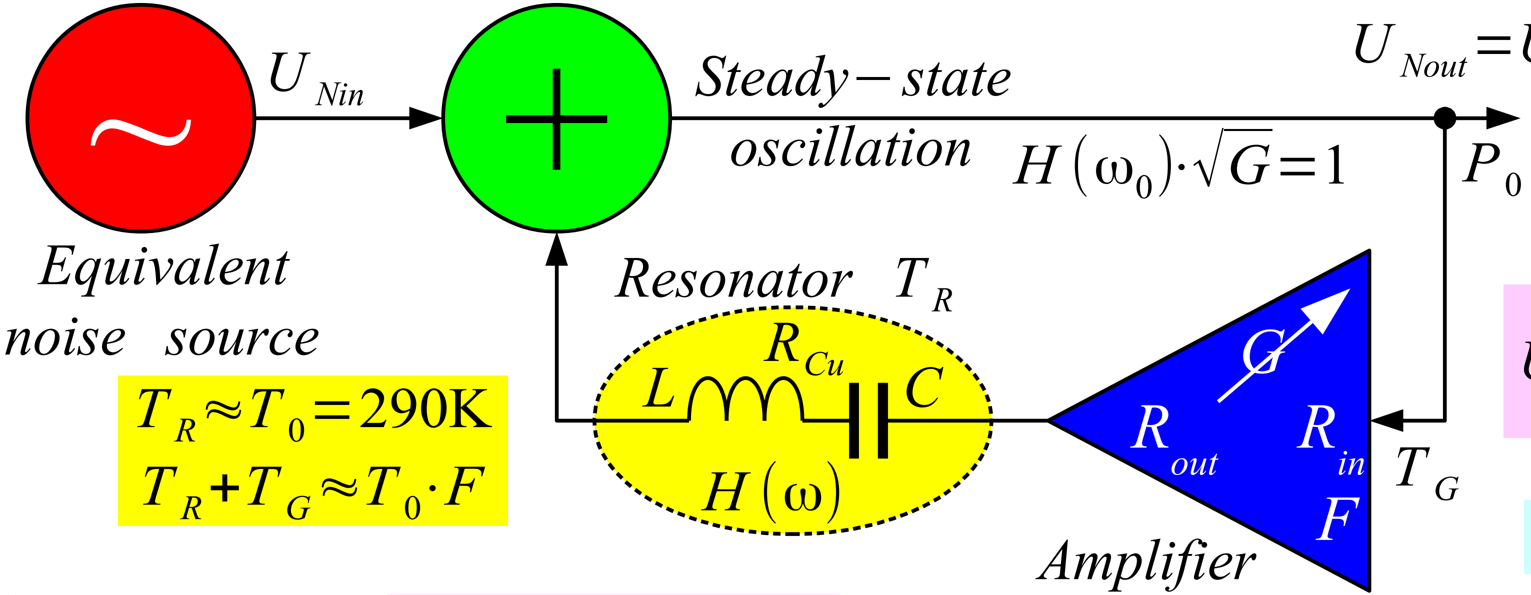
Shannon

$$C=\Delta f\log_2\left(1+\frac{P_S}{N_0\Delta f}\right)$$

24 – BER for different modulations



25 – Forward Error Correction (FEC)



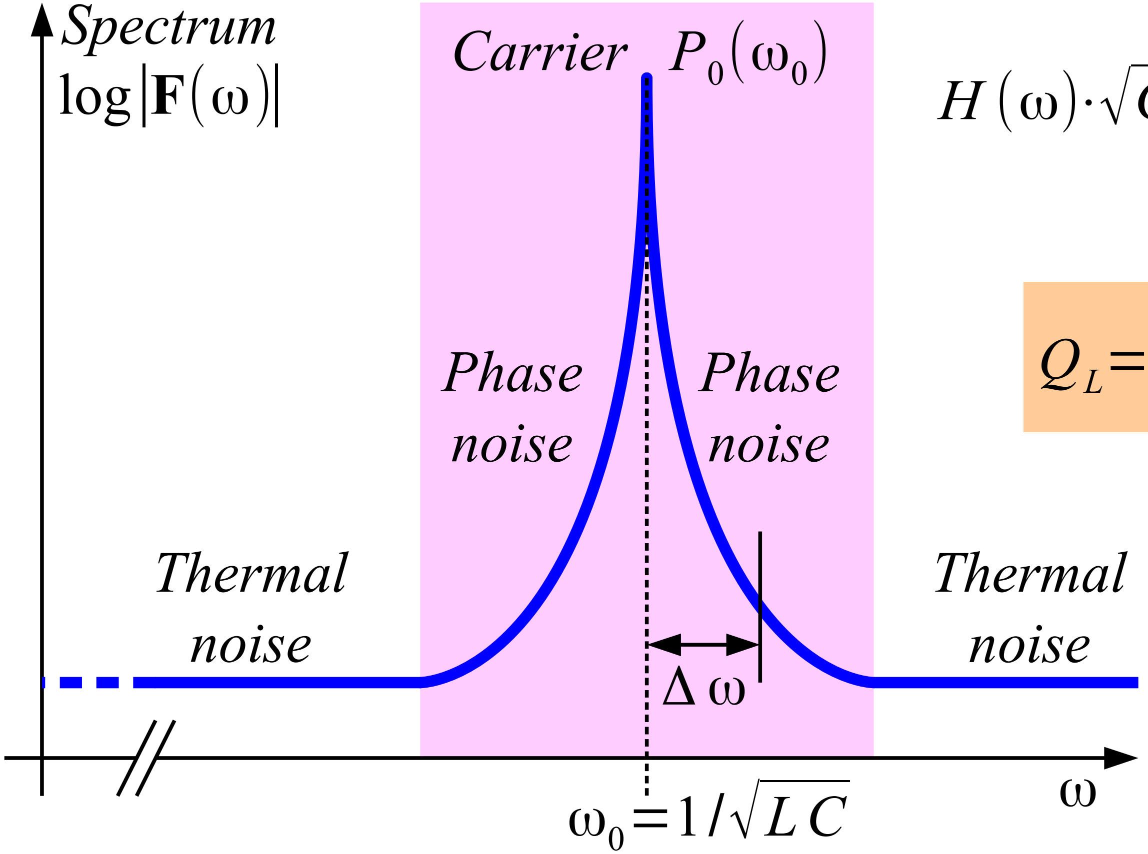
$$T_R \approx T_0 = 290\mathrm{K}$$
$$T_R + T_G \approx T_0 \cdot F$$

$$U_{Nout} = U_{Nin} + H(\omega)\cdot\sqrt{G}\cdot U_{Nout}$$

Valid at $P_{Nout} \ll P_0$

$$U_{Nout} = \frac{U_{Nin}}{1 - H(\omega)\cdot\sqrt{G}}$$

$$\Sigma R = R_{out} + R_{Cu} + R_{in}$$



$$H(\omega)\cdot\sqrt{G} = \frac{\Sigma R}{\Sigma R + j\omega L + \dfrac{1}{j\omega C}} \approx \frac{1}{1 + j2Q_L\dfrac{\Delta\omega}{\omega_0}}$$

$$Q_L = \frac{\omega_0 L}{\Sigma R}$$

$$U_{Nout} \approx U_{Nin}\cdot\left(1 + \frac{\omega_0}{j2Q_L\Delta\omega}\right)$$

$$P_{Nout} \approx P_{Nin}\cdot\left[1 + \left(\frac{\omega_0}{2Q_L\Delta\omega}\right)^2\right]$$

Amplitude and phase noise

$$P_{Nout} \approx P_{Nin}\cdot\left[1 + \left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]$$

26 – Oscillator phase noise

27 – Leeson's equation

Saturation removes amplitude noise $P_\varphi = P_{Nout}/2$

$$\frac{dP_{Nin}}{df}=N_0=k_B(T_R+T_G)\approx k_B T_0 F$$

$$L(\Delta f)=\frac{1}{P_0}\cdot\frac{dP_\varphi}{df}=\frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}\cdot\left(1+\frac{f_C}{|\Delta f|}\right)\quad[\mathrm{Hz}^{-1}]$$

Valid at $L(\Delta f)\cdot\Delta f\ll 1$

Phase noise only

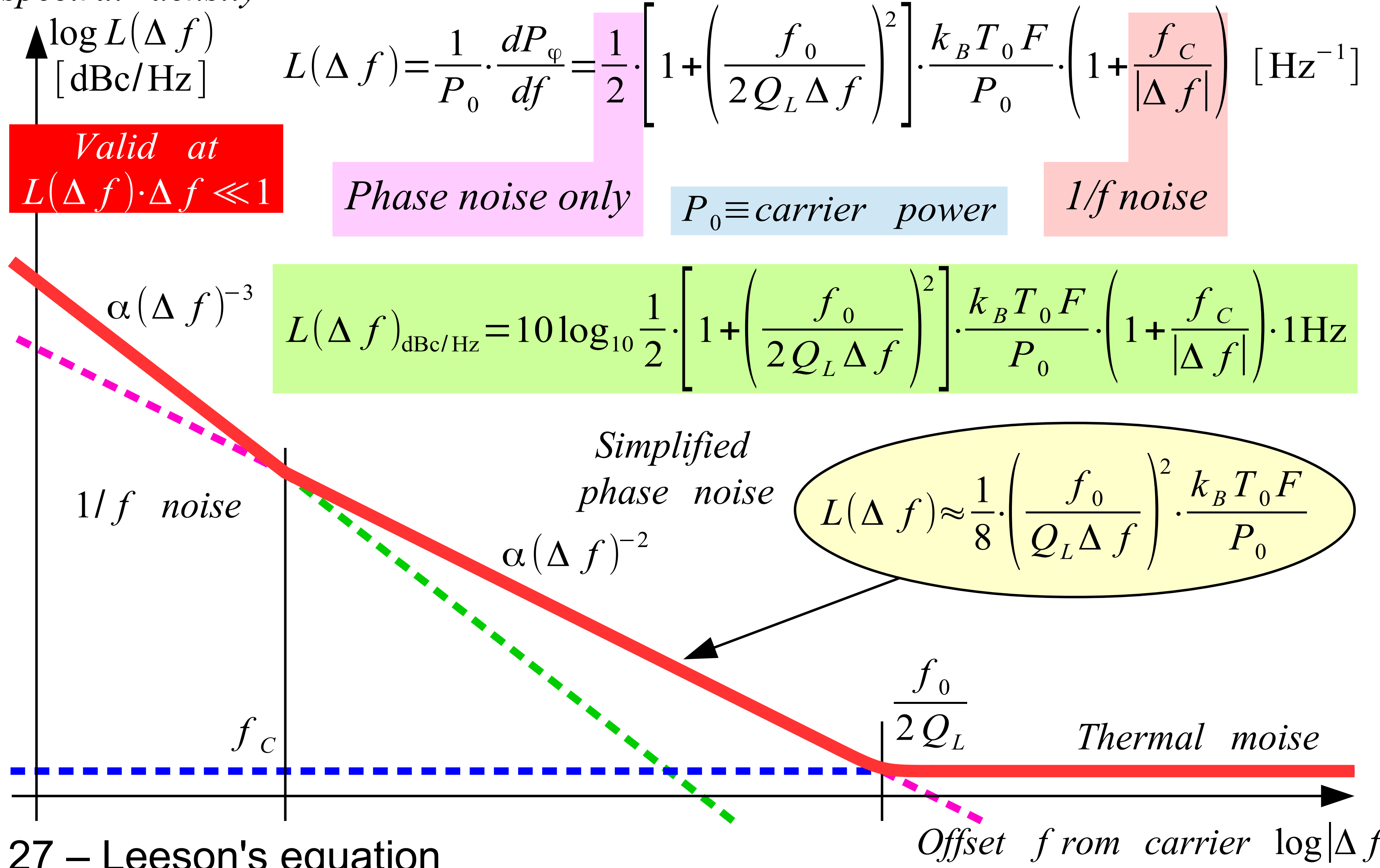
$P_0\equiv$ *carrier power*

1/f noise

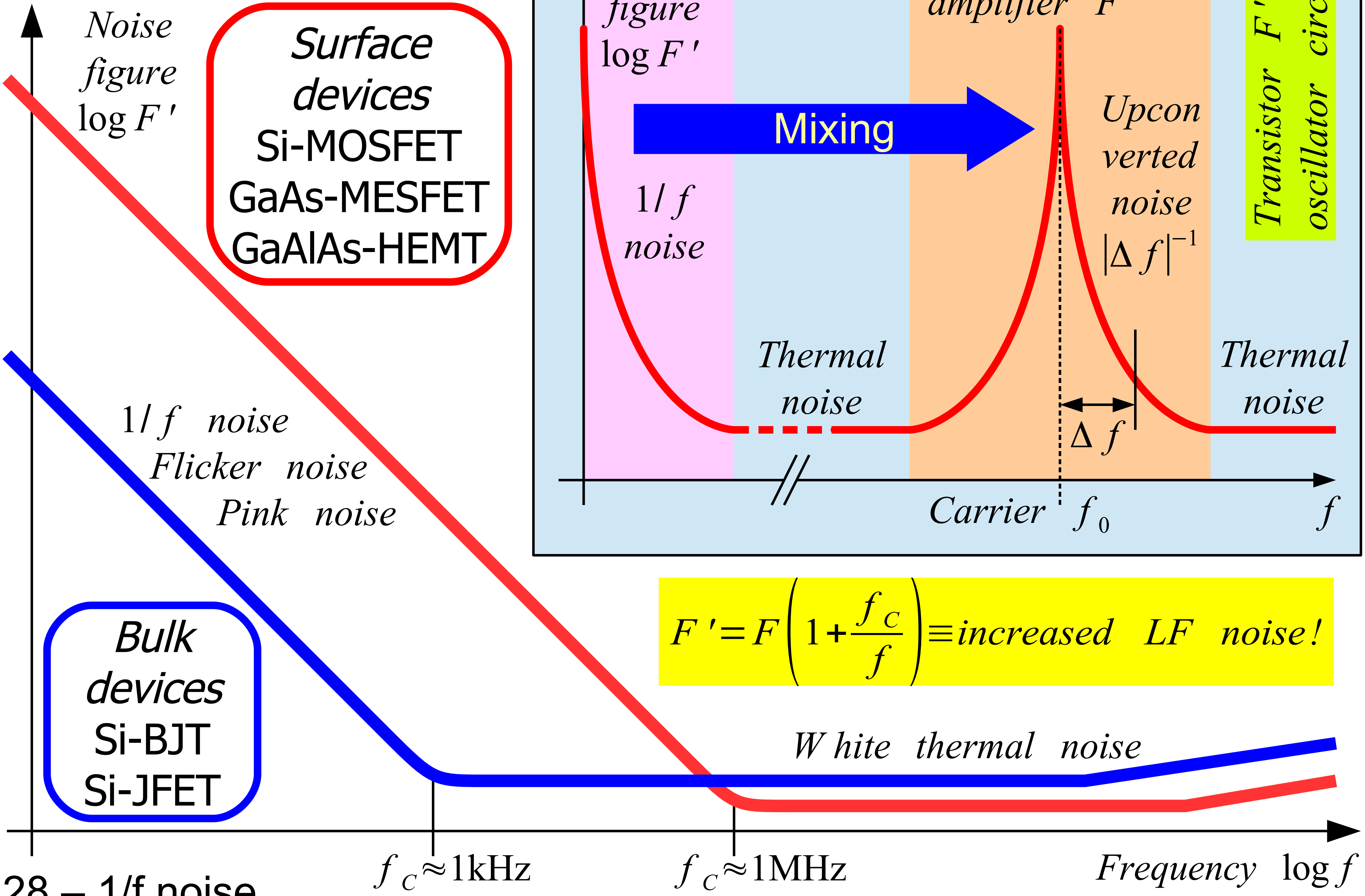
$$L(\Delta f)_{\mathrm{dBc/Hz}}=10\log_{10}\frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}\cdot\left(1+\frac{f_C}{|\Delta f|}\right)\cdot 1\mathrm{Hz}$$

Simplified phase noise

$$L(\Delta f)\approx\frac{1}{8}\cdot\left(\frac{f_0}{Q_L\Delta f}\right)^2\cdot\frac{k_B T_0 F}{P_0}$$



$1/f$ *noise usually does not have a clear explanation!*



Surface devices
Si-MOSFET
GaAs-MESFET
GaAlAs-HEMT

Bulk devices
Si-BJT
Si-JFET

$$F' = F\left(1 + \frac{f_C}{f}\right) \equiv \textit{increased LF noise!}$$

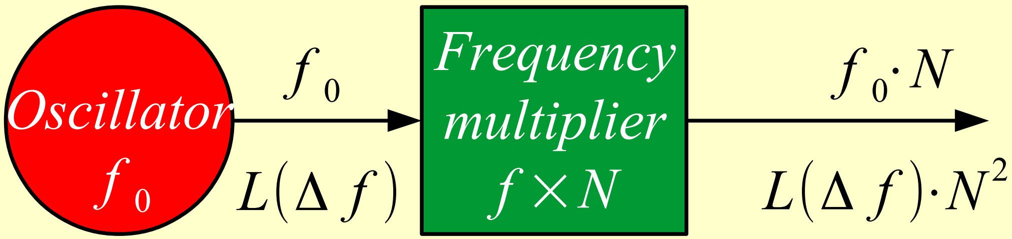
28 – 1/f noise

The loaded resonator quality Q_L defines the oscillator phase noise!

$$L(\Delta f)=\frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_BT_0F}{P_0}\cdot\left(1+\frac{f_C}{|\Delta f|}\right)$$

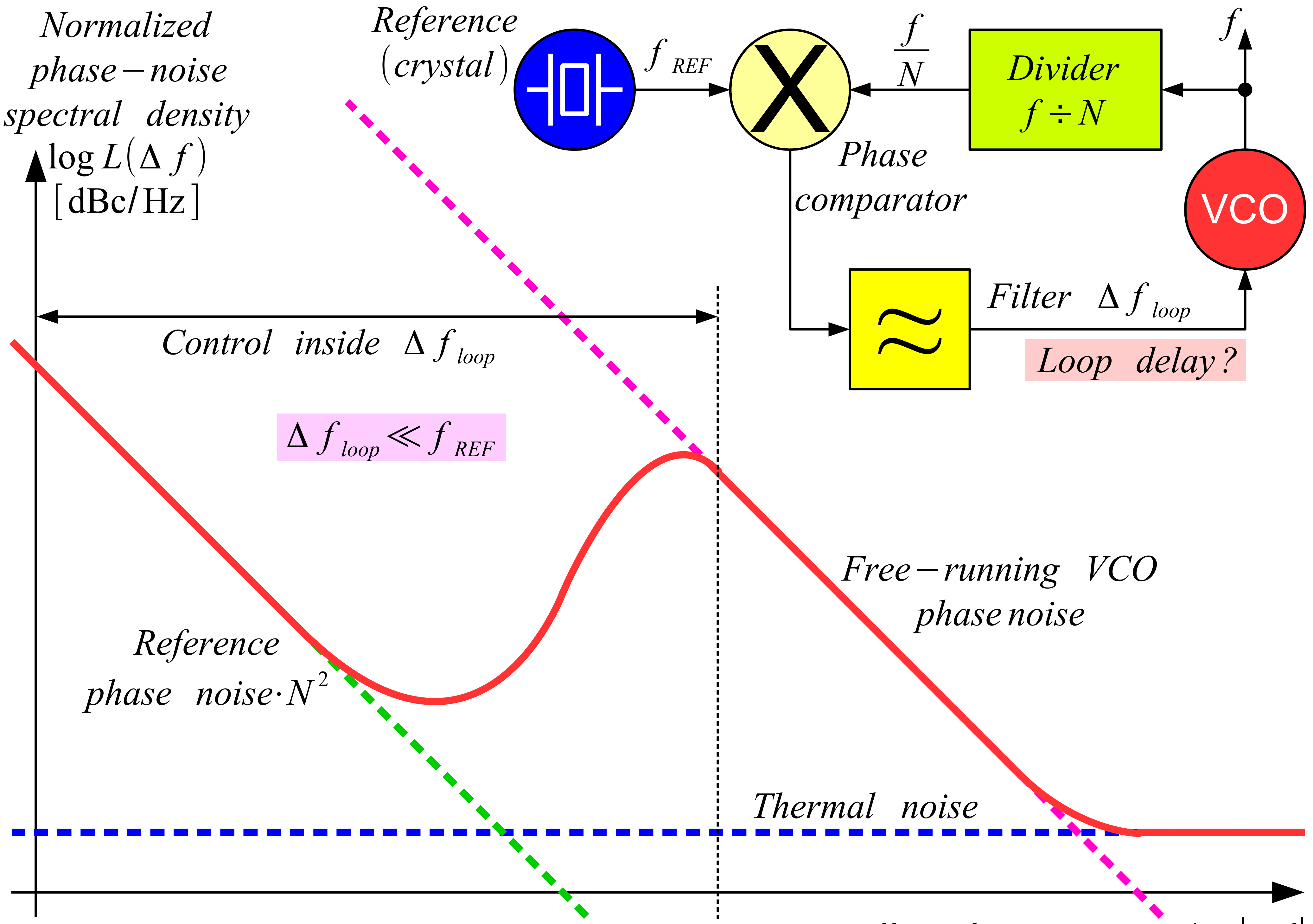
Variable-frequency oscillators	Q_L
RC VCO	~1
BWO tube	~1
Varactor-tuned LC VCO	10↔30
YIG ($Y_3Fe_5O_{12}$) oscillator	300↔1000

Fixed-frequency oscillators	Q_L
RC multivibrator	~1
LC resonator	30↔100
Cavity resonator	1000↔3000
Ceramic dielectric resonator	1000↔3000
AT-cut quartz crystal (fundamental mode)	3000↔10000
AT-cut quartz crystal (third/fifth overtone)	10000↔30000
Electro-optical delay line ($)	~10^5 (noisy!)
Sapphire dielectric resonator ($$$)	~3.10^5
Red HeNe LASER	~10^8



The phase noise multiplies with the square of the frequency multiplication factor!

29 – Resonator quality Q

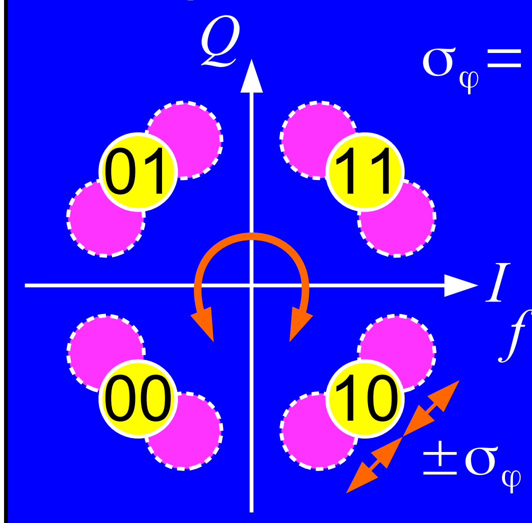


30 – Phase-Locked Loop (PLL)

31 – Effects of phase noise

Example QPSK

$$\sigma_{\varphi}=\sqrt{2\int_{f_{MIN}}^{f_{MAX}} L(\Delta f)\,d\Delta f}$$



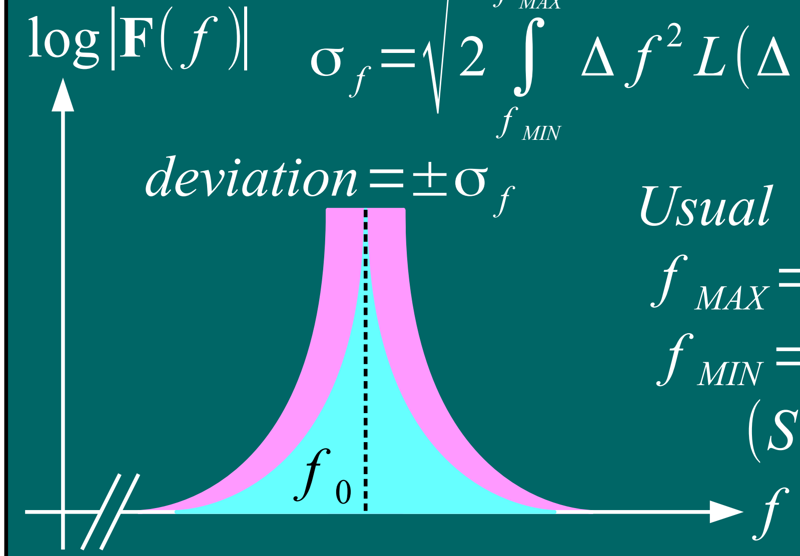
$$f_{MAX}=\Delta f_{modulation}$$

$$f_{MIN}=\Delta f_{carrier-recovery}$$

Modulation constellation rotation

Spectrum

$$\sigma_{f}=\sqrt{2\int_{f_{MIN}}^{f_{MAX}} \Delta f^{2} L(\Delta f)\,d\Delta f}$$



Usual choice

$f_{MAX}=3\text{kHz}$

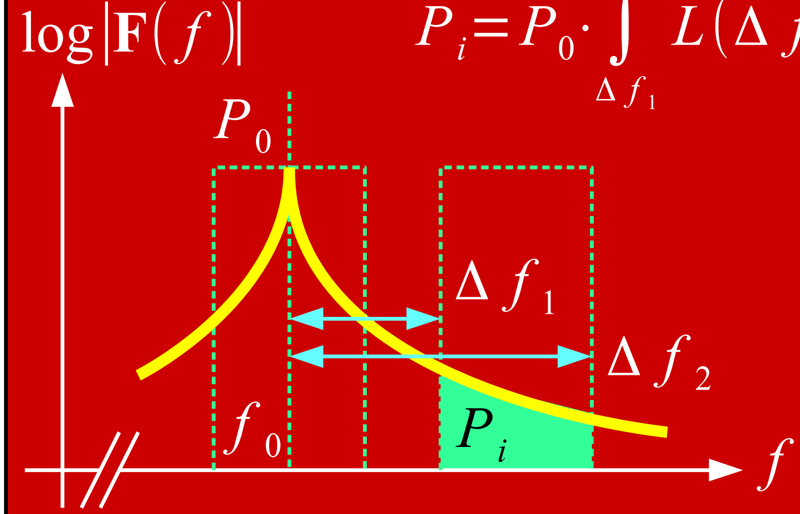
$f_{MIN}=50\text{Hz}$

$(S/N)_{speech}$

Residual FM

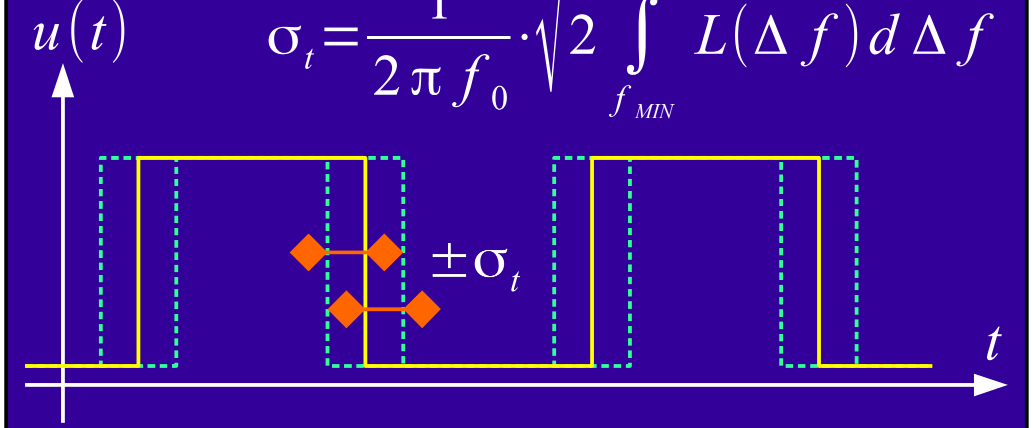
Spectrum

$$P_{i}=P_{0}\cdot\int_{\Delta f_{1}}^{\Delta f_{2}} L(\Delta f)\,d\Delta f$$



Adjacent-channel interference

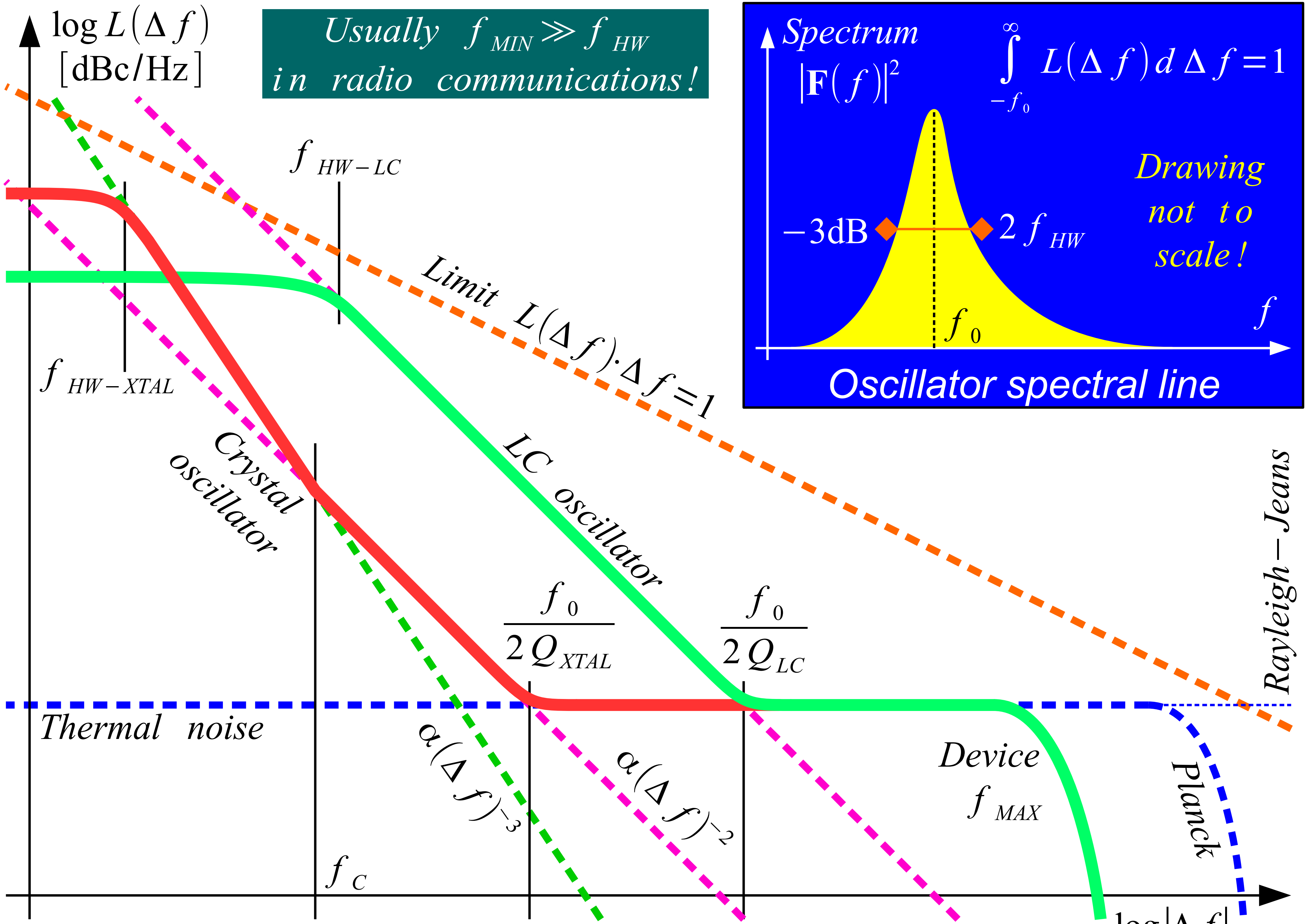
$$\sigma_{t}=\frac{1}{2\pi f_{0}}\cdot\sqrt{2\int_{f_{MIN}}^{f_{MAX}} L(\Delta f)\,d\Delta f}$$



$f_{MAX}\approx f_{clock}$

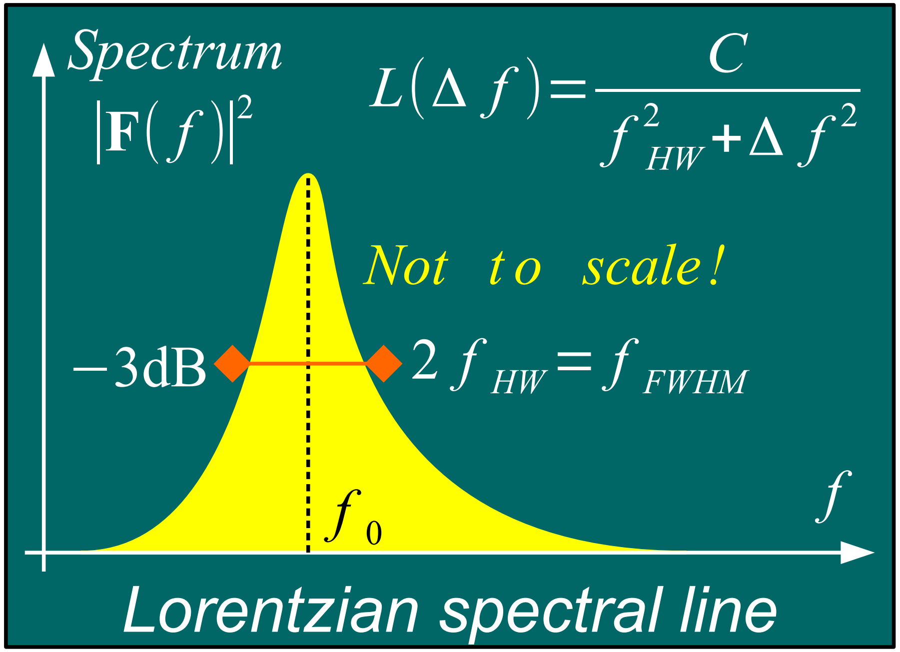
$f_{MIN}=\Delta f_{clock-recovery}$

Clock jitter



Usually $f_{MIN} \gg f_{HW}$ in radio communications!

32 – Phase noise without approximations



Lorentzian spectral line

Flat thermal noise can be neglected: device f_{MAX} or Planck law

LC−oscillator 1/f noise can be neglected

$$L(\Delta f) = \frac{1}{8}\cdot\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{1}{f_{HW}^2 + \Delta f^2}\cdot\frac{k_B T_0 F}{P_0}$$

Lorentzian line in Leeson's equation

$$\int_{-f_0}^{\infty} L(\Delta f)\, d\Delta f = 1 \approx \int_{-\infty}^{\infty} L(\Delta f)\, d\Delta f = \frac{1}{8}\cdot\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{k_B T_0 F}{P_0}\int_{-\infty}^{\infty}\frac{1}{f_{HW}^2 + \Delta f^2}\, d\Delta f =$$

$$= \frac{1}{8}\cdot\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{k_B T_0 F}{P_0}\cdot\left[\frac{1}{f_{HW}}\cdot\arctan\frac{\Delta f}{f_{HW}}\right]_{\Delta f=-\infty}^{\Delta f=\infty} = \frac{k_B T_0 F}{8 P_0}\cdot\left(\frac{f_0}{Q_L}\right)^2\cdot\frac{\pi}{f_{HW}}$$

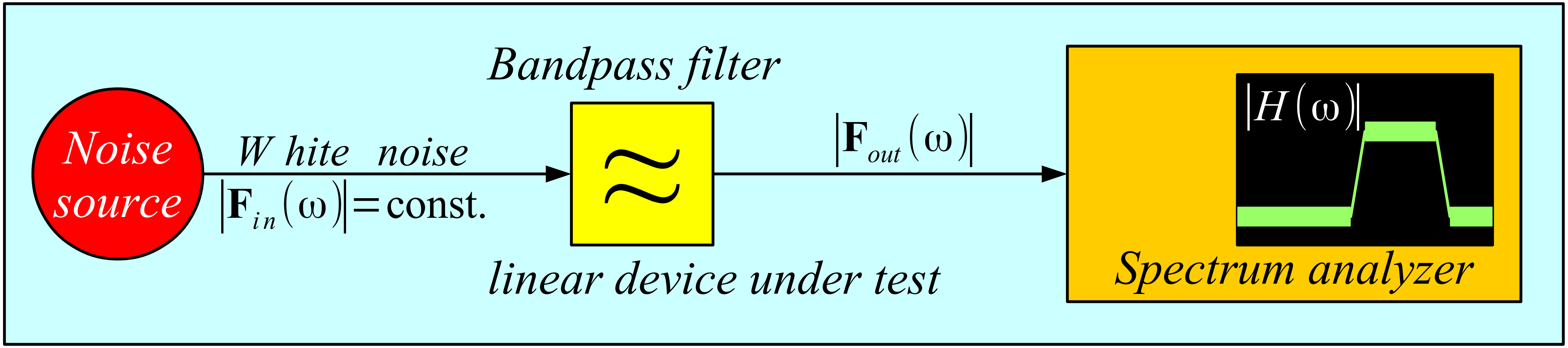
$$f_{HW} = \frac{\pi k_B T_0 F}{8 P_0}\cdot\left(\frac{f_0}{Q_L}\right)^2$$

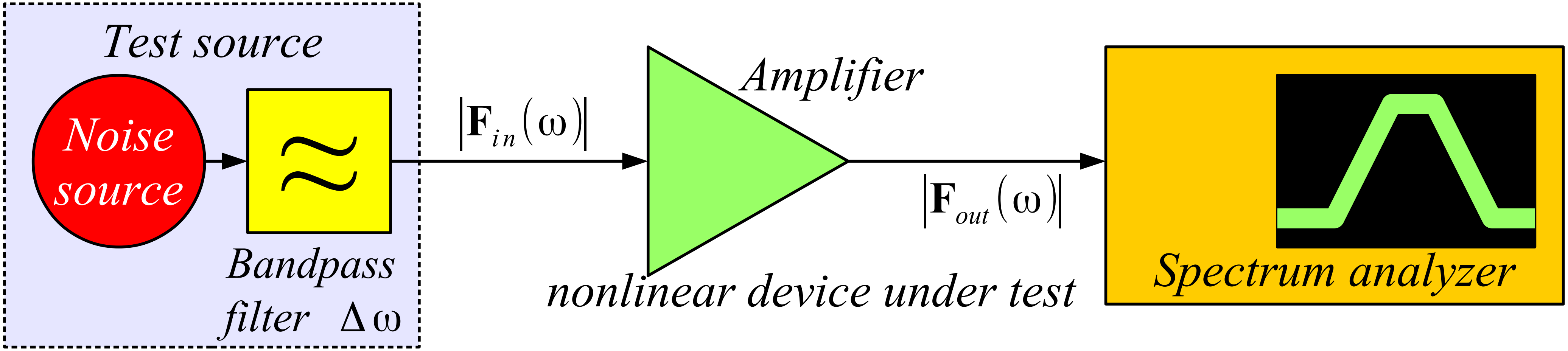
Example $f_0 = 3\text{GHz}$ $Q_L = 10$
$P_0 = 0.1\text{mW}$ $F = 10\text{dB}$
$f_{HW} = 14\text{Hz}$ $f_{FWHM} = 28\text{Hz}$

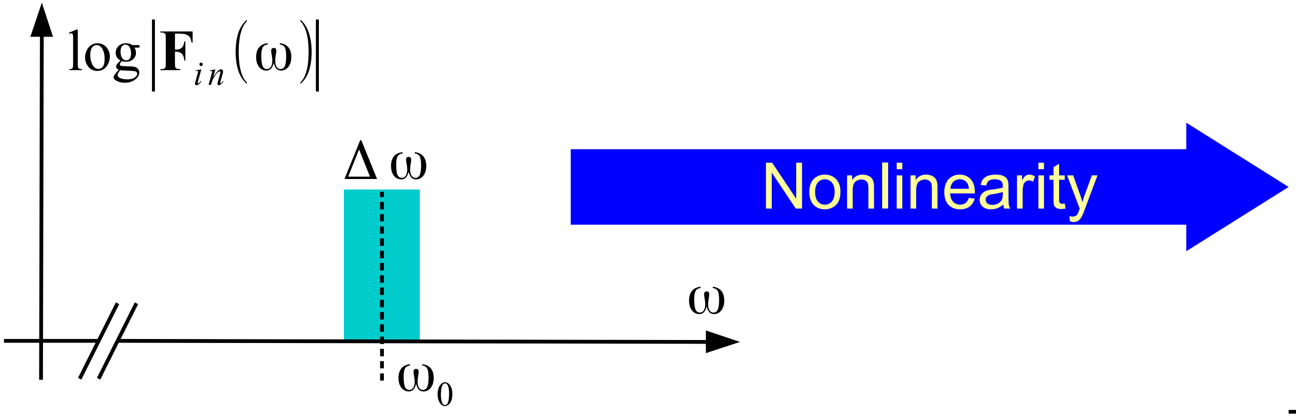
$$C = \frac{k_B T_0 F}{8 P_0}\cdot\left(\frac{f_0}{Q_L}\right)^2 = \frac{f_{HW}}{\pi}$$

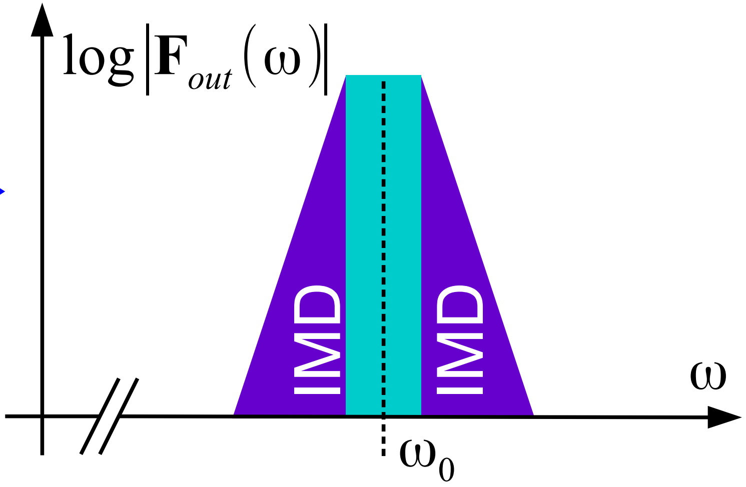
$$L(\Delta f) = \frac{f_{HW}/\pi}{f_{HW}^2 + \Delta f^2}$$

33 – Width of Lorentzian spectral line

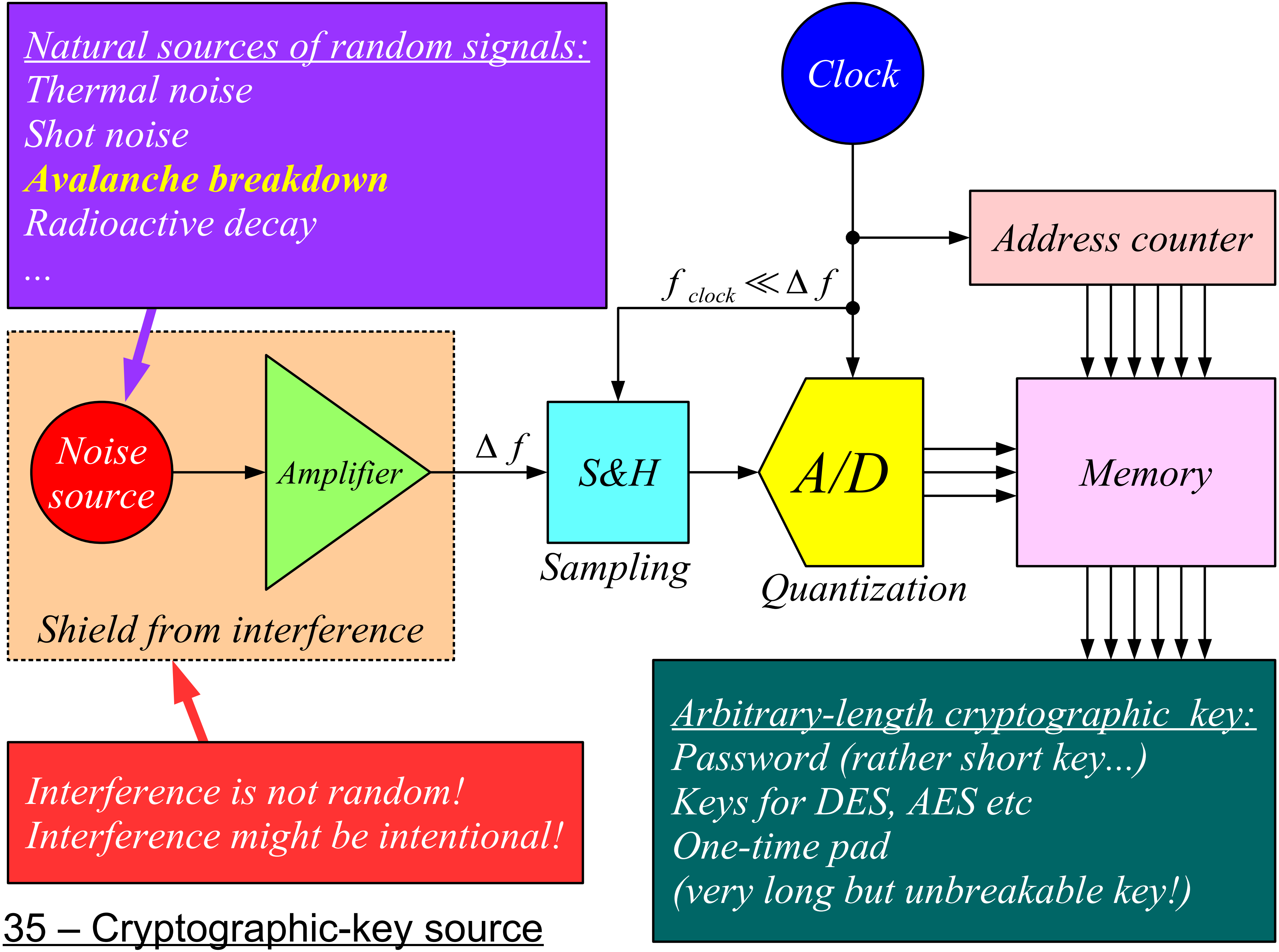








34 – Noise as test signal



Natural sources of random signals:

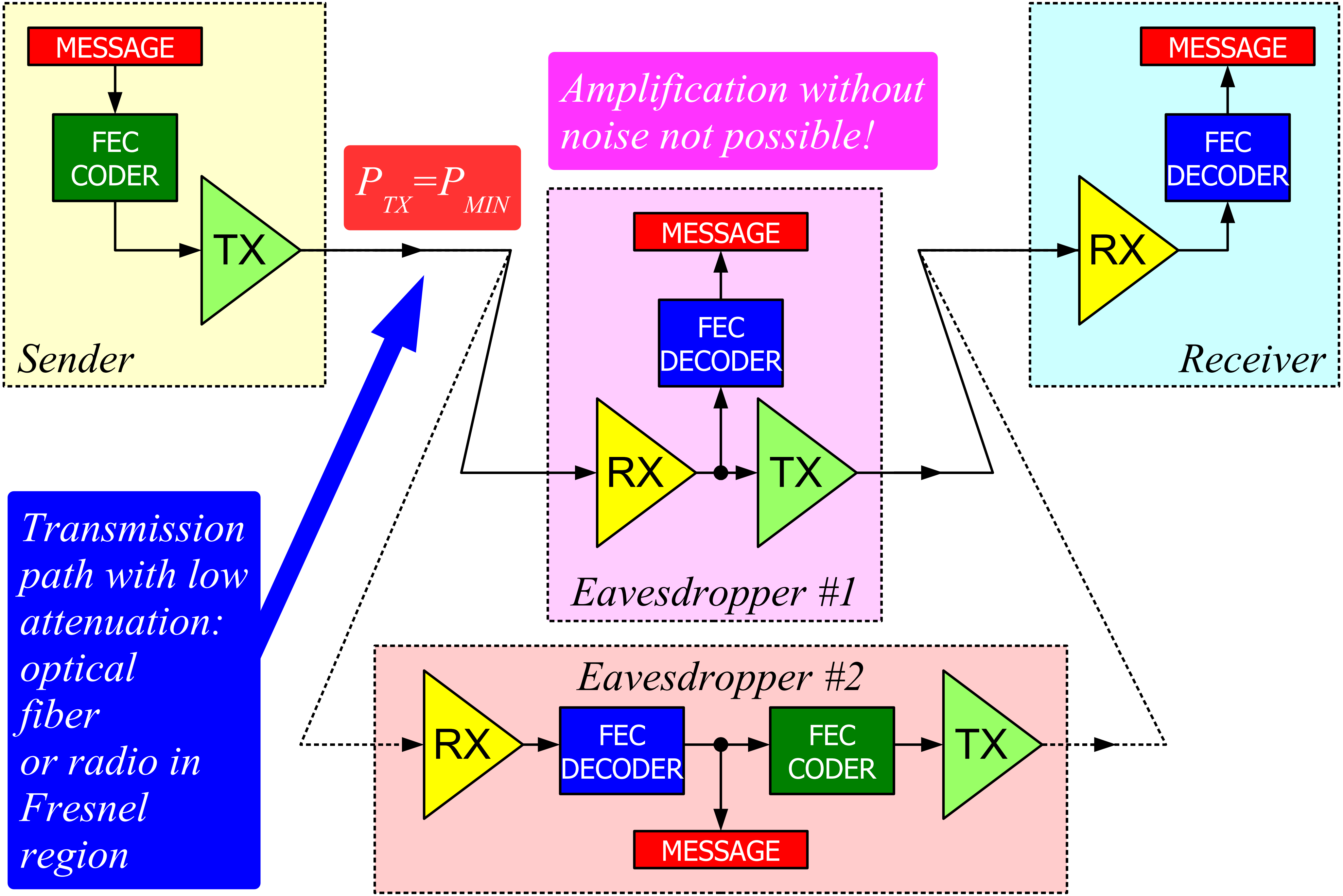
Thermal noise
Shot noise
Avalanche breakdown
Radioactive decay
…

Interference is not random!
Interference might be intentional!

Arbitrary-length cryptographic key:

Password (rather short key...)
Keys for DES, AES etc
One-time pad
(very long but unbreakable key!)

35 – Cryptographic-key source



Sender

MESSAGE

FEC CODER

TX

$P_{TX} = P_{MIN}$

Amplification without noise not possible!

Eavesdropper #1

MESSAGE

FEC DECODER

RX

TX

Receiver

MESSAGE

FEC DECODER

RX

Transmission path with low attenuation: optical fiber or radio in Fresnel region

Eavesdropper #2

RX

FEC DECODER

FEC CODER

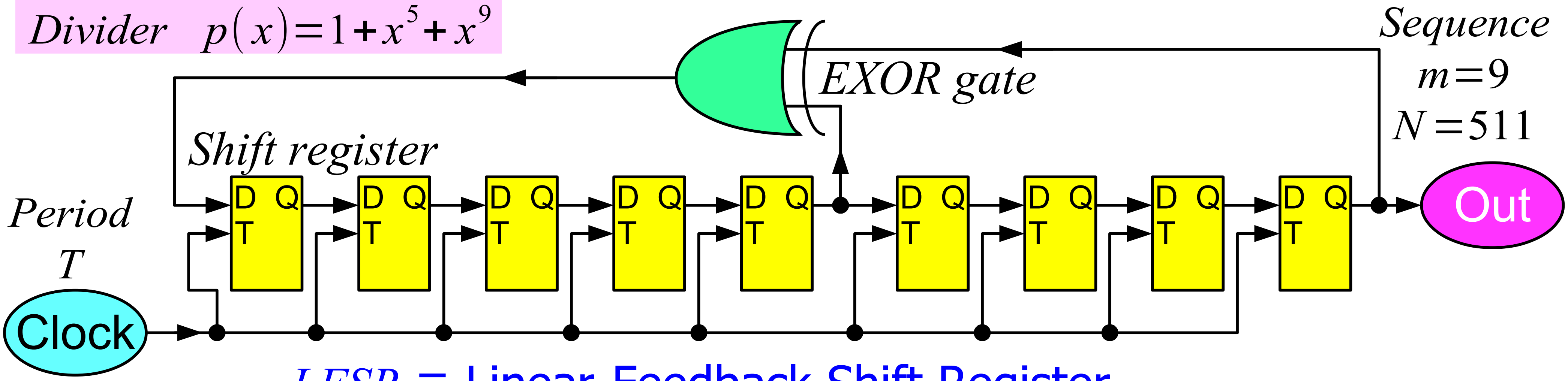
TX

MESSAGE

FEC adds delay!

36 – Noise cryptography

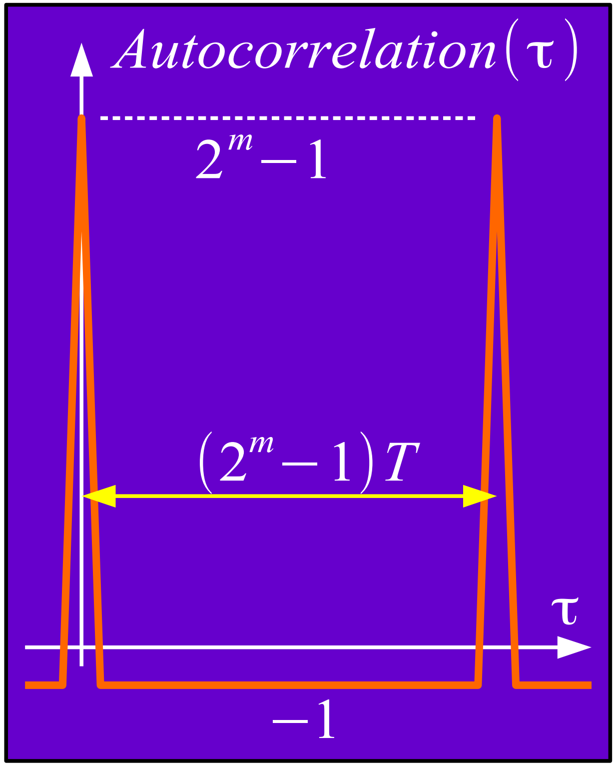
Divider $p(x)=1+x^5+x^9$

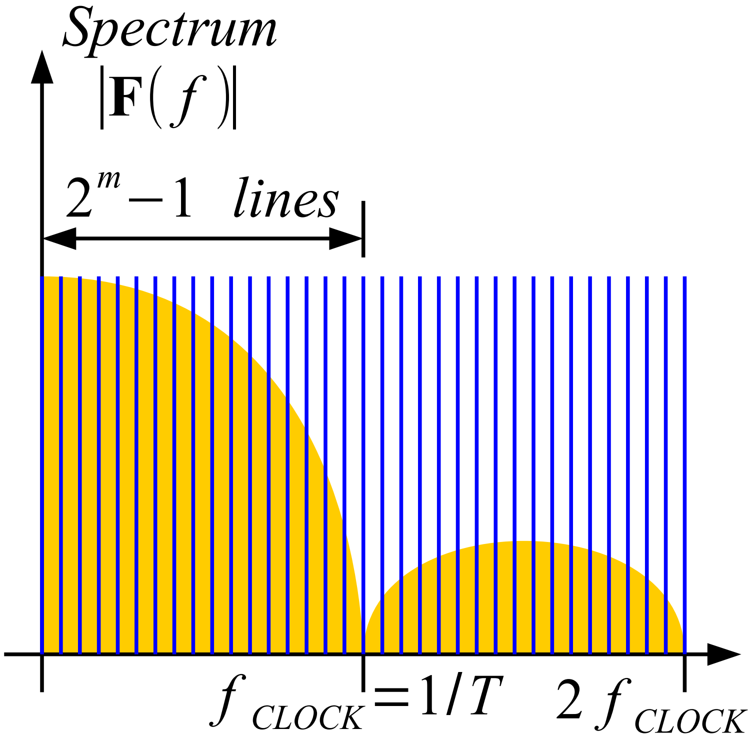


LFSR ≡ Linear-Feedback Shift Register

Primitive polynomial $p(x)=1+x^l+x^m$ → *max sequnce length* $N=2^m-1$

2^{m-1} ones and 2^{m-1}-1 zeros
arranged in groups of
1X m ones, m-1 zeros
1X m-2 ones and zeros
2X m-3 ones and zeros
4X m-4 ones and zeros
……………….
2^{m-5} groups 111 and 000
2^{m-4} groups 11 and 00
2^{m-3} individual 1 and 0





37 – LFSR pseudo-random sequences

Sound and appear as white noise!

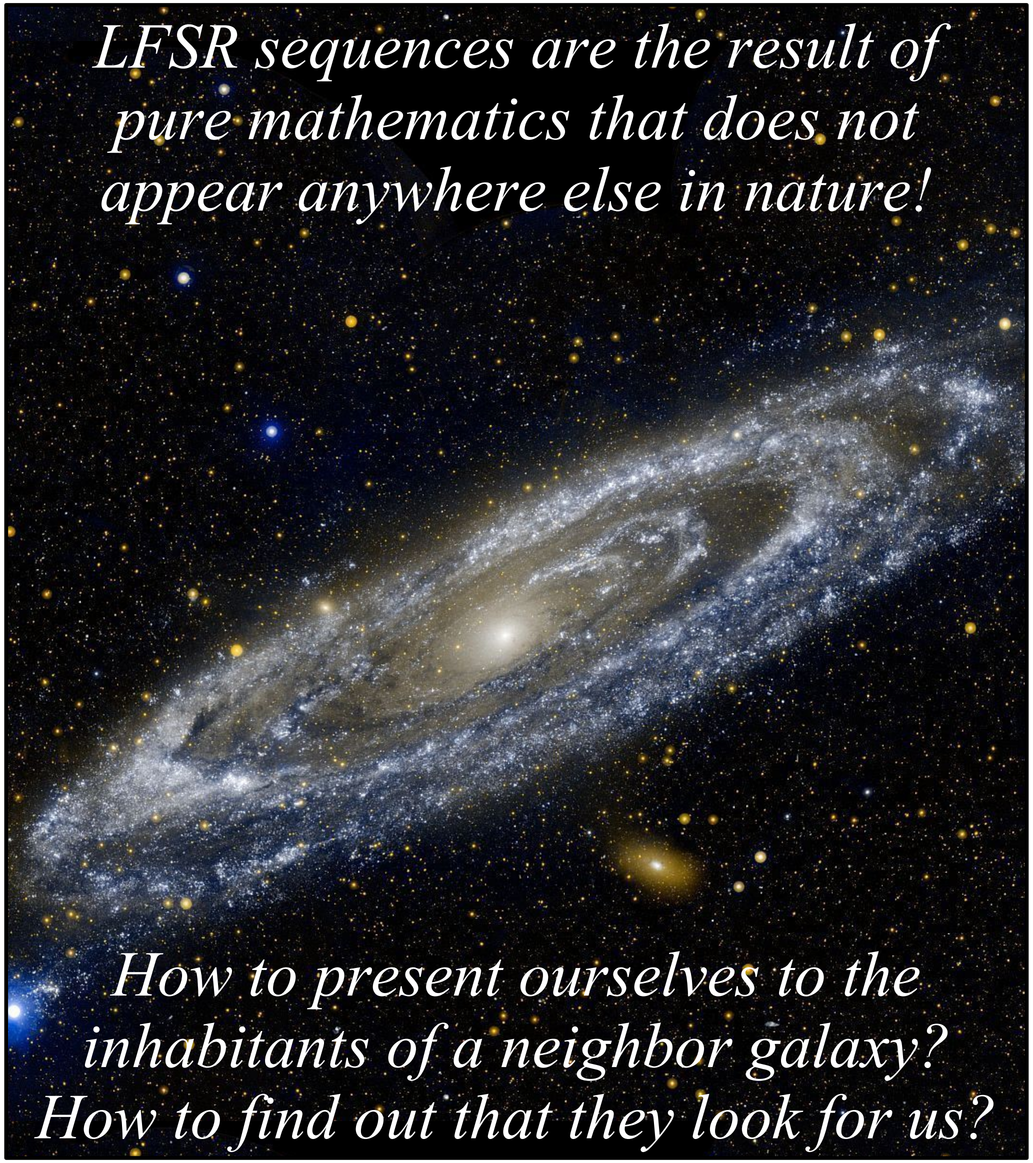
Two-valued autocorrelation with a single very pronounced peak:
- synchronization headers for data frames
- spreading sequences in CDMA
- accurate time transfer in radio navigation (GPS, GLONASS)

Perfect frequency spectrum of uniformly-spaced lines and simple generation/checking:
- test sequences for all kinds of telecommunication links
- data scrambling (randomization) as part of line coding

Peak-to-average power ratio:

$$LFSR: \frac{P_{MAX}}{\langle P \rangle} \approx 1 \qquad Noise: \frac{P_{MAX}}{\langle P \rangle} \rightarrow \infty$$

LFSR pseudo-random sequences are of NO cryptographic value: algorithm Berlekamp-Massey 1969



LFSR sequences are the result of pure mathematics that does not appear anywhere else in nature!

How to present ourselves to the inhabitants of a neighbor galaxy?
How to find out that they look for us?

38 – Use of pseudo-random sequences