Microwave and Millimeter Wave Radiometry

A microwave radiometric sensor is a device for the detection of electromagnetic energy which is noise-like in character. The spatial as well as spectral characteristics of observed energy sources determine the performance requirements imposed on the functional subsystems of the sensor. These subsystems include an antenna, receiver, and output indicator. Natural or non-man-made sources or radiation may be either spatially discrete or extended. In the frequency domain, these sources may be either broadband or of the resonant line type. Sensor design and performance characteristics are primarily determined by the extent to which spatial and frequency parameters characterize the radio noise source of interest to the observer.

A microwave radiometric sensor is frequently referred to as a temperature measuring device, since the output indicator is calibrated in degrees Kelvin. The reason why microwave radiometers are calibrated in temperature units and the modes of operation that are most frequently used are described in the sections immediately following.

A. TEMPERATURE CALIBRATION OF THE OUTPUT INDICATOR

The physical reasoning in support of calibrating the output indicator of a microwave radiometer in degrees Kelvin can be derived from thermodynamic considerations and certain well-known properties of an antenna.

The amount of energy absorbed by an antenna and presented at the input terminals of the receiver depends upon the orientation of the antenna, the polarization of the wave, and the impedance match of the receiving system. Since all antennas are polarized, regardless of design, the maximum amount of energy accepted by an antenna, from a randomly polarized wave, is one-half of the total energy content of the wave. If we assume that an antenna is perfectly matched and that the incoming wave is randomly polarized with a power flux density S, then the absorbed power P_A is given by the expression

$$P_A = \frac{1}{2}SA \qquad\qquad\qquad (A-1)$$

where A is the effective antenna aperture area.

In (A-1), the flux density S of the radiation is assumed to be from a source of small angular size and is measured by the flow of energy from the source through unit area in the wave front at the observing point. If energy dE in the frequency range dv flows through area dA in time dt (where dt is long compared to the period of one cycle of the radiation), then the flux density S is given by the expression

$$S = \frac{dE}{dAdvdt} \qquad\qquad\qquad (A-2)$$

which has the dimensions of power per unit area per unit bandwidth.

Now consider a transmission line, one end of which is terminated with a matched load and the other end of which feeds an antenna in an absorbing medium. If we were to replace the antenna by its equivalent two-terminal network and assume that it is a purely resistive impedance and equivalent to the load impedance, then a transmission line terminated in a matched antenna may be treated in a manner similar to a transmission line terminated with a resistive load, as shown in Figure A-1. If the extent of the absorbing medium is sufficient to completely absorb all radiation from the antenna, the medium and the matched termination must then be at the same temperature T.

THERMAL RESERVOIR AT TEMPERATURE T
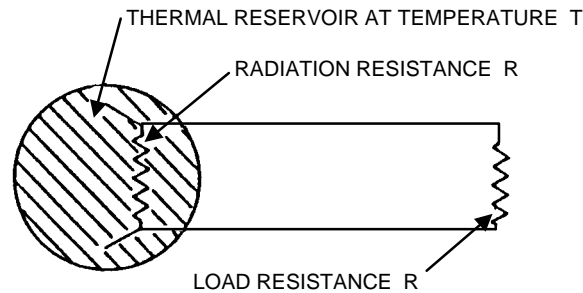
RADIATION RESISTANCE R

LOAD RESISTANCE R

Figure A-1: Equivalent circuit of an antenna immersed in an absorbing medium at temperature T. In equilibrium, the temperature of the load resistance is the same as the temperature of the absorbing medium.

From Johnson noise power considerations, the termination will radiate a power $kTdv$ to the antenna. If the antenna, in turn, did not accept $kTdv$ of radiation from the medium and transfer this power to the load, there would be a net transfer of thermal energy from one region to another at the same temperature without application of work, in violation of the laws of thermodynamics. This suggests that in the microwave and millimeter wave portion of the spectrum, the power delivered to the input of a receiving system by an antenna immersed in an absorbing medium at temperature T is independent of the frequency of observation.

This conclusion can also be reached (Figure A-2) by noting that the medium appears as a blackbody to the radiation resistance of the antenna, i.e., it absorbs all incident radiation and its radiation brightness b in the frequency interval dv in accord with Planck's law is

$$\beta dv = \frac{2hv^3}{c^2} \frac{dv}{[exp(-hv \, / \, kT) \, - \, 1]} \tag{A-3}$$

where

h = Planck's constant
k = Bolzmann constant
c = velocity of light

and the brightness β is the power per unit area per unit solid angle per unit bandwidth.
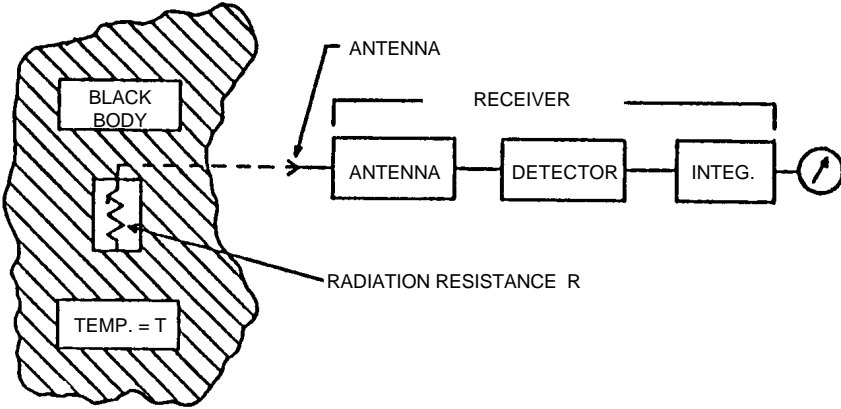
Figure A-2: Simplified block diagram of an antenna and receiver. When the antenna is immersed in a blackbody at temperature T, the receiver input is equivalent to a resistive load R immersed in a thermal bath at temperature T.

From the definition of flux density S, it is evident that

$$S = \int \beta \delta \Omega \tag{A-4}$$

where $\delta \Omega$ is the solid angle increment. This definition of flux density holds for any source of radiation over all solid angles.

Equation (A-3) indicates that the power received from a blackbody, measured at the input terminals of the receiving system, is frequency dependent. Hence, the equivalence of blackbody radiation and Johnson noise power appears inconsistent. The answer to this paradox lies in the characteristic frequency response of any antenna system and the frequency characteristic of blackbody brightness in the millimeter and microwave portions of the spectrum. This can be most easily seen by recalling that the power absorbed by an antenna, from a randomly polarized source when operating in the frequency range dv, is

$$P_A = \frac{1}{2} A(\theta, \phi) \int \beta \, dv \, d\Omega \tag{A-5}$$

where q and f describe the direction of the incoming wave and A(q, f) is the antenna aperture receiving cross section in that direction. Hence, for an extended source

$$P_A = \frac{1}{2} \iint A(\theta, \phi) \, d\Omega \beta \, dv \tag{A-6}$$

If the extended source of radiation is characterized as a blackbody (*i.e.,* **Planck Distribution**), the power received by the antenna would then be expressed in the form

$$P_A = \frac{1}{2} \iint A(\theta, \phi) \, d\Omega \frac{2hv^3}{c^3} \frac{dv}{[exp(hv \, / \, kT) \text{-} 1]} \tag{A-7}$$

In that portion of frequency spectrum where the energy of the photon hv is much less than the random thermal energy per degree of freedom kT at temperature T, the expression for blackbody brightness Eq. (A-3) reduces to the simplified expression

$$\beta dv \approx \frac{2kT}{\lambda^2} dv \quad \text{(Rayleigh-Jeans)} \tag{A-8}$$

where λ is the wavelength of observation. Hence, in the microwave portion of the spectrum Eq. (A-7) reduces to

$$P_A = \frac{kT}{\lambda^2} dv \int A(\theta, \phi) d\Omega \tag{A-9}$$

The average effective cross section, $\overline{A}$, for any antenna immersed in a source of uniform brightness may be expressed in the form

$$\overline{A} = \frac{1}{4\pi} \int A(\theta, \phi) d\Omega = \frac{\lambda^2}{4\pi} \tag{A-10}$$

hence,

$$\int A(\theta, \phi) \delta\Omega = \lambda^2 \tag{A-10}$$

Introducing equation (A-10) in equation (A-9) we arrive at the conclusion that the power received by an antenna immersed in a blackbody at temperature T is frequency independent and equivalent to the Johnson noise power $kTdv$. As a consequence, the noise power received by a microwave or millimeter radiometer is conventionally described in terms of equivalent black body temperature units.

The transition region in the frequency spectrum at which the energy of the photon is comparable to the random thermal energy per degree of freedom is, of course, temperature dependent. Approximate values are shown in Table A-1. It is apparent from Table A-I that microwave and millimeter wavelength measurements of the earth terrain and atmosphere (ambient 290K) at frequencies below 300 GHz (λ = 1 mm) fall well within the region of the spectrum where hv is less than kT.

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Table A-1

TEMPERATURES AND CORRESPONDING WAVELENGTHS

AT WHICH THE ENERGY OF THE PHOTON hv IS EQUAL TO kT

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TEMPERATURE (K)	WAVELENGTH (µm)	WAVELENGTH (mm)	FREQUENCY (GHz)
300	70	0.07	4300
77	200	0.2	1500
20	900	0.9	333
4	6,000	6.0	50
1.5	10,000	10.0	30

In summary, the power received by an antenna immersed in a blackbody at a temperature T is frequency independent and equivalent to the Johnson noise power that would be radiated by an antenna if it were terminated in a matched resistive load at the same temperature T. These two fundamental sources of noise power are equivalent at microwave frequencies due to the inverse wavelength squared dependence of blackbody brightness, which is offset by the wavelength squared dependence of the antenna cross section. Hence, the noise power per unit cycle received by an antenna and presented at its output terminals is directly proportional to the effective blackbody temperature which characterizes the source or sources in which the antenna pattern is immersed. The proportionality factor is Bolzmann's constant. k.

Since most natural sources are not blackbodies, their "signal temperature," measured by a radiometric sensor, refers to the power level that would be received from a blackbody at a temperature which would provide an equivalent power level at the output terminals of the antenna.

This temperature concept is useful in describing the functions of the antenna and receiver in a microwave radiometric sensor. The antenna extracts noise power from the radiation incident on its aperture and presents that noise power at its output terminals from the foregoing analysis, the received noise power can be described in terms of an effective blackbody temperature.

If we assume that the effective blackbody temperature of the composite noise power presented at the output terminals of the antenna is T_A and that portion associated with useful signal power is T_s,

then

$$T_A = T_s + \Sigma T_i$$

where ΣT_i represents a summation of effective noise temperatures from the undesired sources of noise power observed by the antenna. The signal-to-noise ratio at the output terminals of the antenna is then $T_s/\Sigma T_i$. the prime function of the receiver is to amplify and detect the input signal which is characterized by the composite temperature T_A. All processes of receiver amplification add noise to the received signal. This added noise is frequently referred to as the internal receiver noise which can be described by an effective temperature T_R referred to the input terminals of the receiver. The ratio of antenna temperature to receiver noise temperature, at the interface between the antenna and receiver, is then $(T_s + \Sigma T_i)/T_R$. Note that the unwanted noise power ΣT_i received by the antenna and presented at the input terminals of the receiver, cannot be differentiated from the desired signal temperature T_s through amplification alone. Spatial differentiation between T_s and ΣT_i are spatially extended. Similarly, the separation of T_s from ΣT_i may be obtained in the frequency domain by the receiver, if either the source of signal temperature or background noise temperature exhibit markedly different frequency characteristics, such as a resonant line superimposed on a broadband continuum. In this case, the receiver can be scanned in the frequency domain to separate the signal temperature from the temperatures contributed by broadband background sources.

The determination of the equivalent noise temperature of a receiving system is related to the method of *noise figure* measurement. The *noise figure* of a receiving system or network is defined as the signal-to-noise ratio at the input, divided by the signal-to-noise ratio at the output, when the receiver or network is terminated in a matched load at a temperature T_o of 290K. From the definition of *noise figure*, F, the *noise figure* of the network shown in Figure A-3 is given by the expression

$$F = \frac{GkT_0 dv + GN}{GkT_0 dv} \tag{A-11}$$

or

$$F = 1 + \frac{N}{kT_0 dv}$$ (A-12)

If we now define the system noise temperature T_R by the expression

$$N = kT_R dv$$ (A-13)

we see that the relationship between system noise temperature and noise figure is

$$T_R = (F - 1)T_0$$ (A-14)

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                         ┌─────────────────────────┐
              ┌──────────┤     NETWORK             ├──────────┐
              │          │  GAIN         - G       │          │
        T₀  ╲R           │  BANDWIDTH    - dv      │         (meter)
              │          │  INTERNAL NOISE - N     │          │
              └──────────┤                         ├──────────┘
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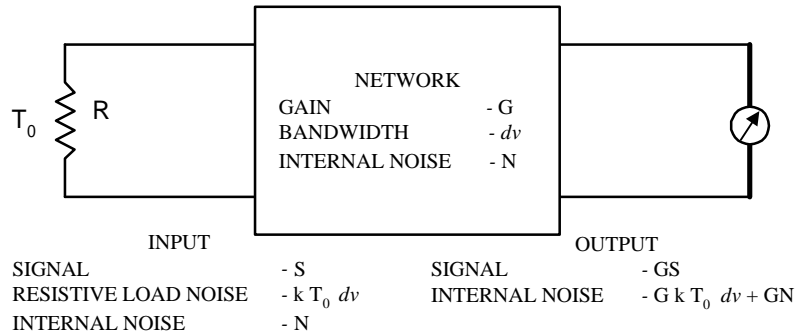| INPUT		OUTPUT	
SIGNAL	- S	SIGNAL	- GS
RESISTIVE LOAD NOISE	- k T_0 dv	INTERNAL NOISE	- G k T_0 dv + GN
INTERNAL NOISE	- N		

Figure A-3: Input and output signal and noise relationships of a four-terminal network with the input terminated in a resistive load which is immersed in a thermal bath at temperature **T**.

B. RECEIVER FUNCTIONS AND TECHNIQUES

The prime function of the receiver in a microwave radiometric sensor is to provide a measure of the input noise power, expressed as an ***antenna temperature*** in equivalent black body temperature units.

As previously noted, the ***antenna temperature*** T_A is, by definition, the temperature to which the radiation resistance of the antenna must be raised in order to produce the same noise power as that contributed by the various sources observed by the antenna. It is also the brightness temperature of a blackbody which, if it completely surrounded the antenna, would provide the same noise power at the receiver input. To describe the method by which the receiver measures the antenna temperature, we will replace the antenna with an equivalent resistive load at the receiver input. If the antenna temperature were T_A, we would obtain the same input noise power to the receiver, by placing the resistive load in a thermal bath at a temperature T_A.

The need for signal amplification becomes readily apparent when one notes that the average noise power per unit bandwidth produced by a resistor at an ambient temperature (290K) is of the order of 10^{-20} watt. Typical detectors require a drive power of about -15dBm. The required input signal amplification must, of course, be increased if temperature changes less than 290K are to be detected and recorded. The receiver must therefore be able to sense a low level change in noise power at its input and provide sufficient stable amplification to drive the output indicator system. Amplification stability is a prime requisite since the receiver must provide a consistent output response for the same input power change. The relatively poor gain stability of present receiving systems is overcome by the use of an input switch or modulator, to be discussed later.

1. *Sensitivity*

The noise power output of a resistive termination is associated with the thermal agitation of electrons within the resistive conductor which produce electronic collisions. As the thermal temperature of the resistor is increased, the thermal agitation increases; and the number of collisions per unit time increases. The resultant noise power output per unit cycle is directly proportional to the absolute temperature of the resistor. As indicated previously, the proportionality factor is Boltzmann's constant k. In this sense, a radio measurement of the thermal temperature of the input resistor may be described as a measurement of the electron collision frequency within the resistor. Since the collisions are random, the number per second will vary; however, the mean of an infinite number of one-second samples will lead to an exact value for the collision frequency. From statistical theory, the probable error in the measurement of a quantity of this type is inversely proportional to the square root of the number of measurements which are made. If the number is infinite, the exact value is determined. If we now measure the electronic collisions within a resistor, using an amplifier of finite bandwidth $\Delta\nu$, the number of independent collisions per second which can be counted is equivalent to the receiver bandwidth. Hence, the error in determining the mean value of the noise temperature (which is proportional to the collision frequency) will be inversely proportional to the square root of the receiver bandwidth. If the averaging process is extended over τ seconds rather than one second, there will be, on the average, $\tau\Delta\nu$ independent collisions in each interval of seconds, therefore

$$\frac{\Delta T_R}{T_R} \approx \frac{1}{\sqrt{\tau\Delta\nu}} \tag{B-1}$$

In most radiometric applications, the magnitude of the signal temperature is negligible when compared with the "receiver noise temperature T_R" which describes the noise power added to the received signal by the various circuits within the receiver.

Simplified functional block diagrams of the most commonly used microwave and millimeter radiometric receiving systems are shown in Figure B-1. The figure depicts the genealogical growth of each receiver from the one preceding. The crystal video receiver is usually the first to be used in a new portion of the frequency spectrum where input signal frequency components required for the other modes of operation are not available. The superheterodyne is the "workhorse" among receivers. The input circuit of the superheterodyne is a "mixer" in which the signal frequency is heterodyned with the local oscillator frequency. The difference or intermediate frequency between the signal and local oscillator is amplified by a tuned intermediate frequency amplifier, referred to as the IF amplifier. The addition of a low noise amplifier forward of the mixer in a superheterodyne mode will establish the receiving system noise temperature by providing adequate gain to overcome the conversion loss of the mixer.
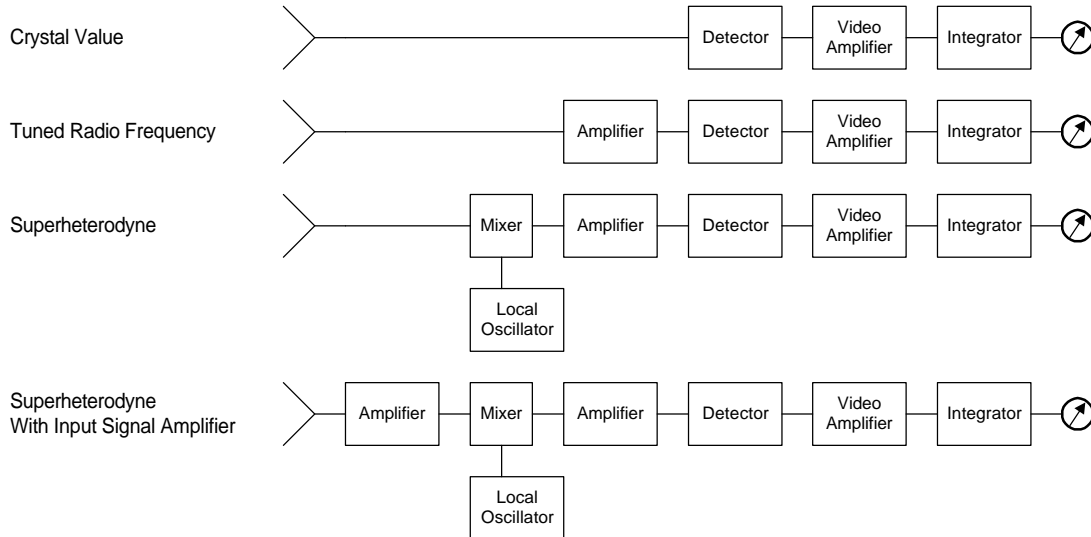
Figure B-1: Simplified block diagrams of commonly used radio receivers.

The sensitivity of a radiometric system, *i.e.*, the minimum detectable signal, is determined by the amplitude of the fluctuations present at the output indicator in the absence of a signal. These fluctuations are attributable to two sources:

(1) The statistical fluctuations in a noise waveform as described by Equation (B-1).
(2) Spurious gain fluctuations associated with the receiving network.

The amplitude of output fluctuations due to the first source can, in principle, be reduced to any desired degree by reducing the post-detection bandwidth (increasing the integration time). In practice, however, the longest usable integration time is limited by the time available for observation of the "signal."

2. Gain Variations and the Dicke Mode

A second source of fluctuations which occur at the receiver output are attributable to receiver gain instabilities. Their significance can be readily grasped by the following example. If we introduce values of $T_R = 1000K$, $\Delta\nu = 2 \times 10^9$ Hz, and $\tau = 1$ sec in Equation (B-1), we obtain an rms value for the amplitude of statistical noise fluctuations at the receiver output of the order of 0.03K. This would be the case, if the receiver were absolutely gain stable. Unfortunately, the best receivers, regardless of type or frequency of operation, exhibit gain instabilities of the order of 1% during a time period comparable to that required for most signal measurements. A receiver with the performance characteristics described above would provide an output fluctuation of 10K, if the gain changed by 1%. As a result, the noise measurement sensitivity of the receiver would be determined by the effect of gain variations rather than by the level of statistical noise fluctuations.

The answer to this dilemma was provided by Dicke[*] in the form of a single pole, double throw switch placed at the input of the receiver, as shown in Figure B-2.

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[*] Dicke, R.H. The measurement of thermal radiation at microwave frequencies. *Rev. Scl. Instr.* 17, 268-275 (1946).
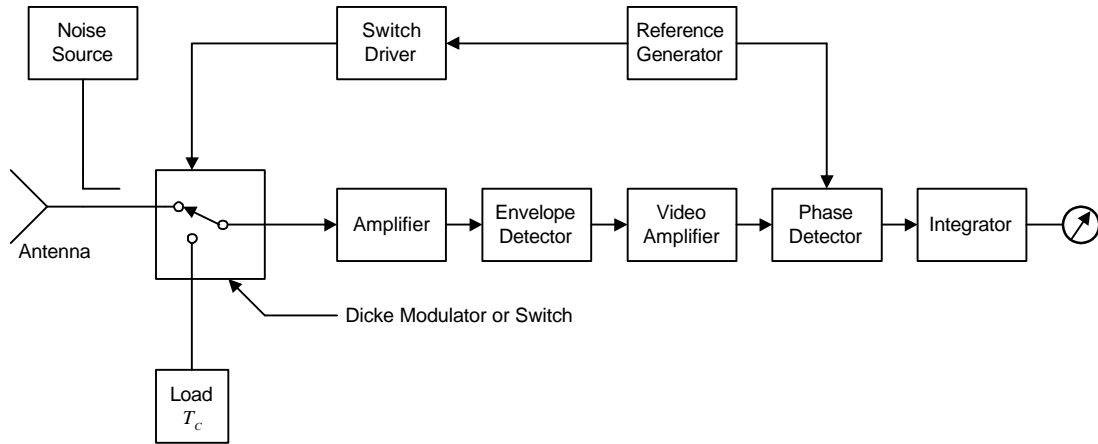
Figure B-2: Simplified block diagram of the Dicke radiometer. A switch or modulator is introduced between the antenna output and receiver input.

One of the input ports of the switch is connected to the antenna output terminal, the other to a resistive load held at a constant temperature T_c. The switch is driven sequentially in a square wave fashion at a frequency considerably higher (typically 30 to 1000 Hz) than that at which a substantial receiver gain variation occurs. With the switch in operation, a signal at the switching or modulation frequency is presented at the input terminals of the receiver with an amplitude proportional to the temperature difference T_A - T_c. Because of the rapid switching rate, any receiver gain variation will operate equally on T_A + T_R during one-half of the switching cycle and on T_c + T_R during the other half, with the result that it operates only on the difference T_A - T_c. If for example, the difference T_A - T_c were 1K, the effect of a 1% receiver gain variation referred to the output indicator system would be 0.01K.

In the example given above, the introduction of the switch provided a marked improvement in the noise measurement capability of a typical receiver by eliminating the effect of receiver gain variations operating on the receiver noise temperature. The gain variations, however, continue to operate on the ***temperature difference*** presented at the two input ports of the switch. This was not an important consideration in early radiometers which had relatively high noise temperatures and narrow bandwidths, leading to sensitivities of the order of a few degrees Kelvin. Present-day broadband radiometric receiving systems, however, have potential sensitivities of the order of 0.05K rms for post-detection time constants of 1 sec or less at frequencies up to 600 GHz. Consequently, the effect of receiver gain variations, operating on an RF input unbalance (large temperature difference between input signal and comparison ports), is of great concern. Several techniques for reducing the RF input temperature unbalance are in common use. These include addition of noise to the signal port of the radiometer, use of a low temperature comparison source, introduction of gain modulation, etc.

Calibration of the output indicator requires the introduction of a precisely known temperature change at the input signal port of the radiometer. This measurement is usually performed under carefully controlled laboratory conditions. This temperature change is frequently generated by the sequential introduction of two very precisely known noise temperature sources at the signal input port of the radiometer. Calibration of two internal noise sources in equivalent temperature units is automatically obtained as a by-product of this laboratory calibration procedure. System gain can be established at any time during a measurement program by introducing a constant and fixed level of noise temperature difference at the radiometer input. The noise level of the gain calibration noise source need not be known precisely. It is far more important that it remain constant and that it be used to establish the level of receiver gain during laboratory calibration of the radiometer response in equivalent temperature units.

This noise source is usually included as an integral part of a radiometer and is referred to as the "calibration or internal noise source."

From the foregoing, it is apparent that one internal fixed calibration noise source provides all of the information required for the precise calibration of the output indicator reading in equivalent temperature units. The internal calibration noise source level should be approximately two orders of magnitude greater than the amplitude of the peak-to-peak fluctuation level at the output indicator for the nominal value of post-detection integration time constant which will be used during the measurement program. This allows opportunity to establish the full-scale output indicator deflection level to an accuracy of at least 1%.

C. MICROWAVE AND MILLIMETER WAVE RADIOMETER APPLICATIONS

The number and diversity of microwave radiometer applications today is truly amazing when one recalls that the Dicke technique was just 50 years old in 1996. Exploited first by the young science of radio astronomy, the power of this instrument has demonstrated its ability to explore the unknown and provide many historic discoveries. Only a small fraction of the knowledge gained was anticipated. It was at first frustrating to learn that the brightness temperature of the sun, measured at low frequencies, was more than 1,000,000K rather than the anticipated 6000K. The radio noise from our own galaxy is markedly different from the anticipated blackbody radiation. Spatially discrete sources of intense radio energy are present in space and can not be identified with optically observed sources. Today, the unanticipated in galactic radio astronomy is considered routine. We are at the dawn of a new era in astronomy, accumulating new knowledge on which we will build a new and deeper understanding in the years ahead.

Spawned by the pioneering and explosive enthusiasm of the young science of radio astronomy, the improvement in microwave radiometric sensor capabilities has been equally startling. The temperature measurement sensitivity achieved today at a wavelength of 1 mm matches the best available 30 years ago at a wavelength of 1 cm. In three decades, the measurement capability at 1 cm has been improved by more than two orders of magnitude. The instrument of 30 years ago filled several consoles and consumed nearly 100 watts of power. Its counterpart today is the size of a matchbox and requires less than one watt of input power.

Paralleling the need for improved sensor capability has been the need for larger antennas of improved surface tolerance to provide greater angular resolution at short wavelengths. With the combined improvements in angular resolution and temperature measurement capability, the latest telescopes are now able to probe the familiar neighbors of our own solar system, mapping the surface of the sun and the moon and measuring the thermal radiation characteristics of the planets.

The pattern of historical development radio telescope application has been the reverse of what one might have anticipated. From the early observation of galactic radio noise, followed by the observation of our neighbors in the solar system, the radio telescope in recent decades has been pointed with greater interest at the planet earth. The first step in this direction was to obtain an improved understanding of the physical processes in our earth's atmosphere. The microwave radiometer has made possible a global picture of water vapor and vertical temperature profiles.