SLOT AERIALS AND THEIR RELATION TO COMPLEMENTARY WIRE AERIALS (BABINET'S PRINCIPLE)*

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(1) SUMMARY

A half-wave slot in a metal sheet may be used as a resonant aerial in a manner similar to a half-wave dipole. The polar diagrams are the same, but the directions of vibration of the electric and magnetic fields are interchanged. If the slot is driven by means of a transmission line connected between opposite edges at the centre, the input impedance at resonance is about 485 ohms. The relation between resonant dipoles and resonant slots is an example of what in optics is known as Babinet's principle. This principle is established in the form required for electromagnetism and quantitative relations are deduced from it.

Arrays of slots may be fed by transmission lines or wave guides to form linear or broadside aerials. Resonant gratings of slots may be used to polarize incident waves or act as band-pass filters. Single resonant slots may be used in wave guides in the same way as parallel-tuned circuits are shunted across transmission lines. All devices adopted in connection with wire aerials have complements for slot aerials. Slots and dipoles may be combined to produce aerials with special polarization properties. Slots (with dielectric plugs) may be used in the skin of aircraft to provide dragless aerials.

In addition, Babinet's principle may be used to reduce new problems to old ones whose solution is already known. These include problems connected with discontinuities in transmission lines, wave guides with corrugated sides, magnetron cavities, and the leakage of electromagnetic radiation through holes in metal walls.

There is a wide variety of problems, both practical and theoretical for which it is always wise to consider the possibility of applying Babinet's principle.

(2) INTRODUCTION

During the war there has been a great increase in the use of centimetre wavelengths, largely in connection with radar, and this development has been accompanied by a considerable increase in the extent to which it is possible to apply the methods of optics to design and analysis of radio equipment. An interesting aspect of this tendency is the use that has been made in radio of what is known in optics as Babinet's principle. This principle is one which permits almost any piece of equipment constructed of wires to be translated into a complementary equipment consisting of slots in metal sheets, there being a close relation between the behaviour of the two equipments. As an example consider a reflecting curtain of parasitic dipoles of the type frequently used in beam aerials at decimetre wavelengths. This is a flat array of parallel half-wave dipoles arranged in a half-wave lattice [see Fig. 1(*a*)]. Regarding this as part of a plane screen, let us now interchange metal and air, thereby obtaining a metal screen in which is cut an array of half-wave slots. Such a screen is illustrated in Fig. 1(*b*): the reason for rotating Fig. 1(*b*) through a right angle in relation to Fig. 1(*a*) will be indicated in the next paragraph. Babinet's principle suggests that there should be an intimate relation between these two complementary metal screens. In fact the characteristic property of the array of dipoles is that it is practically a perfect reflector for waves incident upon it at the resonant frequency of the array and with the electric field vibrating parallel to the dipoles, the array being otherwise largely transparent. The characteristic property of the array of half-wave slots in the metal sheet is that it is almost perfectly transparent for waves incident upon it at the resonant frequency of the array and with the magnetic field vibrating parallel to the slots, the array being otherwise largely reflecting. Not merely are statements such as the above true to a high degree of accuracy, but valuable quantitative relations exist, for example, between the driving-point impedance of a centre-fed half-wave slot and that of a centre-fed half-wave dipole that would just fit into the slot. It is the purpose of this paper to describe some of these relationships and to indicate ways in which they have been or might be of practical value. The paper is based on material originally issued by the Telecommunications Research Establishment in secret form in December, 1941.

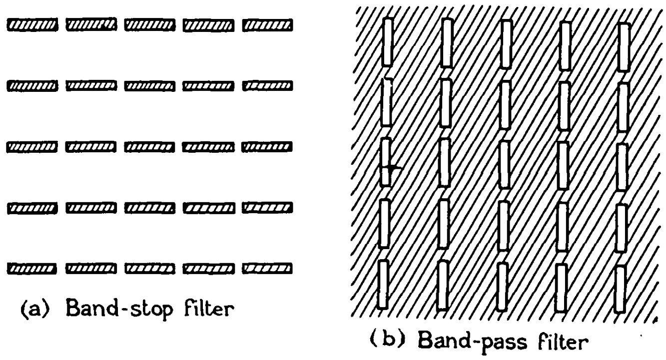

Fig. 1.—Resonant gratings.

Shading denotes metal.
Electric field vibrates horizontally.

Babinet's principle as used and proved in optics is actually too crude an approximation to be of much practical value in radio engineering, and a vital though simple extension of the principle is required for electromagnetic applications. That the optical version of the principle is inadequate for radio engineering may be seen from the fact that no mention is normally made of the polarization of the exciting electromagnetic field in optical applications. But it is obvious that in radio applications involving wires and slots, voltage must be developed *along* wires but *across* slots. Thus for perfect reflection at the resonant frequency by the array of dipoles in Fig. 1(*a*), the electric field of the incident wave must be parallel to the dipoles, whereas for perfect transmission by the array of slots in Fig. 1(*b*), it is the magnetic field that must be parallel to the slots so that voltage is developed across the slots. These vital considerations of polarization do not seem to have arisen in optics. Possibly conducting screens whose geometrical structure is comparable with or less than the wavelength have never had to be considered in optics. But these cases, involving as they do vital resonance conditions, are the ones of major interest in radio.

The version of Babinet's principle required for electromagnetic

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applications is given in Section 3. Applications to the impedance relations for complementary metal screens are given in Section 4. The particularly important case of a half-wave resonant slot and its relation to the complementary half-wave dipole is discussed in Section 5. Finally, in Section 6 some indication is given of the way in which slots have been or might be used in practice.

(3) BABINET'S PRINCIPLE

Babinet's principle as used in optics may be described as follows. Consider a thin plane screen which is black in the sense that it is non-reflecting. Suppose that the screen is pierced by holes of any size, shape or distribution, and that in front of the plane of the screen, there is a source (or distribution of sources). Let the screen obtained by interchanging holes and obstructions be called the complementary screen. Then Babinet's principle states that the disturbance produced at any point behind the plane of the screen for each of two complementary screens adds up to give the disturbance that would be produced with no screen. Obviously this is true on the basis of the simple ray theory since the shadows formed by complementary screens are complementary. It is also true, however, when diffraction is taken into account. This is often proved approximately in books on optics by a somewhat crude application of Huygens' principle. However, Babinet's principle is far more rigorous than this particular proof would suggest. In fact it would be completely rigorous but for the fact that a black screen is not quite a unique concept.[1] In electromagnetism, however, we use metal screens at which the boundary conditions are unique and well known. Consequently Babinet's principle in electromagnetism is absolutely rigid provided only that the screen is:

(i) plane,
(ii) perfectly conducting,
(iii) indefinitely thin.

There is, however, an additional point of vital importance involved in Babinet's principle for electromagnetism. To satisfy the boundary conditions at the surface of the screen we have to introduce image-sources which look out of image-space into object-space through the face of the screen. When adding up the fields produced behind the two complementary screens to obtain the field that would be produced with no screen, the image-sources associated with the complementary screens must cancel each other out. Consequently, for Babinet's principle to hold, images in complementary screens must vibrate in anti-phase. This means that, if one screen is a perfect conductor of electricity, its complement must be what we may describe theoretically as a perfect conductor of magnetism. A perfect conductor of magnetism is, of course, non-existent in nature. But we may overcome this difficulty as follows. After passing to the complementary screen, interchange electric and magnetic quantities everywhere. The complementary screen thus reverts to a perfect conductor of electricity, while the directions of vibration of the electric and magnetic vectors in the surrounding electromagnetic field are everywhere interchanged. This means that, if the source associated with the original metal screen is an elementary electric dipole (or small condenser), that associated with the complementary metal screen must be an elementary magnetic dipole (or small coil). Two sources related in this way may be called conjugate. Babinet's principle for electromagnetism may now be stated as follows:—

Consider two complementary metal screens satisfying conditions (i), (ii) and (iii) above, and at corresponding points in front of each screen let there be conjugate sources (or systems of sources). For the first screen and source let u_1 be the ratio of the field produced behind the plane of the screen to that which would be produced with no screen. Let u_2 be the corresponding ratio for the complementary screen and conjugate source. Then Babinet's principle states that

$$u_1 + u_2 = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Babinet's principle may also be stated in terms of the waves reflected by the screens as follows. For the first screen and source let v_1 be the ratio of the reflected field produced in front of the plane of the screen to the reflected field that would be produced with both screens present, that is, with a complete infinite metal screen. Let v_2 be the corresponding ratio for the complementary screen and conjugate source. Then

$$v_1 + v_2 = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In what follows it will be assumed unless otherwise stated that, when we pass from a given metallic screen to the complementary metallic screen, we at the same time pass from the given sources to the conjugate sources in the sense described above.

As an application of Babinet's principle consider the complementary screens illustrated in Fig. 1. Fig. 1(*a*) represents a curtain of identical regularly-spaced wires such as is often used behind a broadside array of aerials. Suppose that the curtain extends indefinitely in all directions, and that an infinite plane wave is normally incident upon it with its electric field vibrating parallel to the wires. Then there is a frequency, corresponding to a wavelength approximately twice the length of a wire, at which the curtain completely reflects the incident wave. If the dipoles are in the form of flat strips all in the same plane we may regard the curtain as a screen to which we may apply Babinet's principle. Take the complementary screen, and rotate it through a right angle as shown in Fig. 1(*b*), so as to secure the effect of interchanging the directions of vibration of the electric and magnetic fields in the incident wave. Then Babinet's principle shows that, at the frequency at which the curtain of Fig. 1(*a*) is a perfect reflector, the grating of Fig. 1(*b*) is perfectly transparent. Moreover, by considering frequencies displaced from the resonant frequency it follows that the selectivity, or Q, of the complementary gratings of Fig. 1 is the same.*

(4) IMPEDANCE RELATIONS FOR COMPLEMENTARY METAL SCREENS

The reflecting and transmitting properties of gratings of the type indicated in Fig. 1 may conveniently be expressed in terms of what is known as their equivalent surface-impedances or admittances, and there is a simple relationship between the equivalent surface-impedances of two complementary gratings.

It is perhaps necessary to explain what is meant by the equivalent surface-impedance or admittance of gratings such as those illustrated in Fig. 1. To do this it is convenient initially to think of a uniform, infinite, plane, thin screen made of material whose surface conductance is G: a square of such material would have a total conductance G between opposite edges. Let this screen be set up in a homogeneous medium whose intrinsic admittance† is Y. Let a plane wave whose electric field is $\mathscr{E}_1$ be incident normally on one side of the screen, and let $\mathscr{E}'_1$ and $\mathscr{E}_2$ be the electric field in the waves reflected and transmitted normally to the screen. Then the relation between $\mathscr{E}_1$, $\mathscr{E}'_1$ and $\mathscr{E}_2$ is

$$\mathscr{E}/(2Y + G) = \mathscr{E}'_1/(-G) = \mathscr{E}_2/(2Y) \quad . \quad . \quad . \quad (3)$$

These are the same formulae as are obtained when a conductance G is shunted across a transmission line of characteristic admittance Y.

Now suppose the screen is not simply resistive but is a grating such as indicated in Fig. 1, so that it possesses stored electromagnetic energy in addition to, or instead of, resistance. Then

* It should be noted however that a thin strip of breadth b (small compared with the wavelength) behaves essentially like a circular wire of diameter $b/2$ from the viewpoint of selectivity.

† Ratio of magnetic field (amp/m) to electric field (volts/m) for a simple plane wave in the medium.

the relation between the incident, reflected and transmitted waves is still given by (3) but G is now complex or even purely imaginary. It is this complex G that is known as the equivalent surface-admittance of the grating.

Let Y_1 and Y_2 be the equivalent surface-admittances for two complementary gratings such as those shown in Fig. 1 and let Y be the intrinsic admittance of the surrounding medium. For free space

$$Y = 1/Z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where

$$Z = 377\,\Omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

approximately. We have for the two gratings

$$\mathscr{E}_1/(2Y + Y_1) = \mathscr{E}'_1/(-Y_1) = \mathscr{E}_2/(2Y) \quad . \quad . \quad . \quad (6)$$

$$\mathscr{E}_1/(2Y + Y_2) = \mathscr{E}'_1/(-Y_2) = \mathscr{E}_2/(2Y) \quad . \quad . \quad . \quad (7)$$

The same formulae are obtained when an admittance Y_1 or Y_2 is shunted across a transmission line of characteristic admittance Y. From equation (1) the sum of the transmission-coefficients given by equations (6) and (7) is unity. This gives

$$Y_1 Y_2 = 4Y^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

or, in terms of the corresponding impedances

$$Z_1 Z_2 = \tfrac{1}{4}Z^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The same result could have been deduced from equation (2) using the reflection coefficients given by equations (6) and (7). We see from equation (9) that the geometric mean of the equivalent surface-impedances of two complementary gratings is equal to half the intrinsic impedance of the surrounding medium. For free space, in accordance with equation (5) the right-hand side of equation (9) is

$$\tfrac{1}{4}Z^2 = 35\,477\ (\text{ohms})^2 \quad . \quad . \quad . \quad . \quad (10)$$

The curtain indicated in Fig. 1(a) is a simple form of band-stop filter for simple infinite plane waves and corresponds to a series resonant circuit shunted across a transmission line ($Y_1 = \infty$ at resonance). The grating indicated in Fig. 1(b) is a simple form of band-pass filter for simple infinite plane waves and corresponds to a parallel resonant circuit shunted across a transmission line ($Y_2 = 0$ at resonance). The slots in Fig. 1(b) function like resonant slots of the type discussed in the next section.

Another example of complementary gratings is shown in Fig. 2. Fig. 2(a) represents a plane grating of equidistant parallel

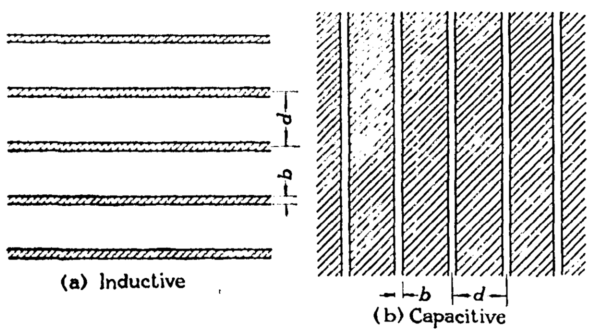

Fig. 2.—Inductive and capacitive gratings.
Shading denotes metal.
Electric field vibrates horizontally.

wires in the form of flat strips of breadth b with their centre-lines distant d apart. Suppose for simplicity that b is small compared with d, which itself is small compared with the wavelength λ. For a plane wave incident normally on this grating with its electric field vibrating parallel to the wires the equivalent surface-impedance is

$$Z_1 = jZ(d/\lambda)\log_e[(2d)/(\pi b)] \quad . \quad . \quad . \quad . \quad (11)$$

This is purely inductive provided that the surrounding medium is loss-free, Z being given by equation (5) in the case of free space. The inductance is of course associated with stored magnetic energy in the vicinity of the wires. The complementary grating is indicated in Fig. 2(b) and has been rotated through a right angle so as to secure the effect of interchanging the directions of vibration of the electric and magnetic fields in the incident wave. The equivalent surface-impedance Z_2 of this grating for a plane wave incident normally with its electric field vibrating across the slots is deduced from (11) by (9). Expressed in terms of admittance, the result is

$$Y_2 = 4jY(d/\lambda)\log_e[(2d)/(\pi b)] \quad . \quad . \quad . \quad . \quad (12)$$

Thus the grating in Fig. 2(b) is purely capacitive, the capacitance being that existing across the slots. That an inductive grating of the type indicated in Fig. 2(b) can be combined with a capacitive grating of the type indicated in Fig. 2(b) to give a resonant grating of the type indicated in Fig. 1(b) is not, of course, in the least surprising: allowance has to be made, however, for interaction between the two gratings.

(5) THEORY OF RESONANT SLOTS

Consider a straight narrow slot, about a half wavelength long, cut in an otherwise infinite, plane, perfectly conducting, thin sheet of metal. Suppose that it is excited by means of a generator connected between opposite edges at its centre as shown in Fig. 3. The metal comprising the two opposite edges of the slot may be regarded as forming a transmission line which is short-circuited except for the half wavelength constituting the slot. The slot radiates considerably more than a corresponding twin-wire transmission line because of the extent to which current associated with the slot spreads over the metal. During one half-cycle, current flows round the ends of the slot as indicated in Fig. 3(a),

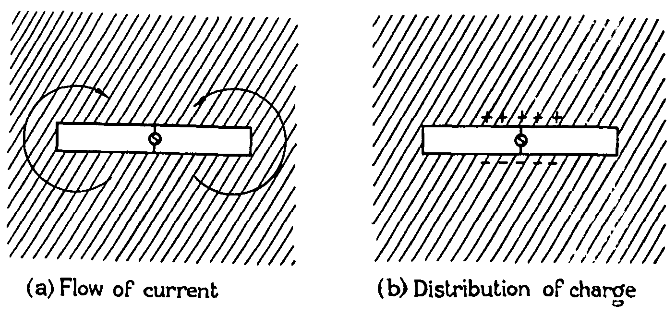

Fig. 3.—Mechanism of half-wave slot.

and charge accumulates on the opposite edges of the slot as indicated in Fig. 3(b): during the next half-cycle the process reverses.

Consider the dipole complementary to the slot shown in Fig. 3. It consists of a narrow rectangular strip of metal fed at the centre with a generator as shown in Fig. 4. Fig. 4(a) shows a view of the dipole at right angles to the plane of the strip, while

(a) Viewed normal to strip (b) Viewed edgewise

Fig. 4.—Dipole complementary to slot of Fig. 3.

Fig. 4(*b*) shows a view in the plane of the strip at right angles to its length. XYZY¹X is a contour round which we shall need to integrate. We may pass from the field associated with this dipole to that associated with the complementary slot by the following procedure, illustrated in Fig. 5.

(i) Interchange electric and magnetic quantities everywhere. We now have a dipole made of a rectangular strip of what we may describe theoretically as a perfect conductor of magnetism. It is excited by a magnetic generator at its centre. This we may regard as taking the form of a loop of wire containing an anti-symmetrical pair of electric generators as indicated in Fig. 5(*a*)

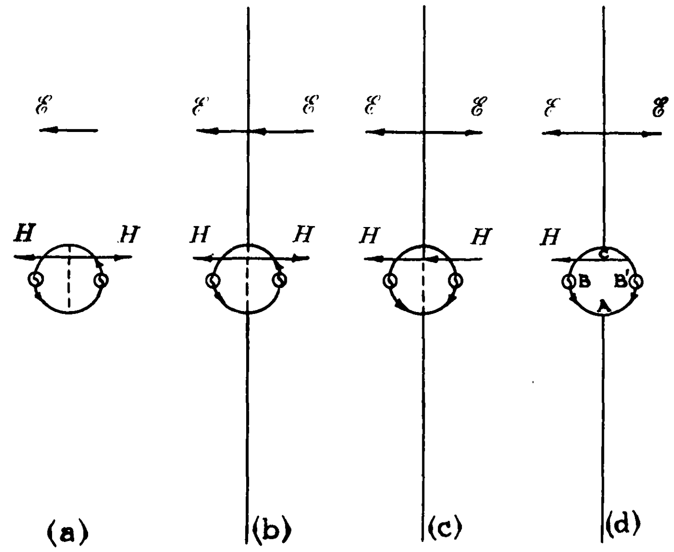

Fig. 5.—Relation between dipole and complementary slot.

which shows a view along the length of the dipole. At the surface of the rectangular strip forming the dipole, the magnetic field H is normal, and points in opposite directions on the two faces of the strip. At other points in the plane of the strip the electric field $\mathscr{E}$ is normal to the plane. These features are illustrated in Fig. 5(*a*) and follow directly from corresponding results for the dipole of Fig. 4 by interchanging electric and magnetic quantities.

(ii) Use the magnetic dipole of Fig. 5(*a*) in conjunction with the complementary metal screen of Fig. 3 as shown in Fig. 5(*b*). The perfect conductor of magnetism in Fig. 5(*a*) and the perfect conductor of electricity in Fig. 3 fit together to form an infinite screen, as indicated in Fig. 5(*b*). The associated electromagnetic field is the same as that of Fig. 5(*a*), since the condition that $\mathscr{E}$ should be normal to the perfect conductor of electricity is thereby satisfied.

(iii) Reverse one of the generators in Fig. 5(*b*) as shown in Fig. 5(*c*), thereby reversing the electromagnetic field on one side of the screen. The electric field $\mathscr{E}$ at the surface of the perfect conductor of electricity now points in opposite directions on the two faces, whereas for the perfect conductor of magnetism it points in the same direction as shown in Fig. 5(*c*).

(iv) Remove the perfect conductor of magnetism in Fig. 5(*c*), leaving a slot in a metal sheet fed at its centre by a pair of generators connected in parallel as indicated in Fig. 5(*d*). This we may do without upsetting the electromagnetic field of Fig. 5(*c*) since there was no discontinuity of magnetic field at the perfect conductor of magnetism in Fig. 5(*c*). ABCB¹A in Fig. 5(*d*) is a contour round which we shall need to integrate. The pair of generators in Fig. 5(*d*) may of course be replaced by a single generator.

It is clear from the procedure described above that the electromagnetic fields of a slot and its complementary dipole are identical except that the directions of vibration of the electric and magnetic fields are interchanged. The polar diagrams for the two cases are therefore identical. It should be noted, however, that this assumes that the slot is cut in a plane metal sheet which extends to infinity. For a finite sheet there would be diffraction at the outer edge (see Section 6).

The relation between the driving-point impedance of a slot and complementary dipole may be calculated as follows. Let

V_1, I_1 = voltage and current delivered by generator exciting dipole.

$Z_1 = V_1/I_1$ = driving point impedance of dipole.

V_2, I_2 = voltage and current delivered by generator exciting slot.

$Z_2 = V_2/I_2$ = driving point impedance of slot.

$\mathscr{E}_1, H_1$ = electric and magnetic fields produced by dipole.

$\mathscr{E}_2, H_2$ = electric and magnetic fields produced by slot.

ds = vector element of length.

From the contours XYZY¹X, ABCB¹A in Figs. 4(*b*) and 5(*d*) we obtain

$$V_1 = \int_{XYZ} \mathscr{E}_1 ds = \int_{XY^1Z} \mathscr{E}_1 ds \quad . \quad . \quad . \quad . \quad (13)$$

$$I_1 = \int_{ABCB^1A} H_1 ds = 2\int_{ABC} H_1 ds \quad . \quad . \quad . \quad . \quad (14)$$

$$V_2 = \int_{ABC} \mathscr{E}_2 ds = \int_{AB^1C} \mathscr{E}_2 ds \quad . \quad . \quad . \quad . \quad (15)$$

$$I_2 = \int_{XYZ^1X} H_2 ds = 2\int_{XYZ} H_2 ds \quad . \quad . \quad . \quad . \quad (16)$$

But, in view of the procedure (i), (ii), (iii), (iv) described above and illustrated in Fig. 5, it follows that

$$\int_{XYZ} \mathscr{E}_1 ds = Z\int_{XYZ} H_2 ds \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$\int_{ABC} \mathscr{E}_2 ds = Z\int_{ABC} H_1 ds \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where Z is the intrinsic impedance of the surrounding medium, given by (5) for free space. From (13), (16) and (17) it follows that

$$V_1 = \tfrac{1}{2}ZI_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and from (14), (15) and (18) that

$$V_2 = \tfrac{1}{2}ZI_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Multiplying (19) and (20) we obtain

$$V_1V_2/I_1I_2 = \tfrac{1}{4}Z^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

or

$$Z_1Z_2 = \tfrac{1}{4}Z^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Thus the geometric mean of the driving-point impedances of a slot and complementary dipole is equal to half the intrinsic impedance of the surrounding medium. This is the same as the relation (9) between the equivalent surface-impedances of complementary gratings. For a half-wave dipole at resonance it is usual to take the centre impedance as

$$Z_1 = 0{\cdot}194Z \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$= 73\,\Omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

for free space from equation (5). It follows from equation (22) that the centre-impedance of a half-wave slot at resonance is

$$Z_2 = 1\cdot29Z \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

$$= 485\,\Omega \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

for free space. If V is the r.m.s. electromotive force across the centre of the slot in kilovolts and W is the power radiated in kilowatts, then

$$W = 1\,000V^2/Z_2 \quad . \quad . \quad . \quad . \quad . \quad (27)$$

$$= 776V^2/Z \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$= 2\cdot06V^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

for free space. This power is, of course, equally divided between the two regions of space on either side of the metal screen, unless steps are taken to the contrary.

It is obvious from Babinet's principle that the selectivity of a slot and its complementary dipole are the same. Equation (22) states in fact that impedance for the one is proportional to admittance for the other, the constant of proportionality being independent of frequency, assuming of course that the surrounding medium is not dispersive. We can reduce the selectivity of a slot by widening it just as we can reduce the selectivity of a dipole by increasing its lateral dimensions.

A slot regarded as a transmission line should have a characteristic impedance, with the aid of which the driving-point impedance at any one point of the slot can be deduced from that at any other point by ordinary theory of transmission lines. In fact, however, there is some difficulty in defining the characteristic impedance of a slot and in calculating it precisely. This arises from the fact that a condenser formed by cutting an infinite-length slot in an infinite metal sheet has a capacitance that is infinite. This is true even if the slot has a finite breadth b, and is due to large contributions from the metal distant from the slot. In practice, of course, it is only metal within a distance of about $\lambda/(2\pi)$ of the slot that really matters. On this basis an approximate characteristic admittance Y_2^0 of the slot may be evaluated as

$$Y_2^0 = (2/\pi)Y \log_e[(2\lambda)/(\pi b)] \quad . \quad . \quad . \quad . \quad (30)$$

Y being the intrinsic admittance of the surrounding medium. In terms of this characteristic admittance, the selectivity of a half-wave slot may be evaluated by the same argument as would be used for a transmission line, the result being

$$Q = \pi Y_2^0/2Y_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

where Y_2 is the reciprocal of (25). The difficulty that arises in specifying the characteristic impedance of a slot corresponds exactly to the difficulty in specifying the characteristic impedance of a single wire. The latter may be taken to be the same as if the wire formed the inner conductor of a concentric transmission-line whose outer conductor is of radius $\lambda/(2\pi)$. Thus, for a wire of diameter $b/2$, or a strip of breadth b, the characteristic impedance is

$$Z_1^0 = [Z/(2\pi)] \log_e[(2\lambda)/(\pi b)] \quad . \quad . \quad . \quad . \quad (32)$$

and in terms of this the selectivity of a half-wave dipole is

$$Q = \pi Z_1^0/2Z_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Z_1 being given by (23). Notice that (30) and (32) satisfy

$$Z_1^0 Z_2^0 = \tfrac{1}{4}Z^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Z_2^0 being the reciprocal of Y_2^0. Notice also that, by virtue of (22) and (34) the selectivities (31) and (33) are equal, as they should be by Babinet's principle.

Suppose we have a half-wave slot in a metal screen and that it is excited, not by a generator, but by a wave incident on one side of the screen. Let W kilowatts be the power transmitted through the slot to the further side of the screen, and V kilovolts the r.m.s. electromotive force developed across the centre of the slot. The relation between W and V may be calculated as follows. Assuming that free vibration swamps forced vibration, the slot functions substantially as if it were excited by a generator radiating $2W$ kilowatts equally divided between the two sides of the screen. We therefore only have to replace W by $2W$ in (27), (28) and (29). This gives for free space

$$V = 1\cdot0\sqrt{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

A grating of resonant slots of the type shown in Fig. 1(*b*) is dealt with in the same way as a curtain of resonant dipoles of the type shown in Fig. 1(*a*). The mutual impedance between two slots is related to the mutual impedance between the two complementary dipoles by equation (22). The interaction between slots and dipoles involves, however, new considerations which have been worked out but which will not be discussed in this paper. In the above discussion particular emphasis has been laid on straight half-wave resonant dipoles and slots. The argument may however be directly extended to any planar wire and complementary slot. The driving point impedances of complementary metal screens at corresponding points are always related by equation (22).

(6) SOME POSSIBLE APPLICATIONS OF SLOTS

Since the above theoretical work was done in 1941, numerous practical applications of slots have arisen in Great Britain and overseas. It is not however the function of this paper to describe these developments in detail, or the techniques that have been worked out to fulfil the conditions involved in the various applications. Some idea of the way in which slots can be used in practice is given below, nevertheless.

It is obviously impracticable to enumerate all the possible ways in which slots could be used. Any plane system of wires used in any application (a simple example would be a twin-wire transmission line) can be regarded as a metal screen to which Babinet's principle can be applied to deduce a complementary system of slots having complementary properties. In fact it is not even necessary that the system of wires should be plane. If it is not plane, the precise quantitative relationship involved in Babinet's principle becomes an approximation, but the qualitative ideas persists largely unchanged (for example, a bent or twisted twin-wire transmission line). Thus almost any equipment normally built of wires could, with sufficient ingenuity, be translated into a complementary equipment incorporating slots. However, in many cases the complementary equipment would be clumsy and of little practical interest, wire-arrangements being preferable. In other cases, however, it is the slots that constitute the more convenient arrangement, while there are some problems for which slots provide the only satisfactory answer.

An obvious application of half-wave slots is to use them for aerials instead of half-wave dipoles. Two practical points immediately arise:

(i) it is not possible for the sheet in which the half-wave slot is cut to be an infinite plane,

(ii) one usually wants the slot to radiate on only one side of the sheet.

The second of these difficulties may be overcome simply, by boxing the slot in on the side from which no radiation is desired. If this is done with a hemisphere of radius about $\lambda/4$ concentric with the centre of the slot, no appreciable reactive load is thrown across the centre of the slot. If the box is such as to throw a reactive load across the slot, this may be balanced out by re-adjusting the length of the slot. One form of box that has been used by Dr. C. H. Westcott of the Telecommunications Research Establishment is constructed by cutting the slot along the generator of a circular metal cylinder: if the diameter of the cylinder regarded as a wave guide is less than that required to prevent transmission of the lowest mode in non-evanescent form, then the ends of the cylinder may be left open without causing appreciable radiation from them. When a half-wave slot is boxed up non-reactively on one side of the sheet so that it can only radiate on the other side, the centre-impedance of the slot is double the value given by equations (25) and (26) and is of the order of a thousand ohms.

The fact that in practice slots have to be cut in metal sheets of finite size has comparatively little influence on the impedance properties of slots, provided there is a reasonable surround of the order of $\lambda/(2\pi)$ or more. The polar diagram of the slot is, however, appreciably modified. Suppose first of all that the slot is radiating on both sides of the sheet, and consider a point in the plane of the sheet, but outside its perimeter. At such a point radiation is received equally from both sides of the sheet and, in accordance with Fig. 5(*d*), the two contributions are in anti-phase. Consequently, zero radiation occurs in all directions in the plane of the sheet, whereas with an infinite sheet zero radiation occurs only in the two directions defined by the alignment of the slot. The larger the metal sheet, the smaller is the range of angles near the plane of the sheet in which the polar diagram is modified in the way described. If the slot is only allowed to radiate on one side of the sheet, then radiation does take place in the plane of the sheet, but the energy density is reduced by diffraction by a factor of two in comparison with the case of an infinite sheet.

An example of the use of a single slot arranged to radiate on one side from a sheet of finite size is provided by the problem of feeding from the focus a paraboloidal mirror cut so that its horizontal dimension is appreciably greater than its vertical dimension. If it is desired to illuminate such a mirror with vertically-polarized waves, we need only place at the focus a vertical half-wave dipole and reflector. But suppose we wish to illuminate the mirror with horizontally-polarized waves. If we merely rotate the dipole through a right angle we then fail to illuminate the sides of the mirror effectively at the same time wasting power above and below the mirror. The obvious procedure in such a case is to use a vertical half-wave slot at the focus. At decimetre wavelengths this can be fed from a conventional transmission line and matching device. At centimetre wavelengths, it could be fed by attaching a wave guide to that face of the slot from which radiation is not desired. In fact, when an H_1-wave guide is allowed to radiate from an open end, this end is merely functioning as a crude low-Q slot.

A grating of the type shown in Fig. 1(*b*) can be energized by means of transmission lines connected across the slots, and used as a broadside array. The grating can be provided with a parallel reflecting sheet, and the entire system constructed as a metal box totally enclosed except for the slots in one face. To avoid throwing reactance across the slots, the reflecting sheet would have to be about $\frac{1}{4}\lambda$ behind the slotted sheet. Reduction of this separation would throw inductive reactance across the slots which could be balanced by lengthening the slots beyond the usual half wavelength. An array of slots of this type could be fitted to an aircraft in such a way that the slots were cut in the skin of the aircraft and plugged with dielectric material. Dragless aerials of this type may become a necessity with high speed jet-propelled aircraft.

An array consisting of a single line of half-wave slots may be conveniently fed at centimetric wavelengths by cutting the slots in one side of a wave guide. This application has proved particularly convenient in practice. Detailed work, both theoretical and experimental, has been done to establish the technique of positioning the slots on a wave guide so as to produce any desired distribution of amplitude and phase, and consequently any desired polar diagram. The pioneer in this application has been Dr. W. H. Watson (see page 747). A set of parallel linear arrays of this type may be fitted together to form a broadside array of the type indicated in Fig. 1(*b*), or alternatively a single linear array can be placed along the focal line of a cylindrical paraboloid.

A parasitic array of half-wave slots of the type indicated in Fig. 1(*b*) may be used to control the polarization of an incident wave. Alternatively, such an array may be used as a band-pass filter to pass only wavelengths in the neighbourhood of twice the length of the slot (and harmonics). In the latter application it may be desirable that the array should cope with incident waves of all polarizations. To achieve this, each element of the array may consist of a pair of perpendicular crossed slots, or alternatively of a circular slot one wavelength in circumference at resonance. The same technique may be used in wave guides. A resonant slot cut in a metal screen placed across a wave guide is the analogue of a parallel-tuned circuit shunted across a transmission line: it forms a band-pass filter. A drawback of this arrangement is that it restricts the ability of the guide to handle high power. If 100 kW is passing through a single half-wave slot, the r.m.s. electromotive force developed across the slot at its centre is about 10 kV from equation (35). This drawback may however be turned to advantage and used as the basis for a system of radar transmission and reception using a common aerial. When the equipment transmits, the resonant slot is short-circuited by the spark which passes across it, and this is arranged to connect the transmitter to the aerial. Otherwise the resonant slot is transparent and the receiver is connected to the aerial. Dr. W. D. Allen was working on these problems of handling centimetric wavelengths in wave guides in 1941 at the Telecommunications Research Establishment, and it was largely his experimental work that inspired the developments outlined in this paper.

There are, of course, slot-complements to all the well known arrangements of wire aerials. These include end-fire arrays, end-fed elements, full-wave elements, V elements and folded elements. The last of these is illustrated in Fig. 6. The driving point impedance of the folded dipole in Fig. 6(*a*) is four times that given by (23) and (24), while that of the folded slot in Fig. 6(*b*) is one quarter of that given by (25) and (26). This is a useful way of reducing the rather high driving point impedance of a half-wave slot. It might seem at first sight that the feeding arrangements for the folded dipole illustrated in Fig. 6(*b*) are unbalanced, since one terminal of the generator is connected to a small insulated strip of metal while the other is connected to the whole of the

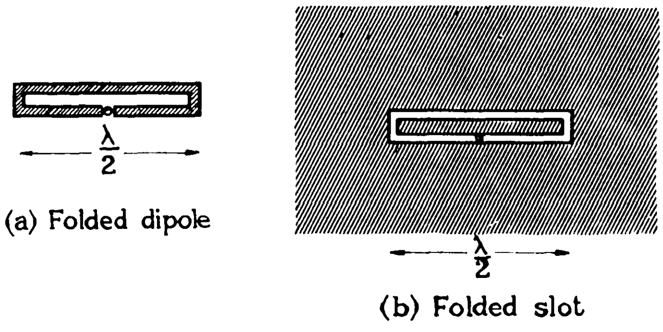

Fig. 6.—Folding technique for modifying input impedance.

rest of the sheet. It must be remembered, however, that the insulated strip is about a half wavelength long and constitutes what is sometimes known as a "dipole-earth."

It would appear that slot aerials are capable of making a contribution to the problem of designing a radiating element that produces circular polarization in all directions of radiation. In theory, this can be achieved merely by feeding a coincident slot and dipole in quadrature, for the separate elements have the same field of radiation except that the directions of vibration of the electric and magnetic fields are interchanged. It might be thought that a dipole and slot in close proximity would interact so violently as to make the arrangement impracticable. It would appear theoretically, however, that this is not so, but the arrangement has not been tried in practice.

There is a possible application of slot-aerials even at broadcasting wavelengths which should perhaps be mentioned even though it is rather far-fatched. It would be feasible to use the earth's surface as a conductor in which to cut a slot-aerial, improving the conductivity of the earth if necessary by conventional "earthing" technique. The procedure would be to cut a trench a half wavelength long and a quarter of a wavelength deep, and apply voltage across the centre of the slot. In the horizontal plane there would be a maximum of radiation at right angles to the slot and a minimum along the slot. There would, of course, be as much radiation vertically as the maximum in the horizontal plane. This would be a drawback for normal broadcasting purposes, but there might be applications in which a polar diagram of this type would be satisfactory, and in which the absence of obstructions above ground-level would be a distinct advantage.

Not only is Babinet's principle capable of suggesting new methods for handling radio waves particularly at centimetre wavelengths, but it is also able to aid in the solution of new problems by reducing them to problems whose solution is already known. One such problem is the leakage of electromagnetic radiation through a hole in a metal wall, which reduces by Babinet's principle to the problem of scattering of radiation by the complementary disc. Another problem in which Babinet's principle serves a useful purpose is that of reflection from a junction in a parallel-plate transmission line where the plates suffer a discontinuous change in separation [see Fig. 7(*a*)]. Let b and d be the separations in the two transmission lines and, for simplicity, suppose that b is small compared with d which itself is small compared with the wavelength λ. Let Z be the intrinsic impedance of the medium in the line of width d. The intrinsic impedance of the medium in the line of width b may if desired be different from Z. For example, it could be $(d/b)Z$ so as to make the characteristic impedances of the two lines equal. In this case, however, the two lines would not be quite matched because of a lumped shunt capacitance C at the junction as shown in Figs. (*a*) and (*b*). The capacitance C may be calculated by an application of Babinet's principle as follows. Using image-technique, the problem of Fig. 7(*a*) may be replaced by that indicated in Fig. 7(*c*). This diagram represents a semi-infinite block of metal, bounded by a plane face, perpendicular to which the block is sliced by slots of breadth b with their centres distant d apart. A plane wave is incident normally on the face of the metal with its electric field vibrating across the slots. A reflected wave is produced in front of the metal as well as transmitted waves down the slots. In addition, there is a capacitive storage field just in front of the metal block which is substantially the same as that occurring in similar circumstances on either side of

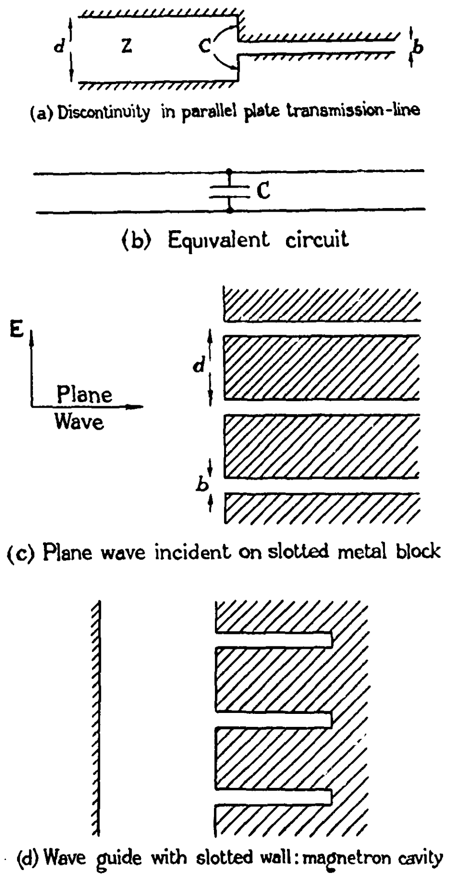

Fig. 7.—Some problems in which Babinet's principle is helpful.

the grating shown in Fig. 2(*b*). This we have already deduced by Babinet's principle from the properties of the grid shown in Fig. 2(*a*). The upshot is that the capacitance C in Fig. 7(*b*) has an admittance $\frac{1}{2}(Y_2/d)$ where Y_2 is given by equation (12).

The argument of the previous paragraph may easily be extended to deal with a wave incident obliquely upon the metal block in Fig. 7(*c*), and then by introducing short-circuits on either side of the face of the block we may solve the problem of a wave guide of the type indicated in Fig. 7(*d*). Wave guides having slotted walls of this type have interesting filter properties. Moreover a cavity of the type shown in Fig. 7(*d*), if bent round into circular form, becomes one of the type used in connection with magnetron oscillators. A slotted block of the type indicated in Fig. 7(c) is in fact an all-metal construction having properties qualitatively similar to a dielectric medium.

Details of all these applications will no doubt be published in due course by various authors. Sufficient has been said however to indicate that the electromagnetic version of Babinet's principle is a powerful technique in giving birth to new ideas and in reducing new problems to old ones whose solution is known.

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