OPTIMUM SPACINGS OF DIRECTIONAL ANTENNAS

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There have been a number of articles (1-4) that have dealt with the stacking of directional antennas. However, there is still a lot of confusion and misunderstanding among radio amateurs. Very often, a radio amateur will find that the expense and effort envolved to obtain the maximum theoretical gain increase of 3 dB on doubling the antenna size has not been achieved in practice. For this reason, it seems advisable to go into the various relationships in more detail.

The basic principles are valid for all types of antenna groups, even if this article is to be based mainly on the Yagi antennas, since this represents the most important application.

1. GAIN OF A DIRECTIONAL ANTENNA

The gain of a directional antenna will, assuming that no losses occur due to side lobes and dissipation, be determined by the beamwidth of the directional characteristic. Kraus (5) used the following equation for this:

$$G_i = \frac{4\pi}{\Theta_E \times \Theta_H} \qquad (1\,a)$$

where Θ_E and Θ_H are the angles between the 3 dB points of the E or H-diagram. G_i is the gain over an isotropic radiator. For angles in degrees, and gain in dB over a dipole, the equation will be as follows:

$$G_d = 10 \log \frac{41253}{\Theta_E \times \Theta_H} - 2.14 \qquad (1\,b)$$

According to this, an increase of gain can only be obtained by decreasing the beamwidth; decreasing the beamwidth by half in one plane will cause a doubling of the power gain (3 dB). If the possibilities of increasing the gain of an individual antenna have been used to the full – for example by lengthening the boom of a Yagi antenna (6) – a further increase of gain can only be obtained by forming groups of antennas. This is to be discussed in this article.

2. THE SUPERPOSITION PRINCIPLE

The radiation characteristic of a group of identical directional antennas results by multiplication of two components:

1. The characteristic of an identical array of (isotropic) elementary radiators
2. The characteristic of one of the directional antennas to be combined (2). Prerequisite is, however, that no interaction takes place.

If the antennas are to be stacked in both planes, it will be necessary to know the individual characteristics of the antenna in both (polarization) planes.

The characteristics of point (isotropic) sources in various arrangements and spacings are to be found in any good antenna handbook. Large arrays can be combined from several sub-groups. However, since mainly groups of two or four antennas are used for amateur radio applications, the following description is to be limited to the relatively clear case of stacking two antennas in the H-plane (two horizontally polarized Yagis stacked one above the other).

2.1. Diagram of two Point-Sources

Two equal-phase point sources will provide the maximum field strength at all points where their waves arrive at identical phase, and a minimum where anti-phase conditions exist, i.e. $(2\,n - 1) \times 180^\circ$ phase shift. In the case of **Figure 1**, the maximum field strength will be obtained in the symmetrical axis, and the first null is exhibited at the angle $\varphi 1$.

As can be seen in the diagram, the following is valid:

$$\sin \varphi 1 = \frac{\lambda}{2 \times D} \qquad (2)$$

The angle for the second null will result when the path difference amounts to $3/2\ \lambda$, $5/2\ \lambda$, and so on.

The prerequisite for the appearance of a null is a spacing D of at least $\lambda/2$, otherwise, a path difference of $\lambda/2$ will not be possible (smaller spacings are thus unsuitable).

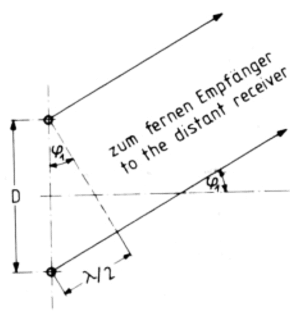

Fig. 1: Superimposition of the beams of two equal-phase point sources

At $D = \lambda/2$, the null will be at $\pm$ 180°, and a polar diagram will have the form of a horizontal Eight. If D is greater than $\lambda/2$, side lobes will appear in the diagram at right angles to the central axis, and will break up into a number of smaller lobes on increasing the spacing D. The first zero-position becomes then nearer and nearer to the central axis.

If D is large with respect to λ, this will result in a large number of narrow lobes. For $D = 10\ \lambda$, for instance, the first null will be at $\pm$ 2.9°, the second at $\pm$ 8.6°, the third at $\pm$ 14.5°.

2.2. Diagram of two Stacked Yagis

If the diagram of the point sources is multiplied by the H-diagram of a Yagi antenna, the zero points will be present in all cases. This can be seen easily since the antenna array will not radiate anything in a direction where the individual antenna does not. It will be found that the characteristic of the individual antenna tends to envelope the overall characteristic. The diagrams taken from (1) for two stacked 3-element Yagis show this principle clearly **(Figure 2)**. The diagrams show the polar diagrams of the relative field strength in the H-plane in a linear scale. Each diagram is referenced to the maximum value, which means that a direct comparison is not possible.

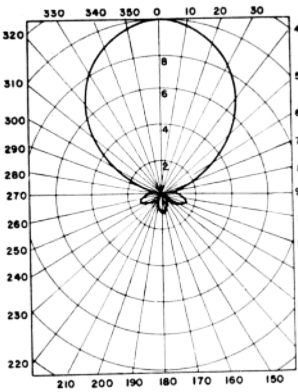

Fig. 2a

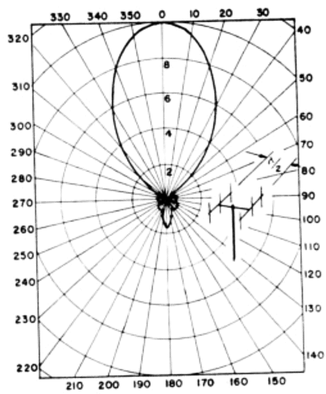

Fig. 2b

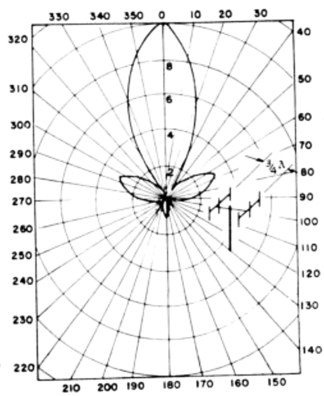

Fig. 2c

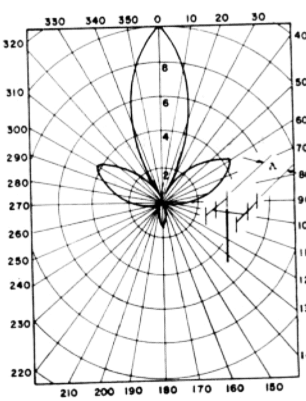

Fig. 2d

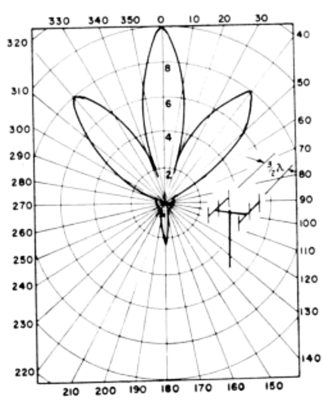

Fig. 2e

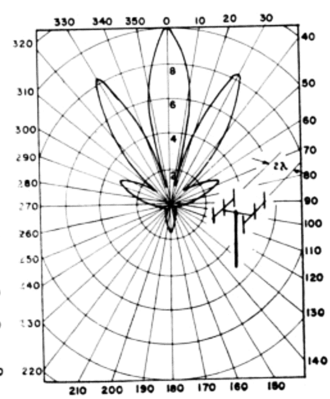

Fig. 2f

Fig. 2: Horizontal diagrams of two vertically polarized, horizontally stacked 3-element Yagis at various stacking distances

3.OPTIMUM SPACING

As is to be expected, large spacings will provide a stacking gain of virtually 3 dB. This is because the field components will subtract over half of the directions in space and add in the other half; this means that the total energy remains equal. The optimum stacking distance is the lowest spacing at which the gain is virtually doubled.

As was seen in the gain equation, it is necessary for the beamwidth of the main beam to be reduced by half. It is therefore necessary for the position of the first null to be selected. Accordingly, the required position is found when the null is placed at the – 3 dB point of the individual characteristic. It is not intended to prove this here, but this will be seen with the aid of Figure 2 :

The beamwidth of the individual antenna (Figure 2a) i.e. the angle between the points of 0.71 of the receive voltage, amounts to approximately 80°. A spacing of $D = 0.75\ \lambda$ will generate zero positions at $\pm$ 41.8° and limit the main beam to somewhat less than 40° as will be seen in Figure 2.

$D = \lambda/2$ (Figure 2b) will lead to a beamwidth of approximately 50°, which is unsatisfactory; $D = \lambda$ will reduce the main beam to approximately 30°, but will result in a very large side lobe (Figure 2d).

If half the beamwidth $\Theta/2$ is inserted for φ in equation (2), the following will be obtained:

$$D_{opt} = \frac{\lambda}{2 \sin \Theta/2} \qquad (3)$$

There are a number of other methods of calculating the optimum spacing D_{opt}, e.g. using the gain calculation by integration of the diagram, or calculating the aperture. The result will be equally accurate and identical.

At the optimum stacking distance, the first side lobe will always be approximately 13 dB down on the main beam. Figure 2 shows how the side lobes become greater on increasing the spacing. It will also be seen clearly why the gain formula will no longer be valid with diagrams having strong side lobes, since the gain will remain constant due to the side lobe losses inspite of the fact that the main beamwidth becomes narrower.

It is interesting to see that the optimum stacking distance of even such short antennas as 3-element Yagis is greater than half a wavelength. According to the author, the stacking gain D amounts to 2 dB at 0.5 λ, to 2.8 dB at $D = 0.75\ \lambda$ ($\approx D_{opt}$) and fluctuates between 2.8 and 2.95 dB at larger values of D.

If the relationship between D_{opt} and the beamwidth Θ is traced, this will result in the diagram given in **Figure 3**, which is well known from other publications.

4. OTHER SPACINGS

It is often advisable in practice to know how a deviation from a calculated value has an effect on the gain.

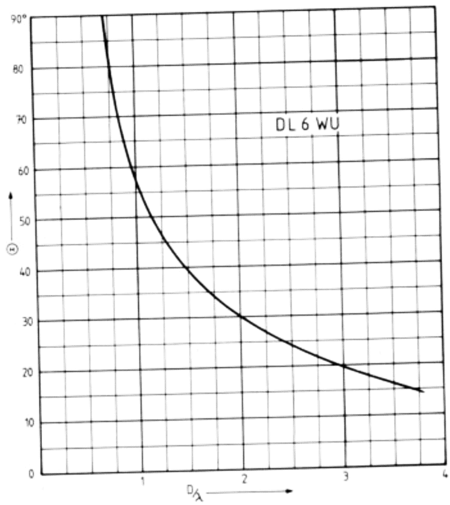

Fig. 3: Optimum stacking distance for two identical directional antennas

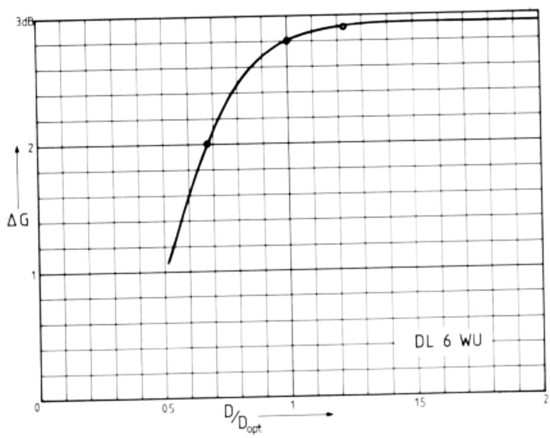

Fig. 4: Stacking gain for two antennas on deviating from the optimum spacing

If the spacing is increased, the side lobes will also increase as previously mentioned, the main beam will become sharper, but the gain will only increase by a fraction of a dB in each stacking plane. This very low increase in gain is not worthwile when considering the given disadvantages and the expense and effort of construction, and can easily be lost in the extra length of the feeders.

The gain will fall off rapidly at spacings that are less than optimum, and the increasing coupling between the antennas will also play its part. **Figure 4** shows the mean values of numerous measured values from various sources.

It may be acceptable in many cases to reduce the spacing to 0.75 of D_{opt}; in this case, the stacking gain will result in approximately 2 dB and virtually no additional side lobes will be generated. Obviously there is little room for compromises.

5. LARGER GROUPS

The previous example was valid for an array of two antennas. If larger groups are to be formed, it will be necessary to add further antennas in the horizontal or vertical plane, or to form a matrix. The superposition principle remains valid, and it is, for instance, possible to class the array of two stacked antennas described in section 3 as an individual antenna. If an array of four antennas has been optimally stacked, this can also be considered as an individual antenna to form even larger groups.

The geometric method pointed out in chapter 3 will give twice the spacing of the individual antennas for the center-to-center spacing of two such arrays of four antennas. This is where the values deviate slightly from another according to the different methods.

However, the gain of very large arrays (more than 16 individual antennas) will mainly be determined by the aperture, which means the actual advantage of Yagi antennas is lost more and more.

Principally speaking, approximately 2.5 dB increase of gain can be expected in practice with each doubling the antenna (7).

6. DEMANDS ON THE INDIVIDUAL ANTENNAS

When designing an antenna array, it will usually be based on a certain individual antenna. The characteristics of this antenna should be known exactly if optimum results are to be achieved.

The most important characteristic is the radiation diagram in the plane to be stacked. One requires the beamwidth (– 3 dB), and the side lobe suppression. A close relationship exists between these magnitudes and the gain (8). If several different antennas having the same gain are available, the most suitable will be the one having the lowest side lobes. Since it must then have the largest beamwidth, it will require the lowest stacking distance. Required is a side lobe suppression of at least 15 dB.

This value is, unfortunately, not exhibited by most long Yagi antennas. Another difficulty is that mostly only the E-plane diagrams are available (in the polarization plane), and the side lobes in the H-plane are always considerably greater. If antennas with strong side lobes are combined to form arrays, this will result in several disadvantages:

In addition to the mechanical problems due to the unnecessarily large spacings there will be the higher pointing accuracies due to the sharper main beam. The number and magnitude of the side lobes in the overall diagram will increase due to the inavoidable side lobes of the array characteristics which are superimposed on the individual characteristic. In the case of EME arrays (large groups of antennas for moonbounce communications), this will increase the possibilitiy of receiving additional noise and interference sources.

There are some examples of long Yagi antennas for 2 m and 70 cm having a clean polar diagram. Virtually all antennas in the 9- to 12-element class possess a sufficiently good side lobe rejection.

7. EXAMPLE

Required is an antenna for the 2 m band having a gain of approximately 18 dB over a dipole. This means that two Yagis of at least 15 dB, or four Yagis of 12 to 13 dB are to be combined to form an array.

It will be seen in Figure 1 of (6) that a 15 dB Yagi will have to be at least 5 λ long, which amounts to over 10 m. This means that a group of four antennas is advisable.

In order to obtain a gain of 12.5 dB, a minimum length of 2.3 λ or approximately 4.5 m is required. A 10-element antenna according to (6) of 4.5 m in length and 12 to 12.5 dB gain would meet these demands. The beamwidths would be in the order of 35°, or 40° respectively. This can be checked by placing these values in the Kraus-equation which results in a gain of 12.55 dB, an acceptable value.

The stacking distances for horizontal polarization can be calculated as follows:

$$\text{Vertical spacing } D_H = \frac{2.08 \text{ m}}{2 \times \sin 20^\circ} = 3.04 \text{ m}$$

$$\text{Horizontal spacing } D_E = \frac{2.08 \text{ m}}{2 \times \sin 17.5^\circ} = 3.46 \text{ m}$$

This antenna array exhibits a beamwidth of 17.5° horizontal and 20° vertical which corresponds to a calculated gain of 18.6 dB, from which approximately 1 dB must be subtracted for the side lobes.

This relatively compact antenna array represents the lower limit for EME communications. The gain is in excess of that of a 64- or 80-element colinear array. Model measurements on the 70 cm band have proved the calculated values; the stacking gain was measured to be 2.7 or 2.8 dB at the given spacings.

8. FEEDING

The author does not intend to discuss all possible methods of feeding the antenna groups, this has been described in detail already elsewhere (7, 9). It is important that all antennas are excited with identical amplitude and identical phase (except for some special cases). The cable lengths within the group should have a low VSWR and should be as short as possible, but of identical length. Low-loss cable should be used.

Each tenth of a dB of line loss will be added to the noise figure of the receiver or preamplifier, which in the ideal case would be located at the interconnection point of the feed.

It is advisable for such practice to be taken into consideration, especially with the low-noise transistors available today, otherwise all the effort and difficulties will not have been worthwile.

The old proverb that »a good antenna is the best RF amplifier« still remains valid, even when used in conjunction with the best preamplifier.

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