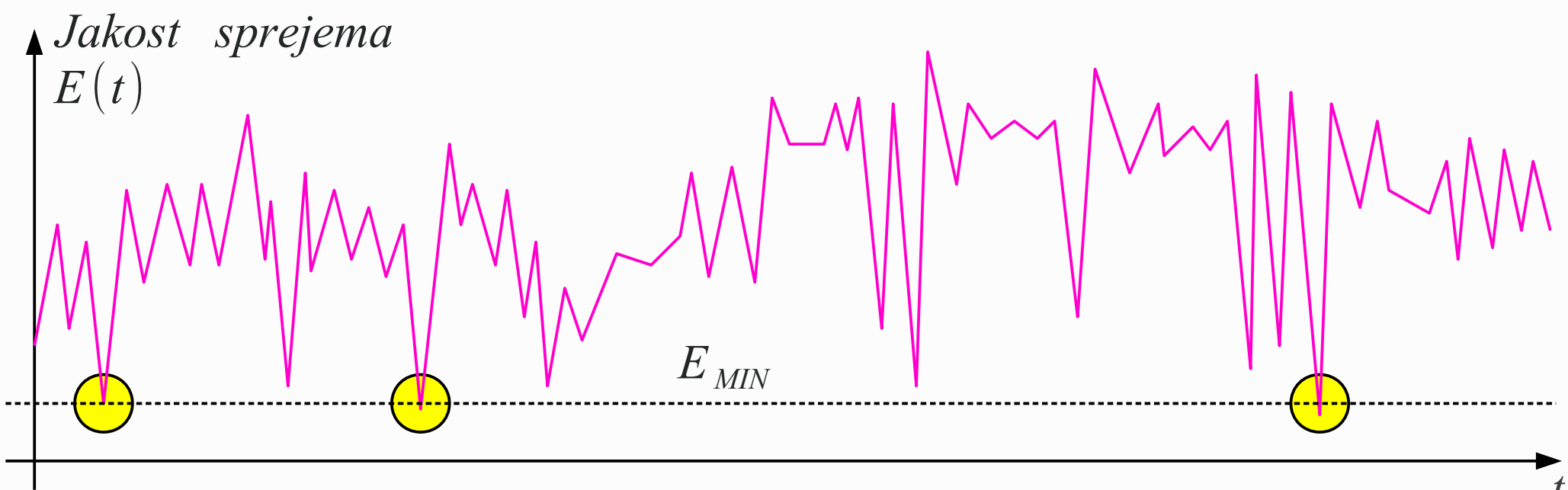


Večpotje

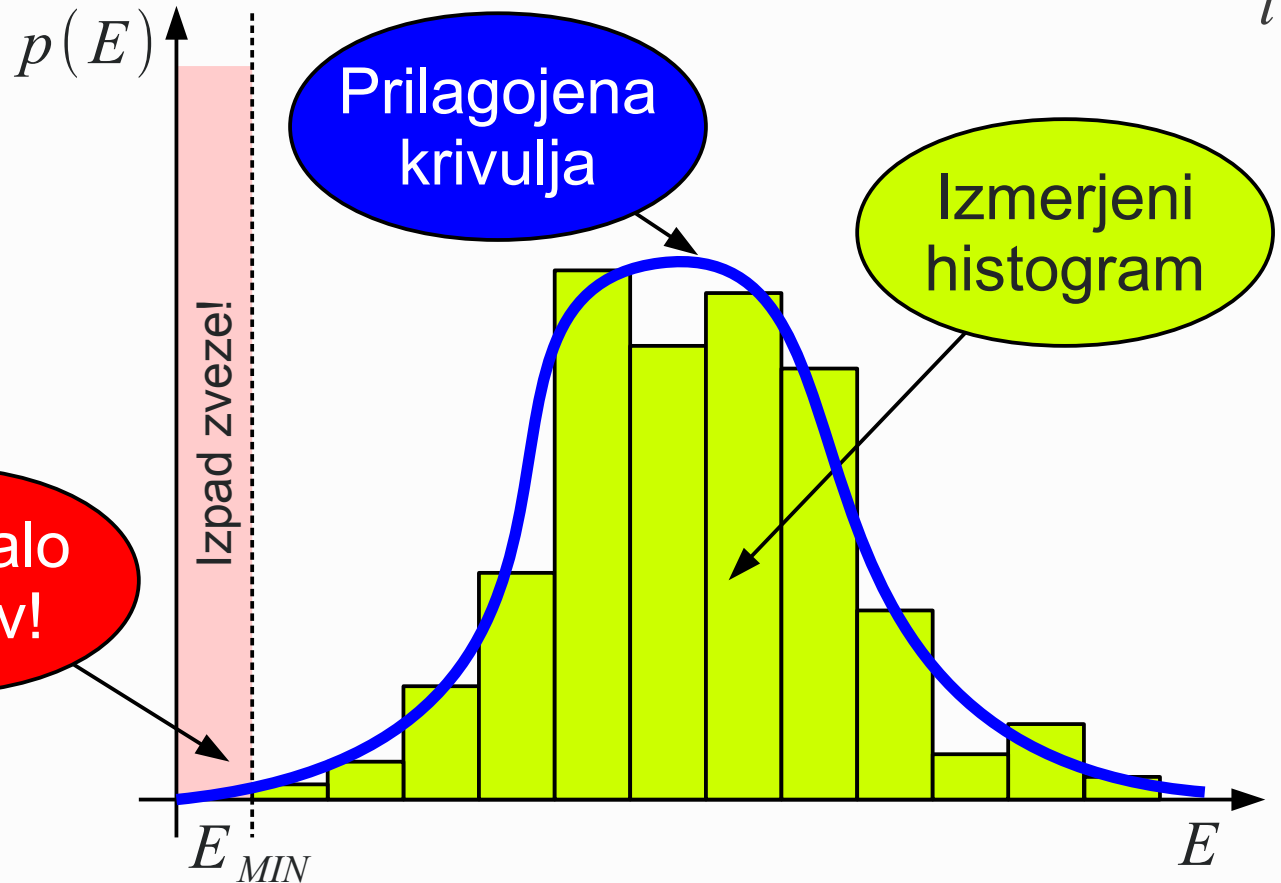


Pogostnost izpada

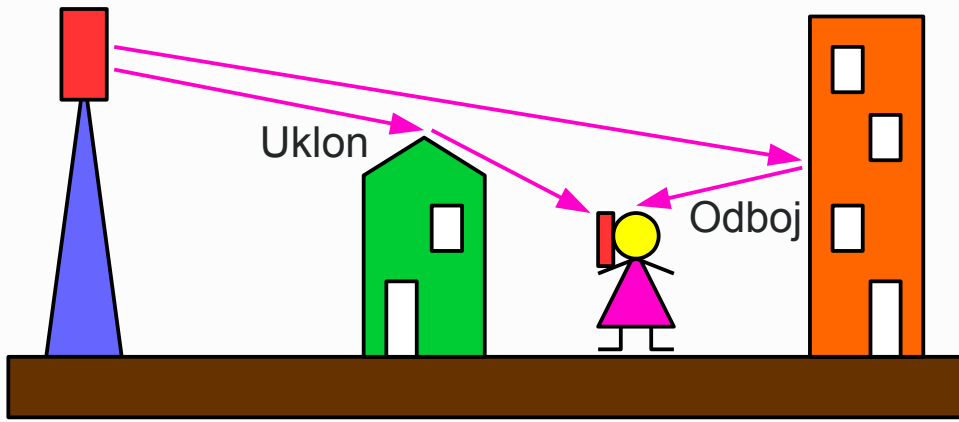
$$P_{izpada} = \int_0^{E_{MIN}} p(E) dE$$

Prilagajanje
krivulje
zahteva
fizikalno
utemeljitev!

Zelo malo
meritev!



Presih in izpad zveze



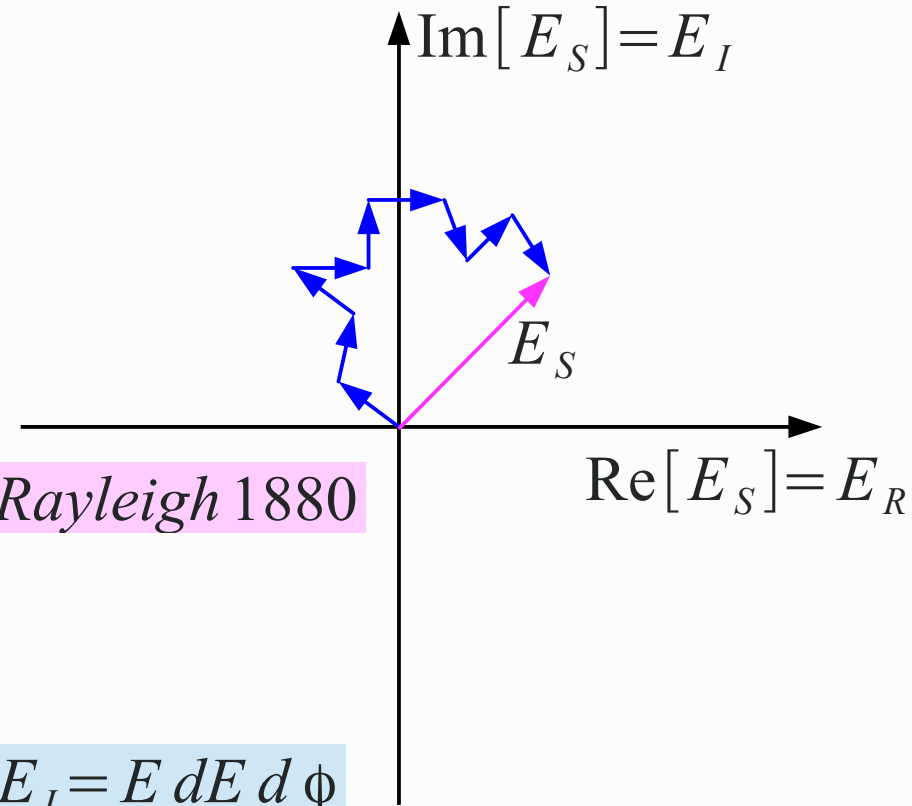
Večpotje brez vidljivosti: Rayleigh ($\langle E^2 \rangle$)
vsota mnogo naključnih malih kazalcev

Gaussova
porazdelitev
komponent

$$p(E_R) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{E_R^2}{2\sigma^2}}$$

$$p(E_I) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{E_I^2}{2\sigma^2}}$$

Lord Rayleigh 1880



$$E_S = E_R + jE_I = E e^{j\phi}$$

$$E = |E_S|$$

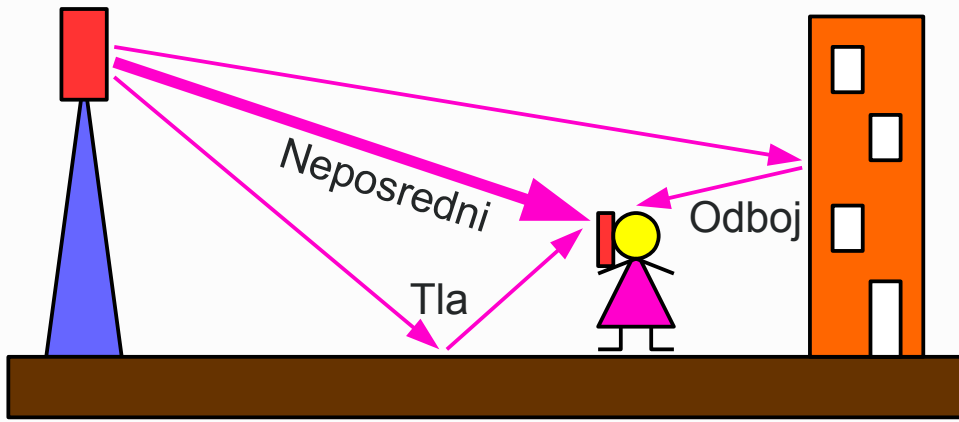
$$dE_R dE_I = E dE d\phi$$

$$p(E_R, E_I) = p(E_R) p(E_I) = \frac{1}{\sigma^2 2\pi} e^{-\frac{E_R^2 + E_I^2}{2\sigma^2}} = p(E) p(\phi)$$

$$\langle E^2 \rangle = 2\sigma^2$$

$$p(E) = \int_0^{2\pi} p(E_R, E_I) E d\phi = \int_0^{2\pi} \frac{1}{\sigma^2 2\pi} e^{-\frac{E^2}{2\sigma^2}} E d\phi = \frac{E}{\sigma^2} e^{-\frac{E^2}{2\sigma^2}} = \frac{2E}{\langle E^2 \rangle} e^{-\frac{E^2}{\langle E^2 \rangle}}$$

Rayleighjeva porazdelitev



Večpotje z neposrednim žarkom: Rice (E_0, σ)
 en velik in mnogo naključnih malih kazalcev

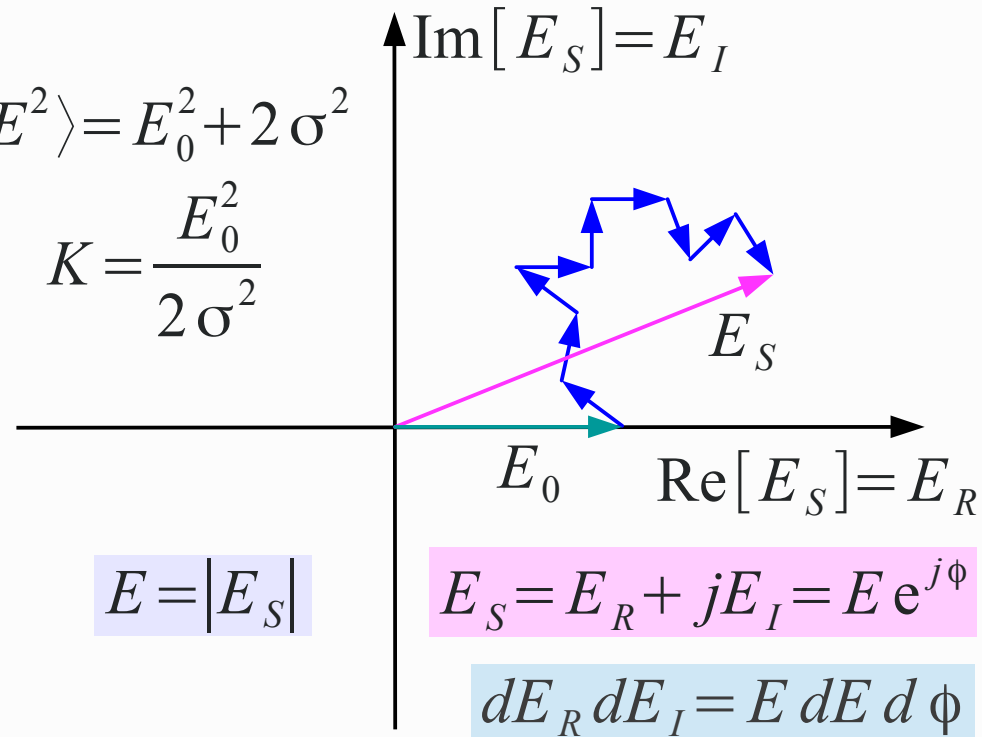
Gaussova porazdelitev komponent

$$p(E_R) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(E_R - E_0)^2}{2\sigma^2}}$$

$$p(E_I) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{E_I^2}{2\sigma^2}}$$

$$\langle E^2 \rangle = E_0^2 + 2\sigma^2$$

$$K = \frac{E_0^2}{2\sigma^2}$$



$$E = |E_S|$$

$$E_S = E_R + jE_I = E e^{j\phi}$$

$$dE_R dE_I = E dE d\phi$$

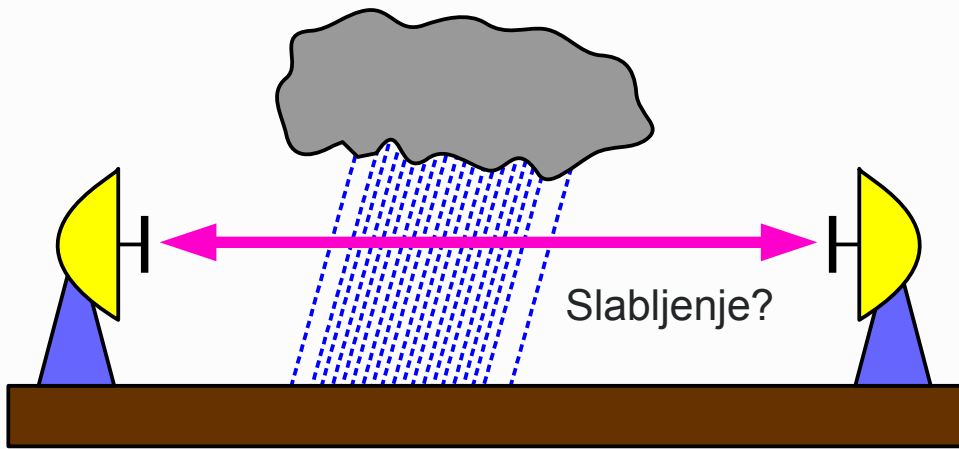
$$p(E_R, E_I) = \frac{1}{\sigma^2 2\pi} e^{-\frac{(E_R - E_0)^2 + E_I^2}{2\sigma^2}} = \frac{1}{\sigma^2 2\pi} e^{-\frac{E^2 + E_0^2}{2\sigma^2}} e^{\left(\frac{E_R E_0}{\sigma^2}\right)} = \frac{1}{\sigma^2 2\pi} e^{-\frac{E^2 + E_0^2}{2\sigma^2}} e^{\left(\frac{E_0 E}{\sigma^2}\right) \cos \phi}$$

Stephen O. Rice 1948

$$p(E) = \int_0^{2\pi} p(E_R, E_I) E d\phi = \frac{E}{\sigma^2} e^{-\frac{E^2 + E_0^2}{2\sigma^2}} I_0\left(\frac{E_0 E}{\sigma^2}\right)$$

$$\int_0^{2\pi} e^{\left(\frac{E_0 E}{\sigma^2}\right) \cos \phi} d\phi = 2\pi I_0\left(\frac{E_0 E}{\sigma^2}\right)$$

Riceova porazdelitev



Vremenski pojavi: log-normalna ($\langle E_{dB} \rangle, \sigma_{dB}$)
 produkt mnogo naključnih prispevkov
 brez interference večpotja

$$p(E_{dB}) = \frac{1}{\sigma_{dB} \sqrt{2\pi}} e^{-\frac{(E_{dB} - \langle E_{dB} \rangle)^2}{2\sigma_{dB}^2}}$$

Francis Galton ~ 1880

Fizikalno utemeljeno?

$$E_{dB} = 20 \log_{10} \left(\frac{|E_S|}{E_{REF}} \right) = 20 \log_{10} \left(\frac{E}{E_{REF}} \right) = \frac{20}{\ln 10} \ln \left(\frac{E}{E_{REF}} \right)$$

$$\sigma_{dB} = \sqrt{\langle (E_{dB} - \langle E_{dB} \rangle)^2 \rangle}$$

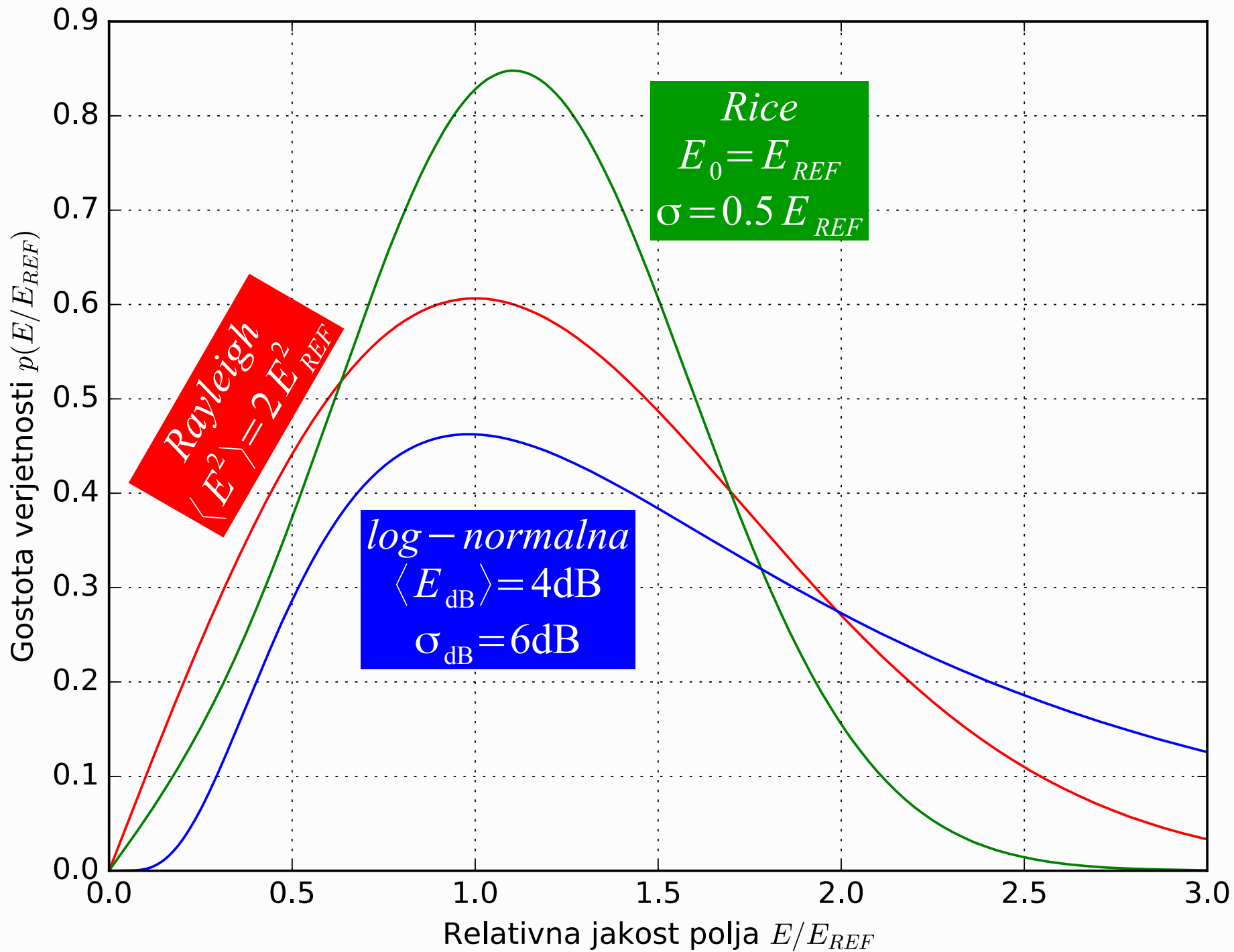
$$\frac{dE_{dB}}{dE} = \frac{20}{\ln 10} \left(\frac{E_{REF}}{E} \right) \frac{1}{E_{REF}} = \frac{20}{E \ln 10}$$

$$p(E_{dB}) dE_{dB} = p(E) dE$$

$$p(E) = p(E_{dB}) \frac{dE_{dB}}{dE} = p(E_{dB}) \frac{20}{E \ln 10} = \frac{20}{E (\ln 10) \sigma_{dB} \sqrt{2\pi}} e^{-\frac{\left[20 \log_{10} \left(\frac{E}{E_{REF}} \right) - \langle E_{dB} \rangle \right]^2}{2\sigma_{dB}^2}}$$

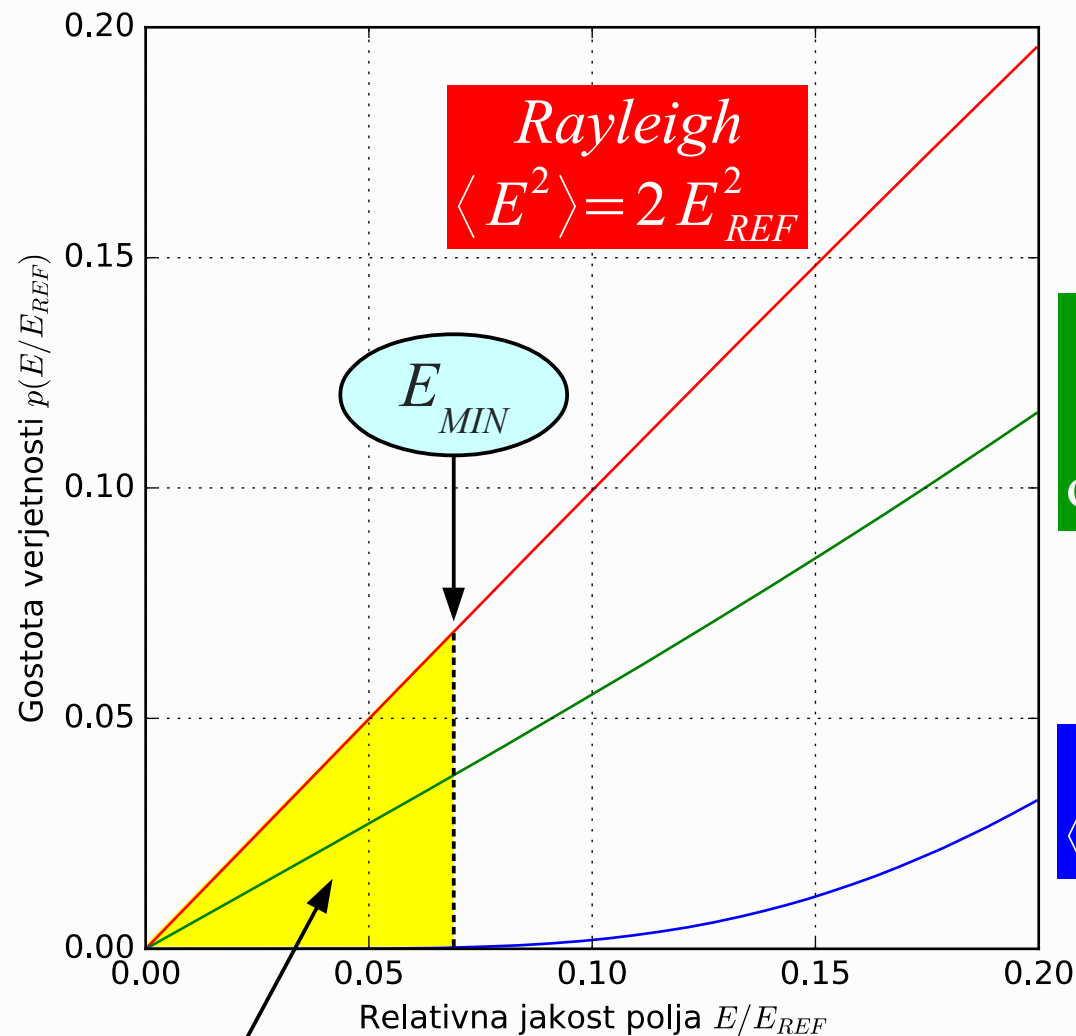
Log-normalna porazdelitev

Rayleighjeva, Riceova in log-normalna porazdelitev



Rayleighjeva, Riceova in log-normalna porazdelitev

Pošten račun: Rayleigh



Rayleigh
 $\langle E^2 \rangle = 2 E_{REF}^2$

Rice
 $E_0 = E_{REF}$
 $\sigma = 0.5 E_{REF}$

log-normalna
 $\langle E_{dB} \rangle = 4\text{dB}$ $\sigma_{dB} = 6\text{dB}$

$$P_{izpada} = \int_0^{E_{MIN}} p(E) dE$$

$$P_{izpada} = \int_0^{E_{MIN}} \frac{2E}{\langle E^2 \rangle} e^{-\frac{E^2}{\langle E^2 \rangle}} dE$$

$$P_{izpada} = 1 - e^{-\frac{E_{MIN}^2}{\langle E^2 \rangle}}$$

P_{izpada}

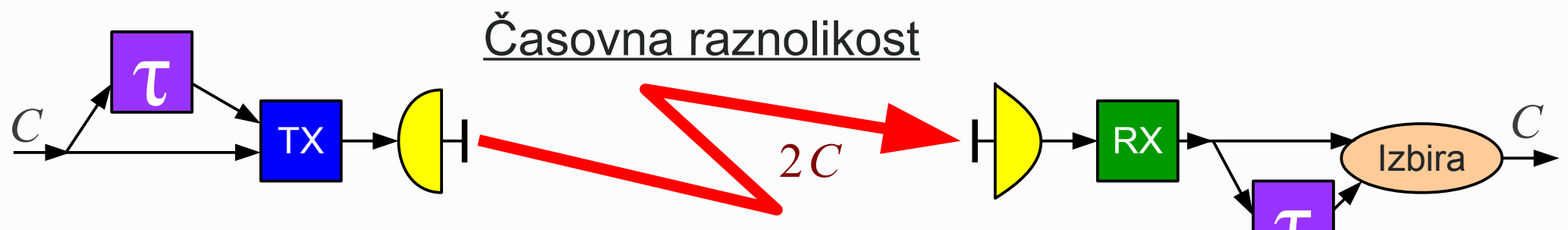
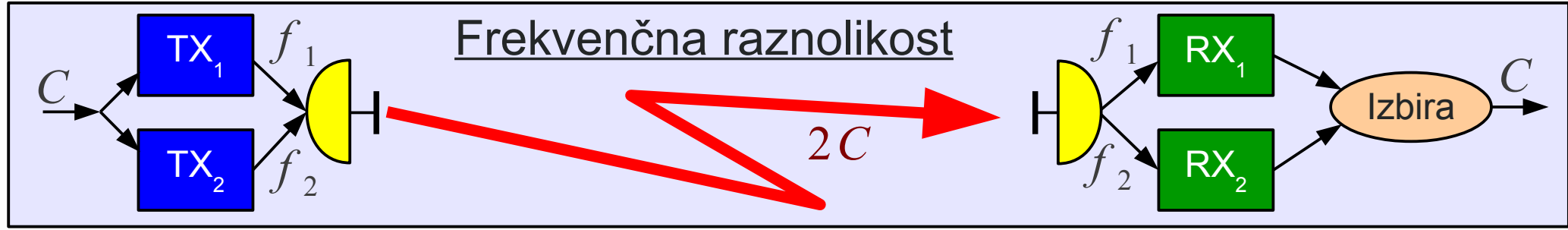
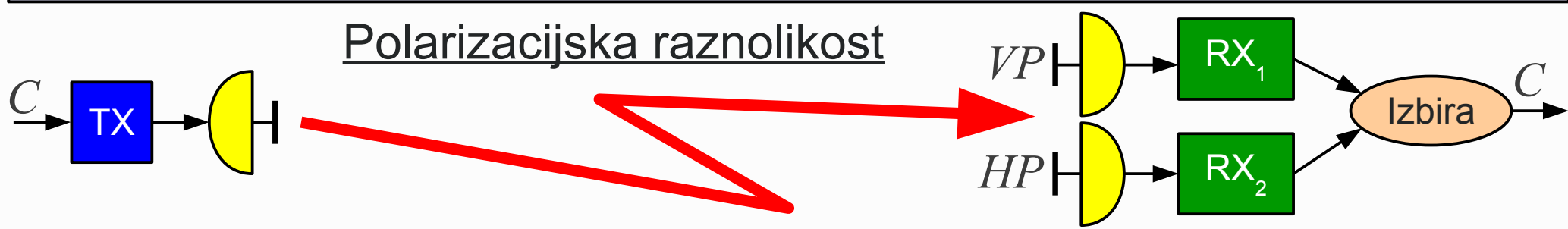
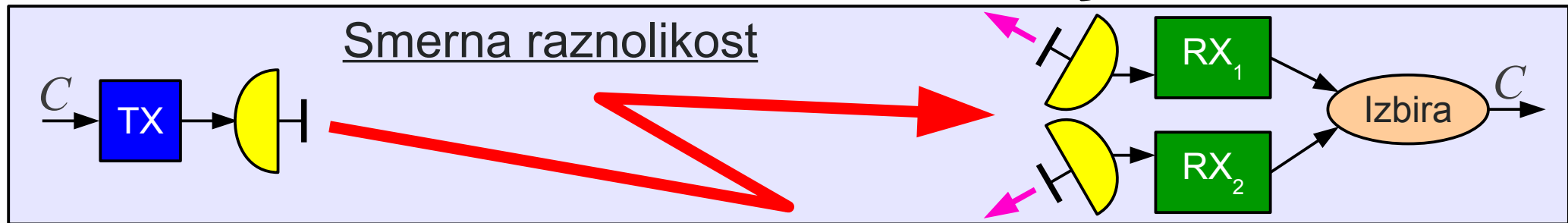
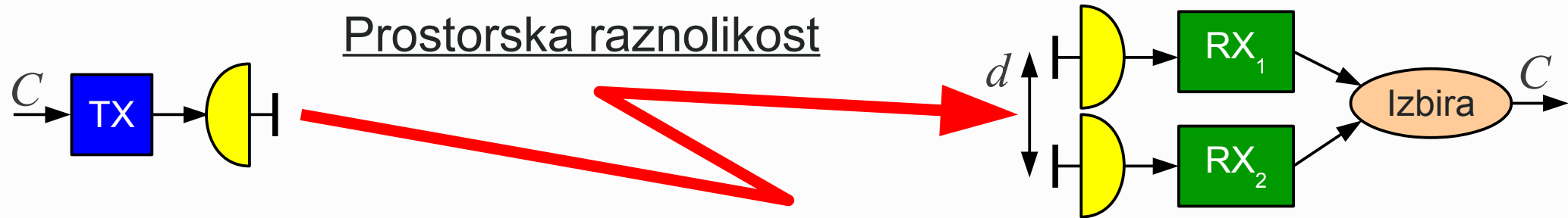
$$P = \alpha E^2 \rightarrow dP = \alpha 2 E dE$$

$$P_{izpada} = \int_0^{P_{MIN}} \frac{1}{\langle P \rangle} e^{-\frac{P}{\langle P \rangle}} dP = 1 - e^{-\frac{P_{MIN}}{\langle P \rangle}}$$

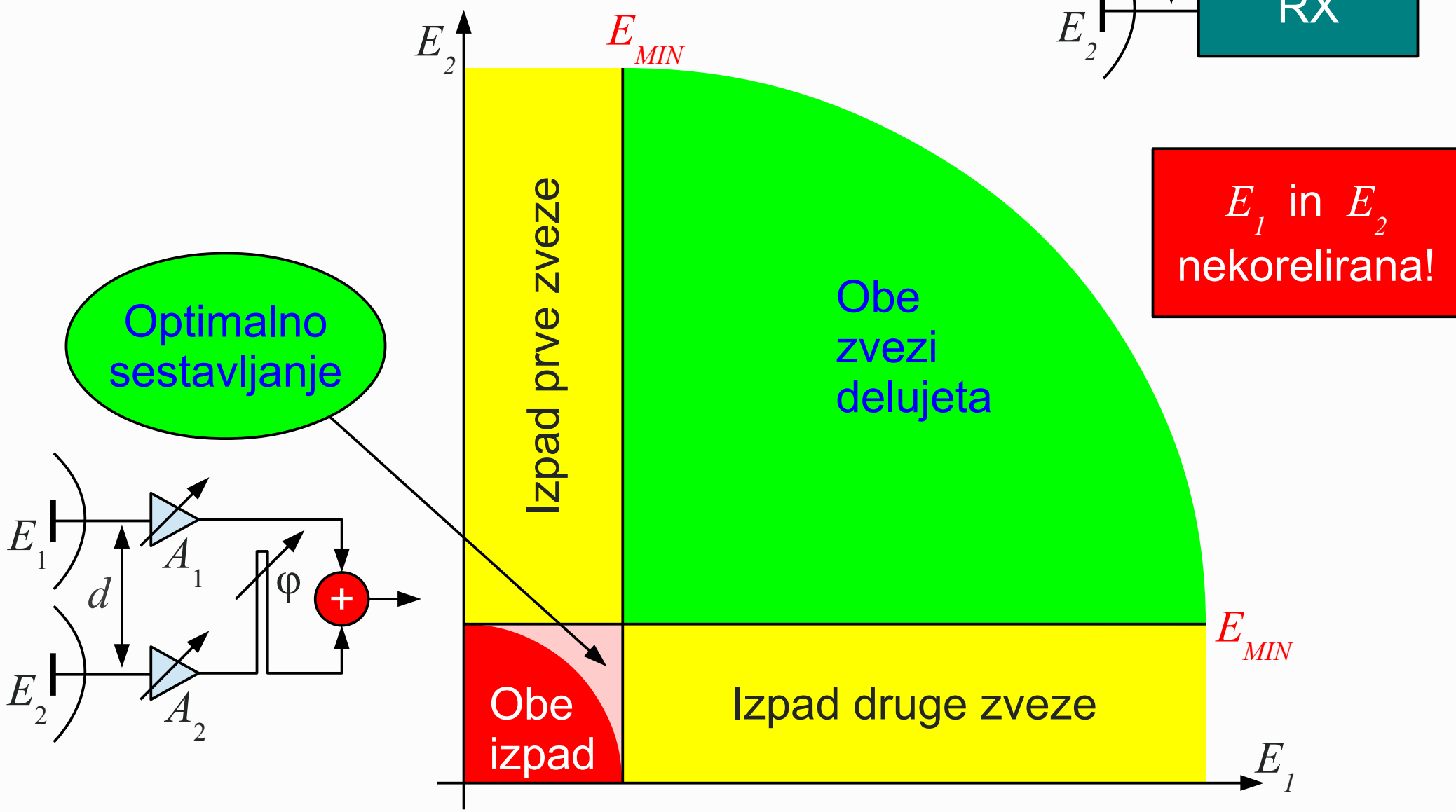
$$P_{MIN} \ll \langle P \rangle \rightarrow P_{izpada} \approx \frac{P_{MIN}}{\langle P \rangle}$$

Zgled: mobilni telefon
 $\langle P \rangle = -90\text{dBm} = 1\text{pW}$
 $P_{MIN} = -105\text{dBm} = 0.032\text{pW}$
 $P_{izpada} \approx 0.032 \approx 3\%$

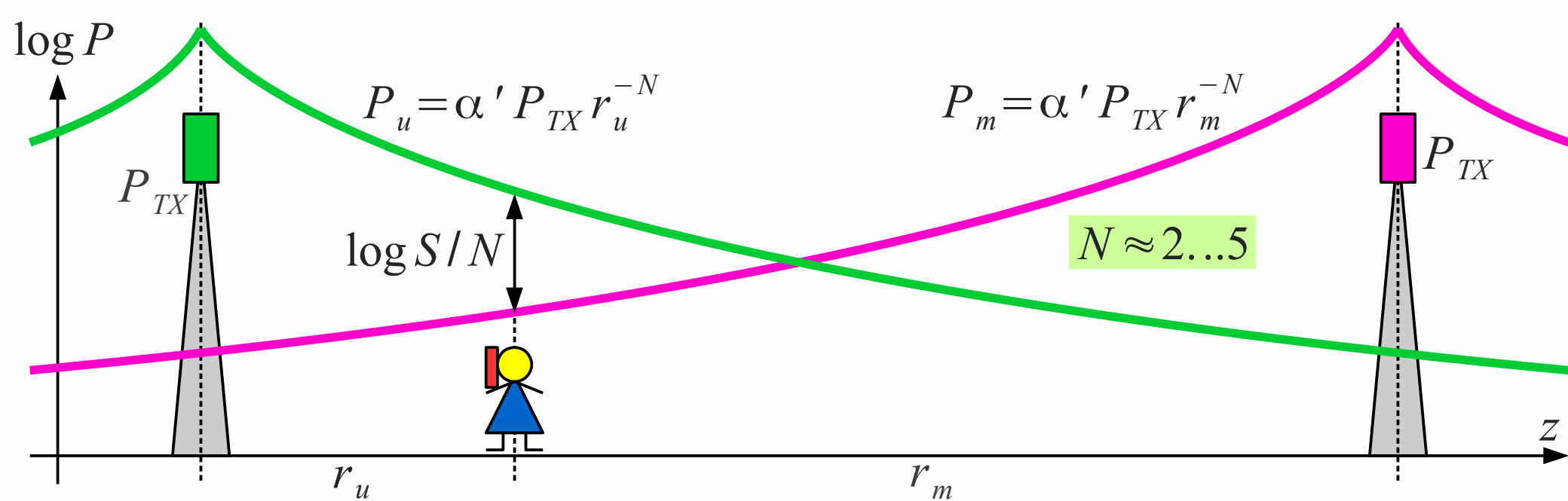
Izračun verjetnosti izpada zveze



Raznolikost kot protiukrep za presih



Pogostnost izpada pri nekoreliranem sprejemu



Mestno okolje brez vidljivosti

$$3 \leq N \leq 5$$

$$P_{RX} = P_{TX} G_{TX} G_{RX} \alpha(\lambda) h_{TX}^2 h_{RX}^2 d^{-N}$$

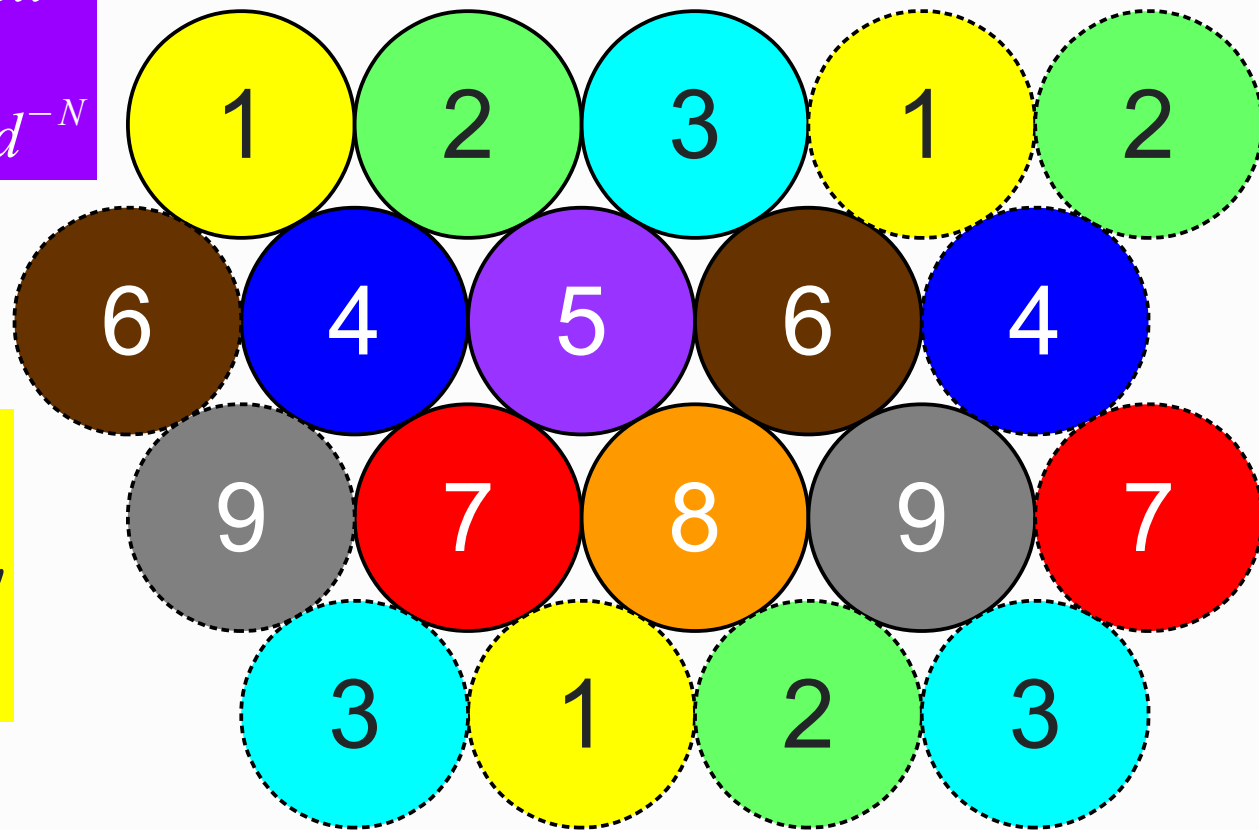
$$S/N = \frac{P_u}{P_m} = \left(\frac{r_m}{r_u} \right)^N$$

Primer: $N=4$

$$S/N = 28\text{dB} = 625$$

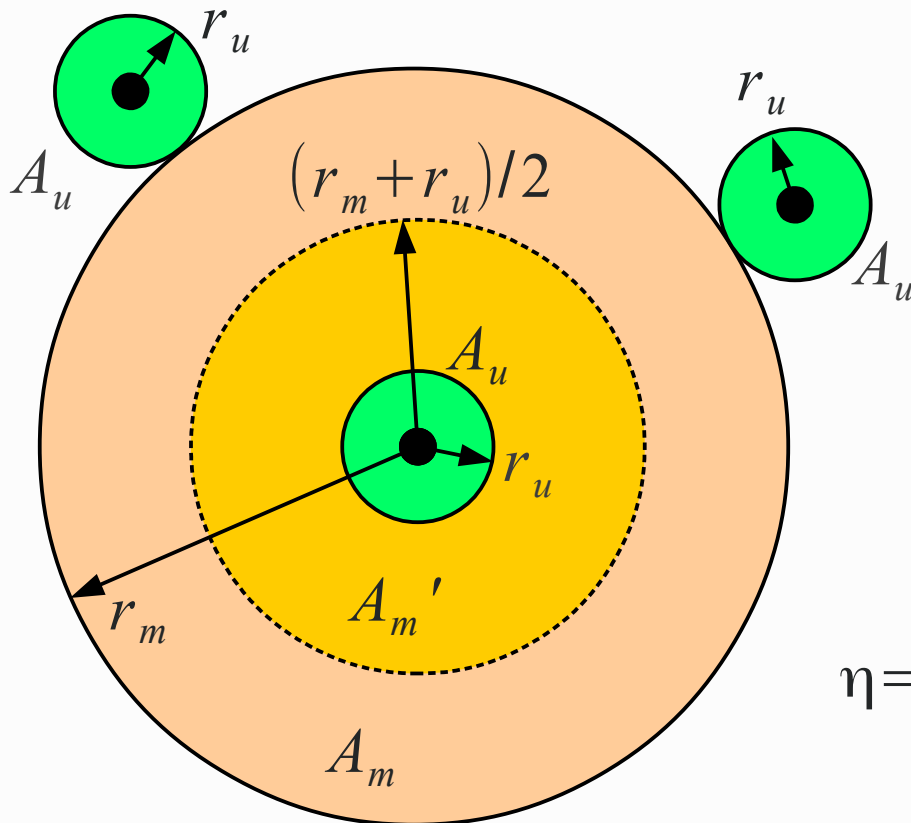
z rezervo presiha!

$$r_m = r_u \sqrt[N]{S/N} \approx 5 r_u$$



Ponovna uporaba spektra

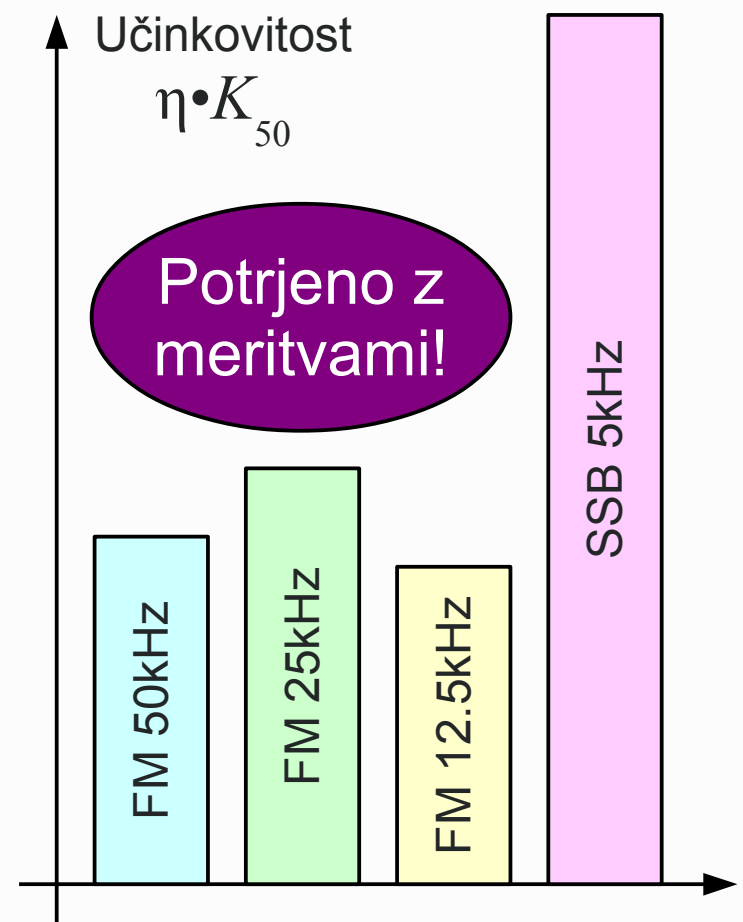
Modulacija	K_{50}	Koleb Δf	$m = \Delta f / B_u$	$3m^2$	r_m / r_u	η	$\eta \cdot K_{50}$	Ocena
FM 50kHz	1	$\pm 15\text{kHz}$	5	75	1.08	0.929	0.929	FM prag?
FM 25kHz	2	$\pm 6\text{kHz}$	2	12	1.70	0.549	1.098	Dober!
FM 12.5kHz	4	$\pm 1.5\text{kHz}$	0.5	0.75	3.40	0.207	0.827	Slab!
SSB 5kHz	10	$B_R = B_u = 3\text{kHz}$		1	3.16	0.231	2.309	Presluh?



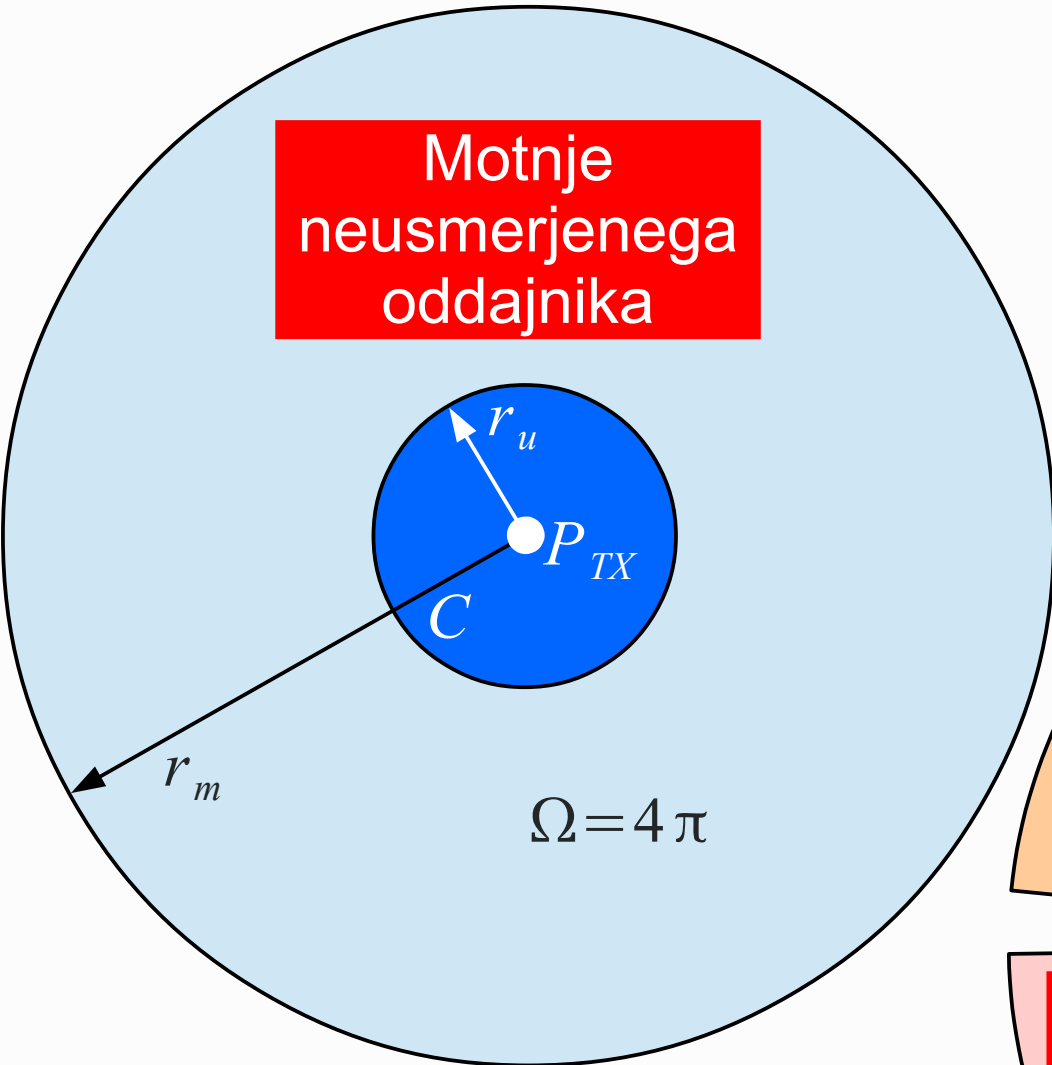
Primer:
 $N = 4$
 $S/N = 100$

$$\frac{r_m}{r_u} = \sqrt[3]{\frac{N \cdot S/N}{3m^2}}$$

$$\eta = \frac{A_u}{A_m'} = \left(\frac{2}{1 + \frac{r_m}{r_u}} \right)^2$$

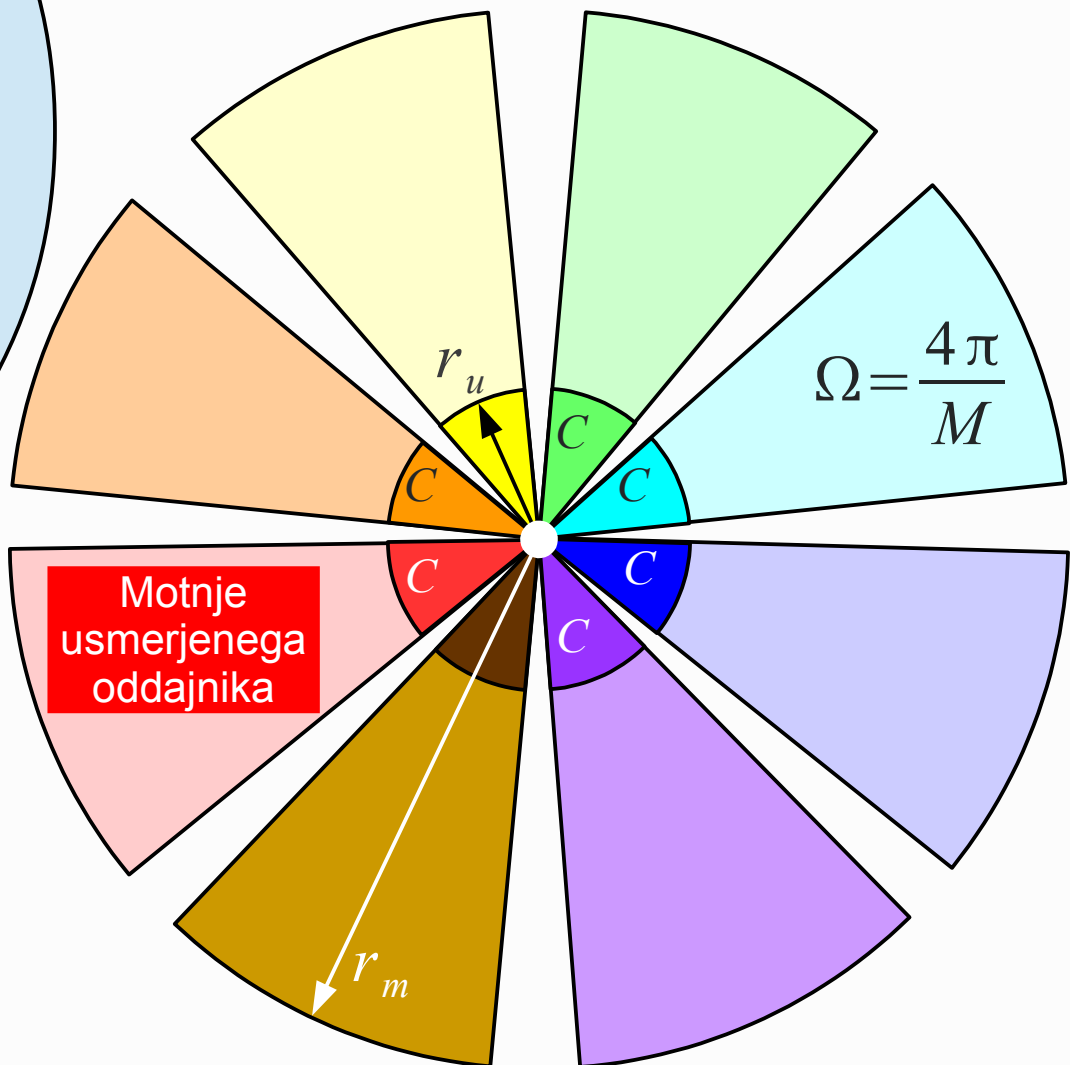


Izbira učinkovite modulacije



Uporaba usmerjenih anten:

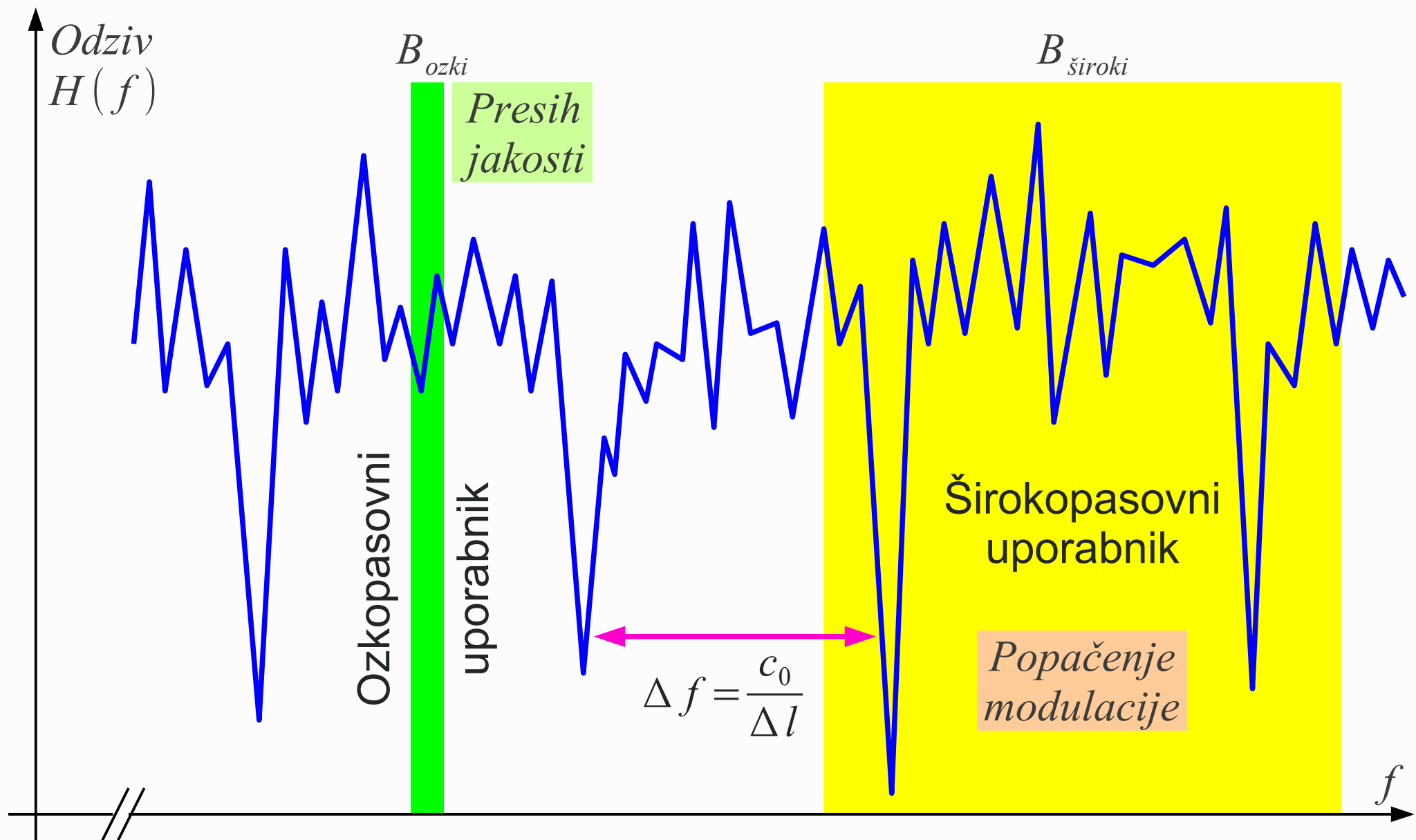
$$D = \frac{4\pi}{\Omega} = M$$

$$\sum C = M C$$


Omejitev EIRP je škodljiva!

Smiselna je omejitev $\sum P_{TXi}$

Povečanje zmogljivosti z usmerjenimi antenami



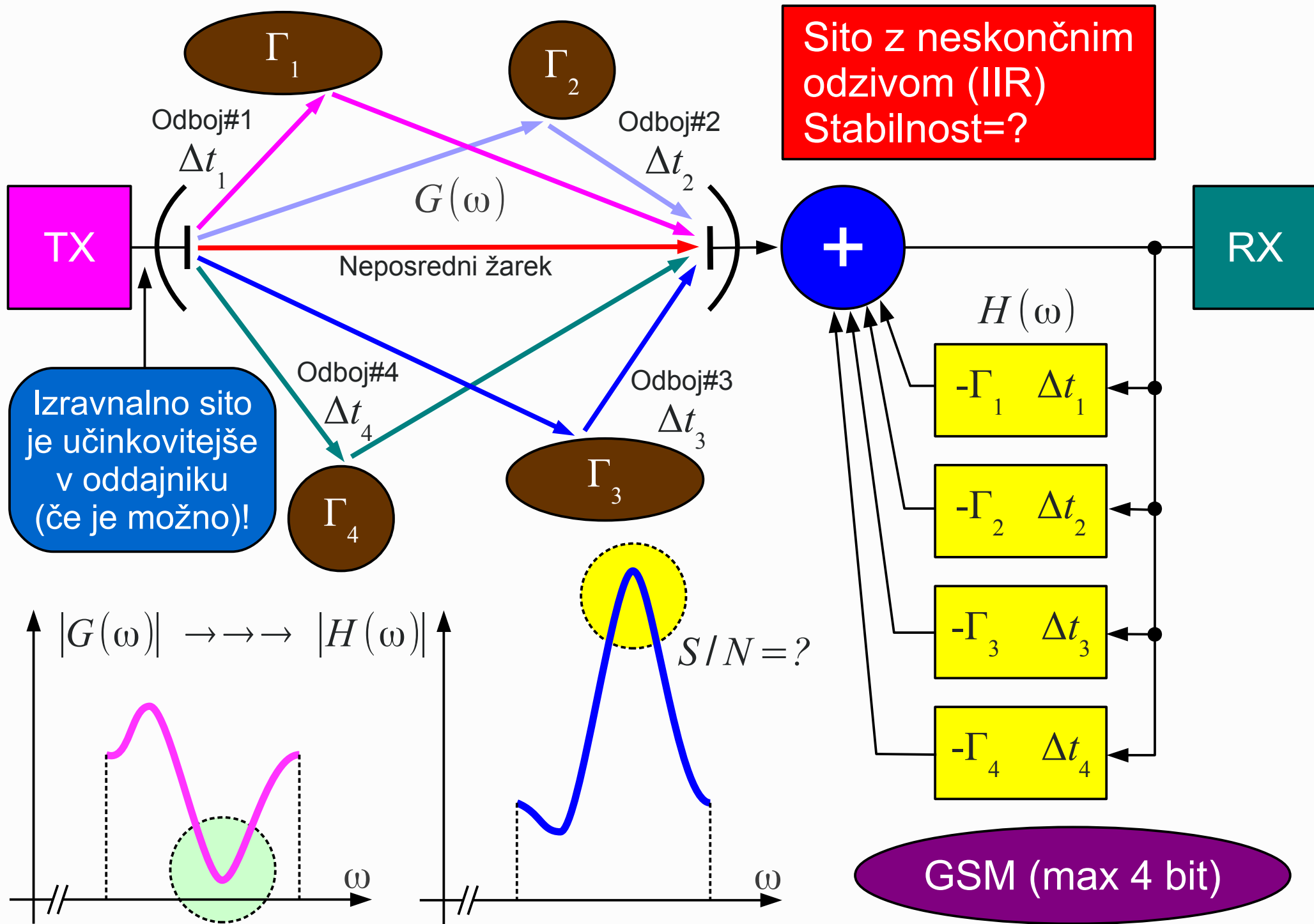
$$B_{ozki} \ll \Delta f \ll B_{široki}$$

Mestno okolje $f_0 \approx 450\text{MHz}$

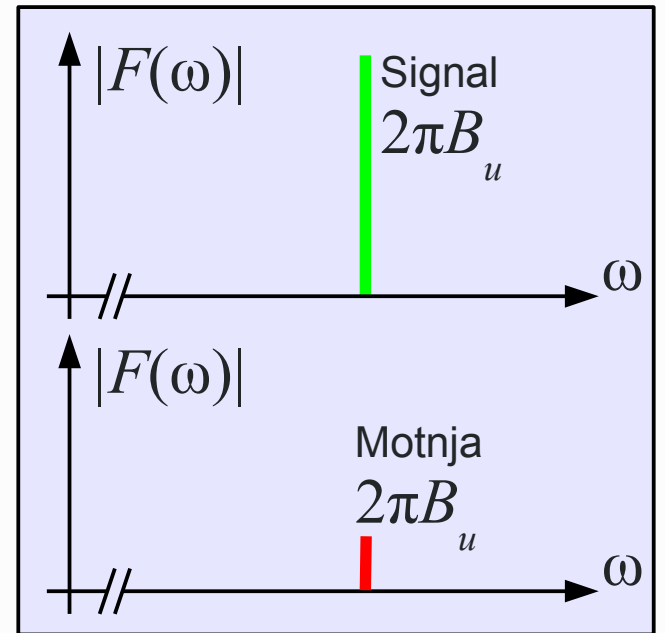
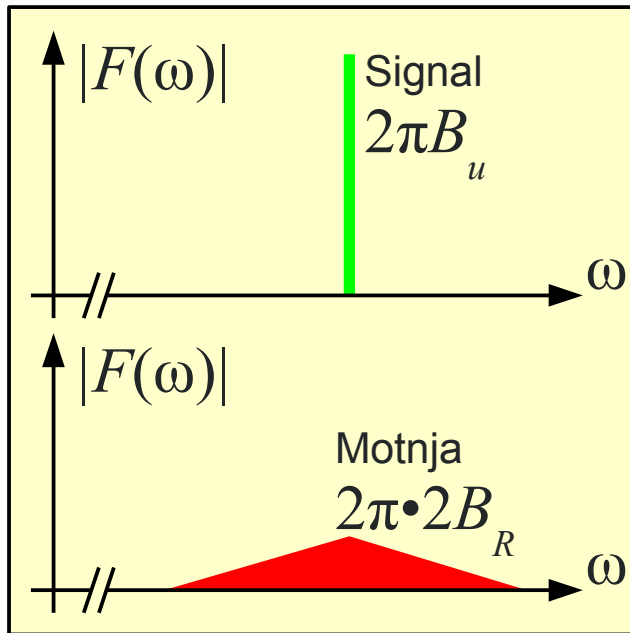
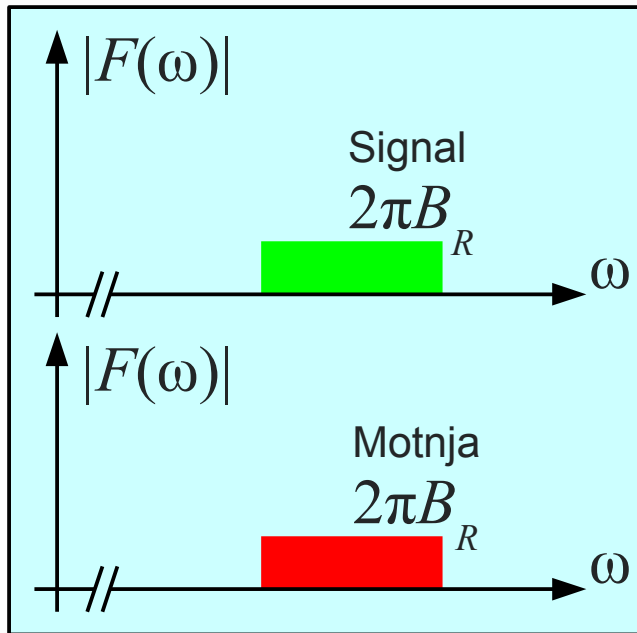
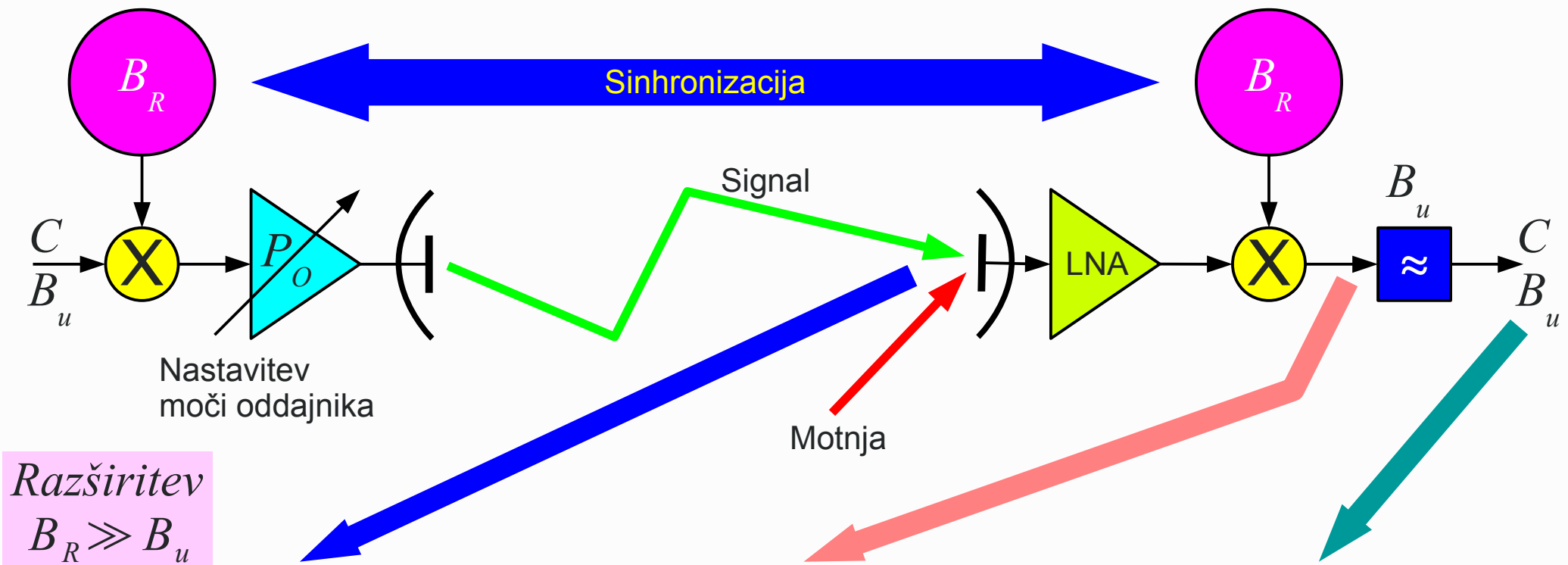
$\Delta l \approx 200\text{m} \dots 1.5\text{km}$

$\Delta f \approx 1.5\text{MHz} \dots 200\text{kHz}$

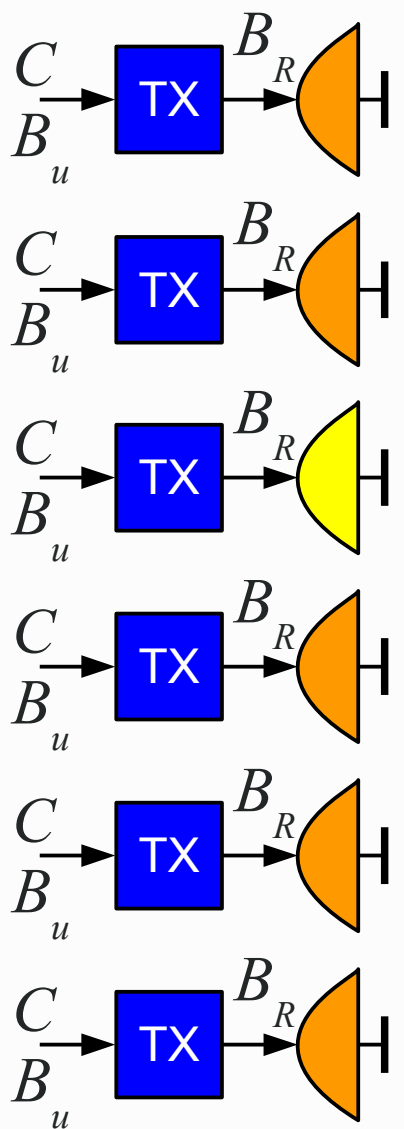
Večpotje v frekvenčnem prostoru



Odpravljanje popačenja večpotja z izravnalnim sitom



Razširjeni spekter (Spread spectrum)

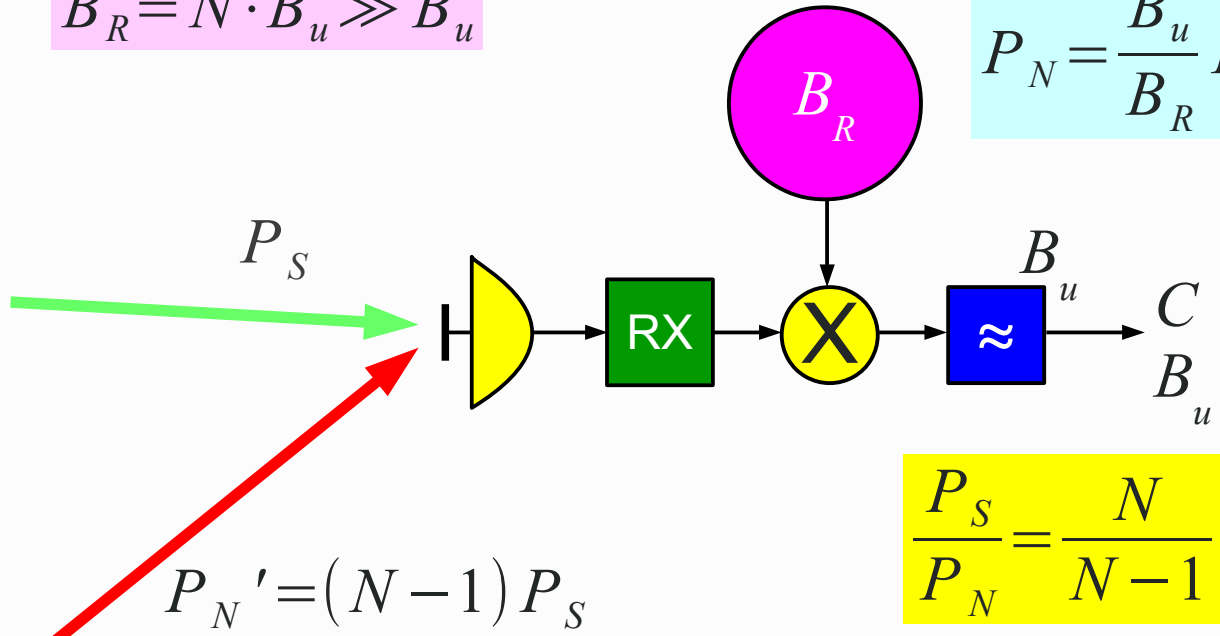


N oddajnikov (uporabnikov)
 različna razširitvena zaporedja

Odpornost na presih: $B_R \geq \Delta f$ večpotja

$B_R = N \cdot B_u \gg B_u$

$P_N = \frac{B_u}{B_R} P_{N'}$



$P_{N'} = (N - 1) P_S$

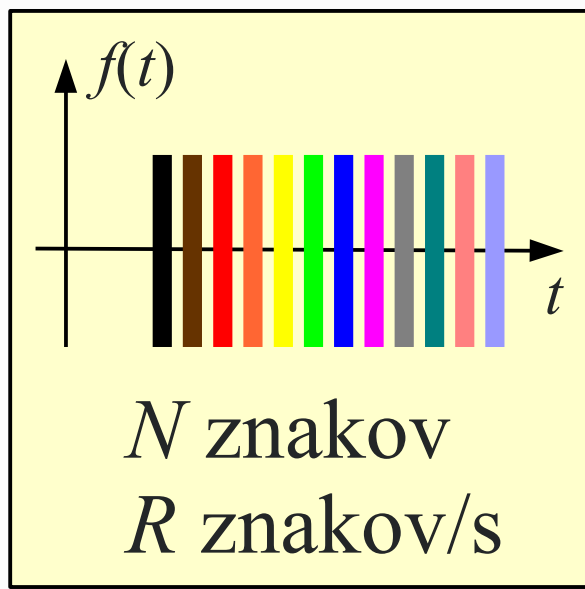
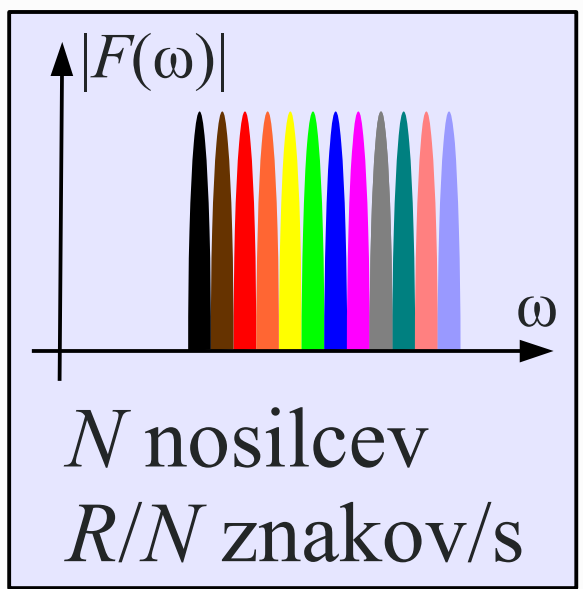
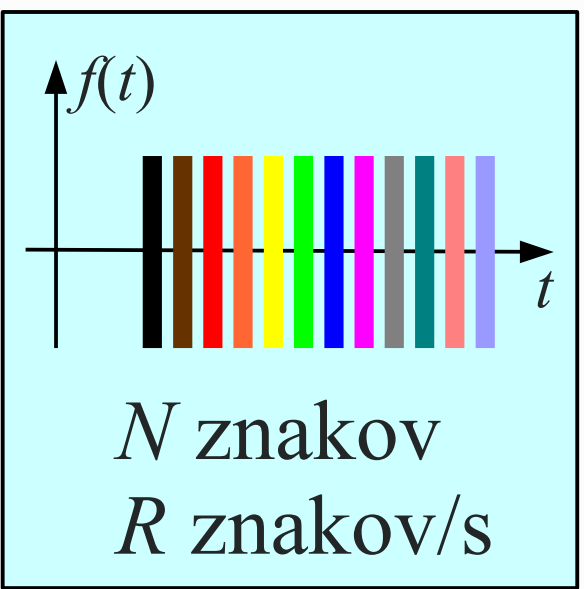
$\frac{P_S}{P_N} = \frac{N}{N - 1} \approx 1$

UMTS
 $\Delta f_{\text{večpotja}} \approx 1.5\text{MHz}$
 $B_R = 5\text{MHz}$

Odporen na presih
 Koristno izrablja večpotje
 FEC je del B_R
 Omogoča kodni sodostop CDMA

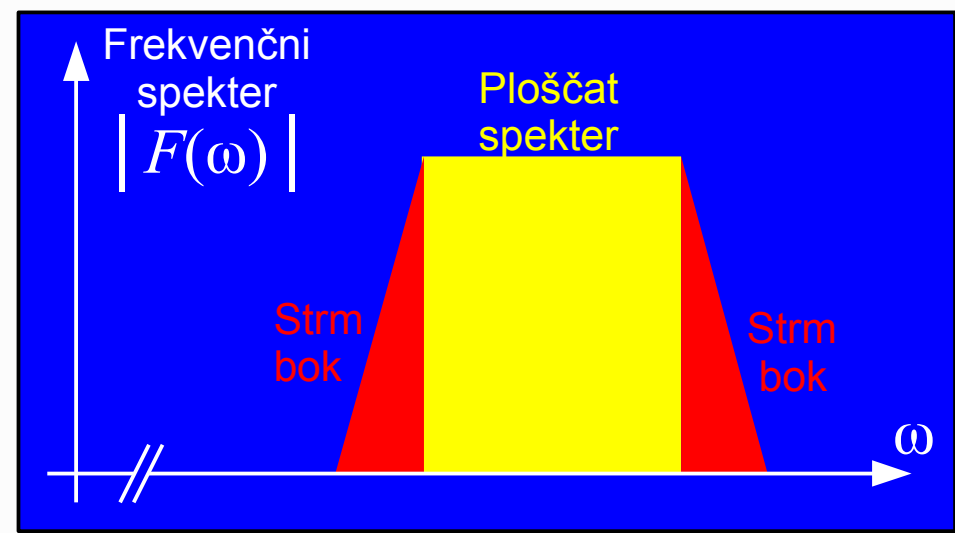
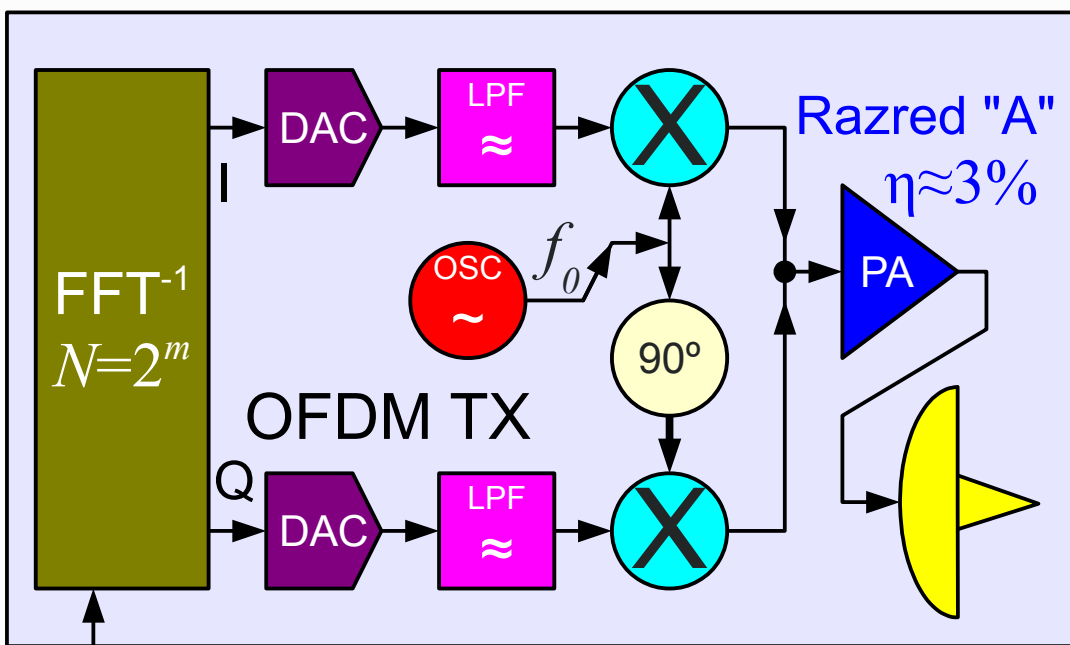
Natančno uravnavanje moči vseh oddajnikov
 Zelo zahtevna sinhronizacija
 Zahteva močen FEC
 Nizka spektralna učinkovitost $C/B \approx 1$ bit

Kodni sodostop (CDMA)



~1950 analogni večtonski modem za ionosferske zveze
 ~2000 številski DFT → OFDM WLAN (WiFi) 802.11a (FFT)

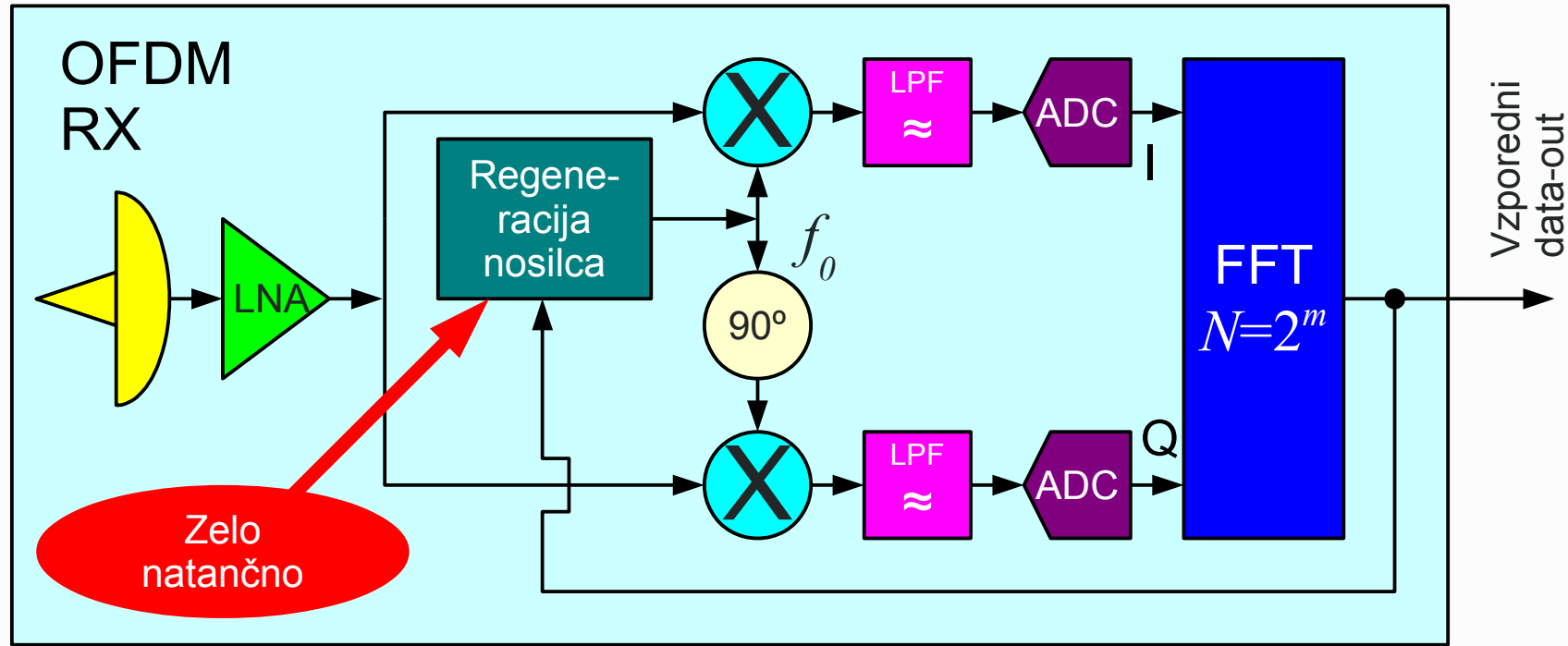
Večtonski modem kot protiukrep za popačenje večpotja



Neobčutljiv na večpotje

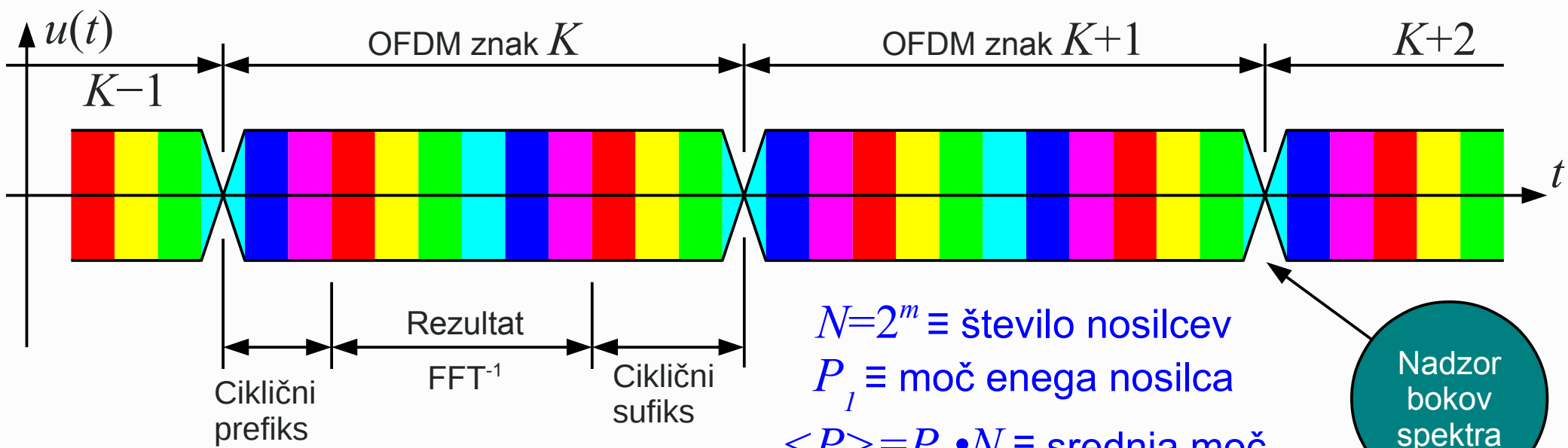
Vzporedni data-in:
BPSK, QPSK,
QAM znaki

WiFi
DVB-T
LTE



Zelo natančno

Orthogonal Frequency-Division Multiplex (OFDM)



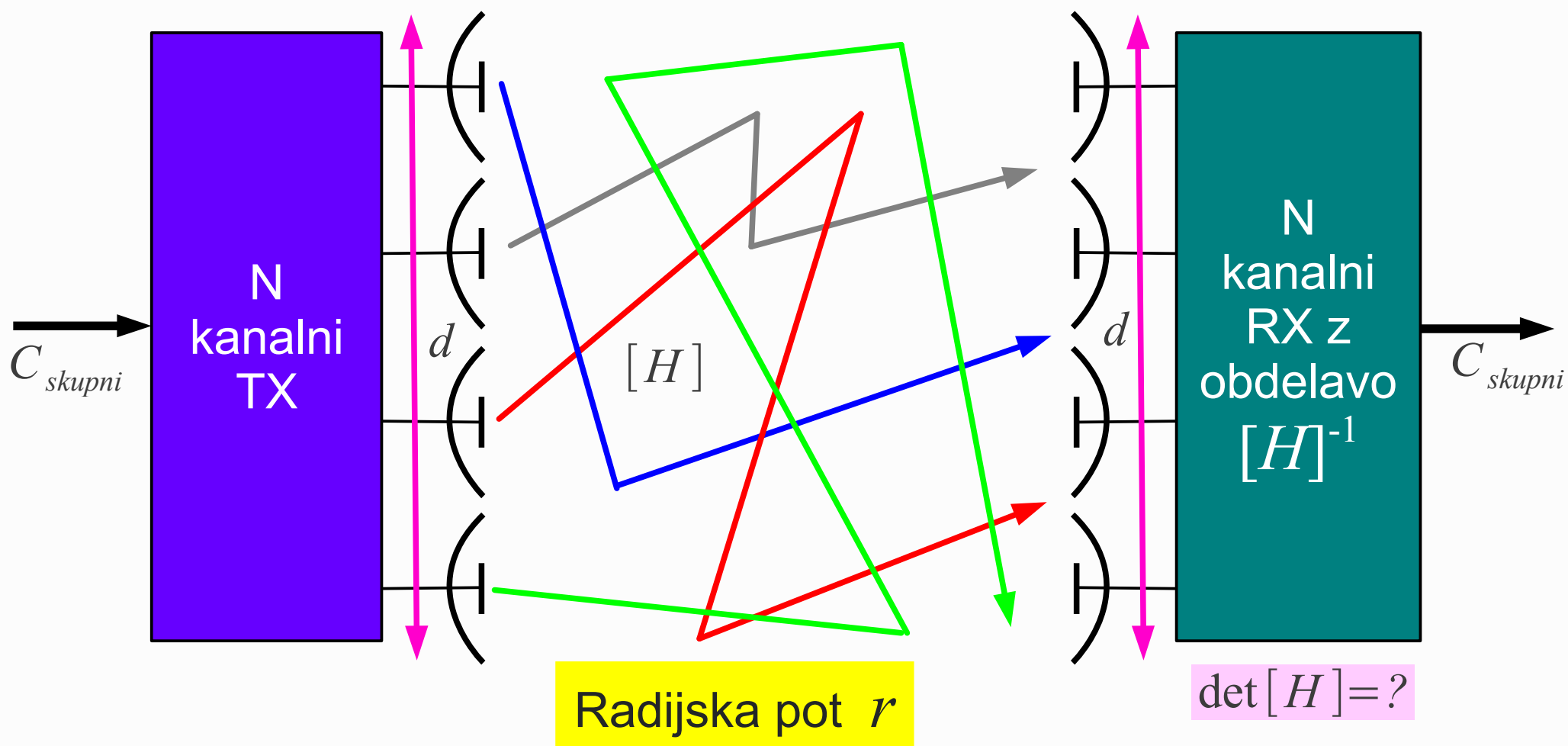
$N=2^m \equiv$ število nosilcev
 $P_1 \equiv$ moč enega nosilca
 $\langle P \rangle = P_1 \cdot N \equiv$ srednja moč
 $P_{MAX} = P_1 \cdot N^2 \equiv$ vršna moč

$$t_{prefiks} + t_{sufiks} \geq \Delta t_{večpotja}$$

- Nastavljiva odpornost na $\Delta t_{večpotja}$
- Skoraj pravokoten frekvenčni spekter
- Zadošča šibek FEC
- Spektralni izkoristek C/B dosega teoretske vrednosti BPSK, QPSK, QAM
- Omogoča enofrekvenčna omrežja SFN (Single-Frequency Network)

- Visoko razmerje $P_{MAX} / \langle P \rangle = N$ pogojuje slab izkoristek oddajnika $\eta \approx 3\%$
- FFT zahteva $N \cdot \log_2 N$ računskih operacij
- Ozkopasovni nosilci zahtevajo visoko frekvenčno stabilnost $\Delta f \leq 10\% R/N$
- Preveliki znaki ~ 12000 bit ($N \approx 2000$, $C/B \approx 6$ bit) za nekatere protokole
- Ozkopasovne motnje rušijo sinhronizacijo

Lastnosti OFDM



Koristna uporaba večpotja!

$$C_{skupni} = N \cdot C_{kanala} = N \cdot B \cdot \log_2 \left(1 + \frac{P_s}{P_n} \right)$$

(+) visoka spektralna učinkovitost: $C/B \geq 10$ bit

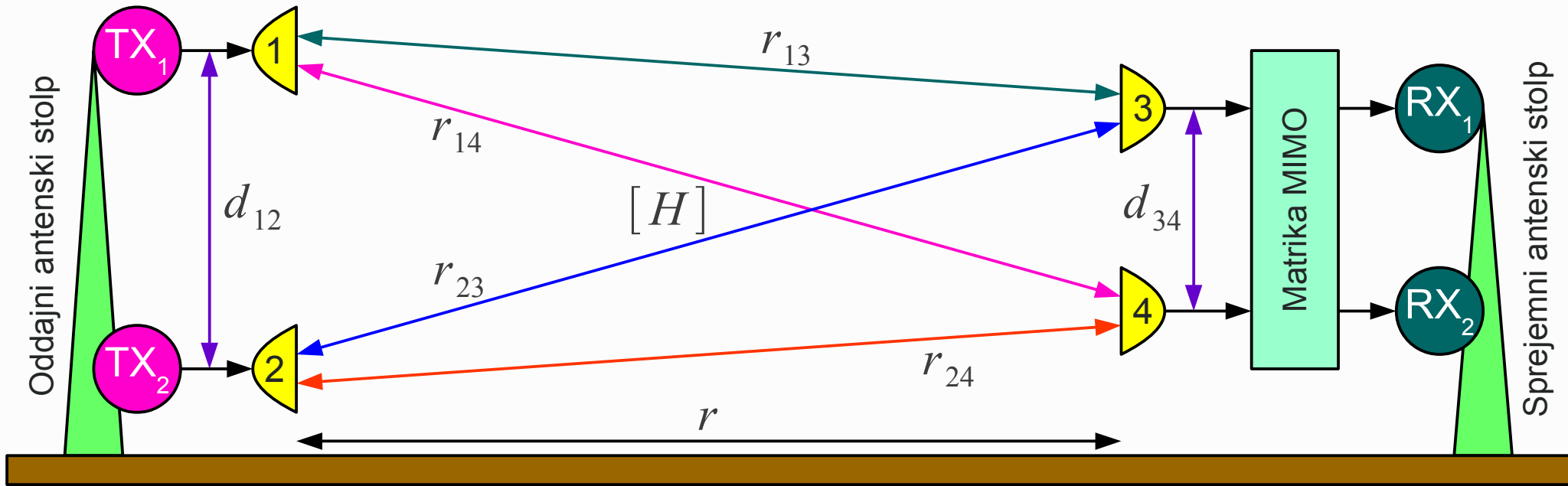
(-) zahteva N oddajnih anten in N sprejemnih anten

(+) preprosta rešitev MIMO 2x2: uporaba obeh polarizacij

(-) več kot dve polarizaciji $\det[H] \neq 0$ le na kratkih poteh $r \leq 2d^2/\lambda$

MIMO (Multiple-In Multiple-Out)

Podvojevanje C/B mikrovalovne zveze \equiv Line-Of-Sight MIMO



Pogoj za $\max \det[H]$: $r_{14} - r_{13} - r_{24} + r_{23} = \lambda/2$

$C/B > 40\text{bit}$

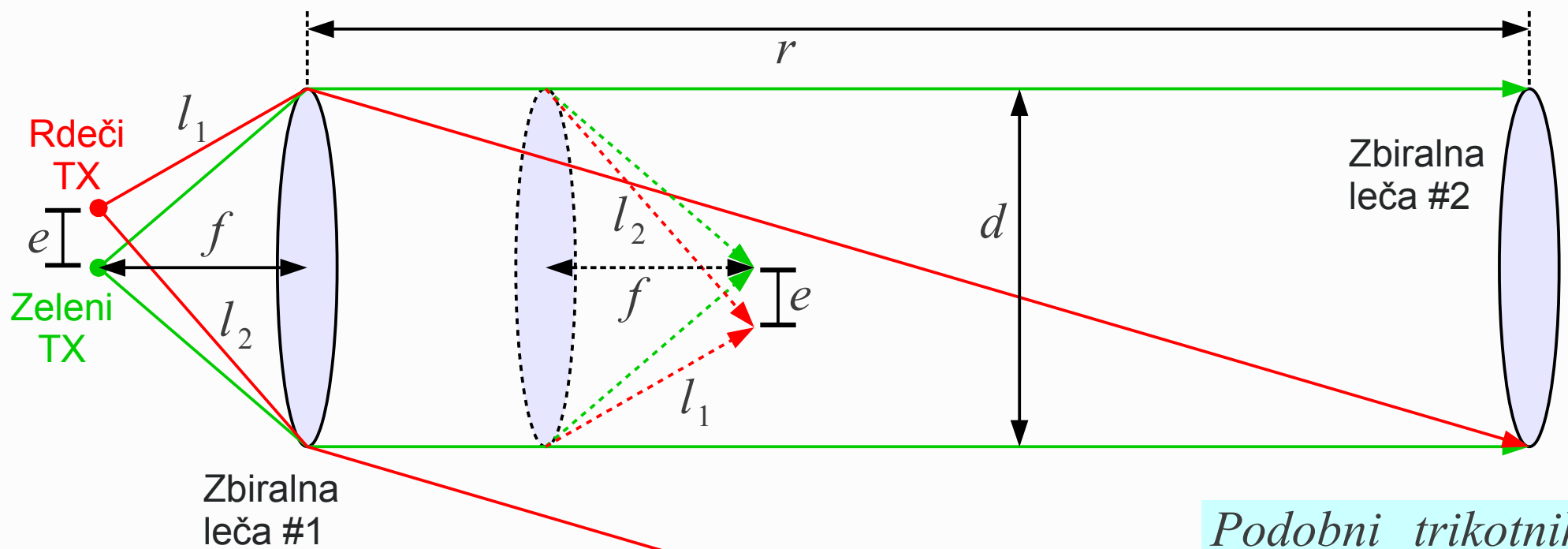
$$r_{13} = r_{24} = \sqrt{r^2 + \left(\frac{d_{12} - d_{34}}{2}\right)^2} \approx r + \frac{d_{12}^2 - 2d_{12}d_{34} + d_{34}^2}{8r}$$

$$r_{14} = r_{23} = \sqrt{r^2 + \left(\frac{d_{12} + d_{34}}{2}\right)^2} \approx r + \frac{d_{12}^2 + 2d_{12}d_{34} + d_{34}^2}{8r}$$

$$r_{14} - r_{13} - r_{24} + r_{23} \approx \frac{d_{12}d_{34}}{r} \rightarrow d_{12}d_{34} = r \cdot \lambda/2$$

MIMO brez večpotja

Zgled:
 $r = 10\text{km}$ $f = 15\text{GHz}$
 $\lambda = c_0/f = 2\text{cm}$
 $\langle d \rangle = \sqrt{d_{12}d_{34}}$
 $\langle d \rangle = \sqrt{r \cdot \lambda/2} = 10\text{m}$
Preizkus:
 $r = \frac{2\langle d \rangle^2}{\lambda} = 10\text{km}$



Podobni trikotniki

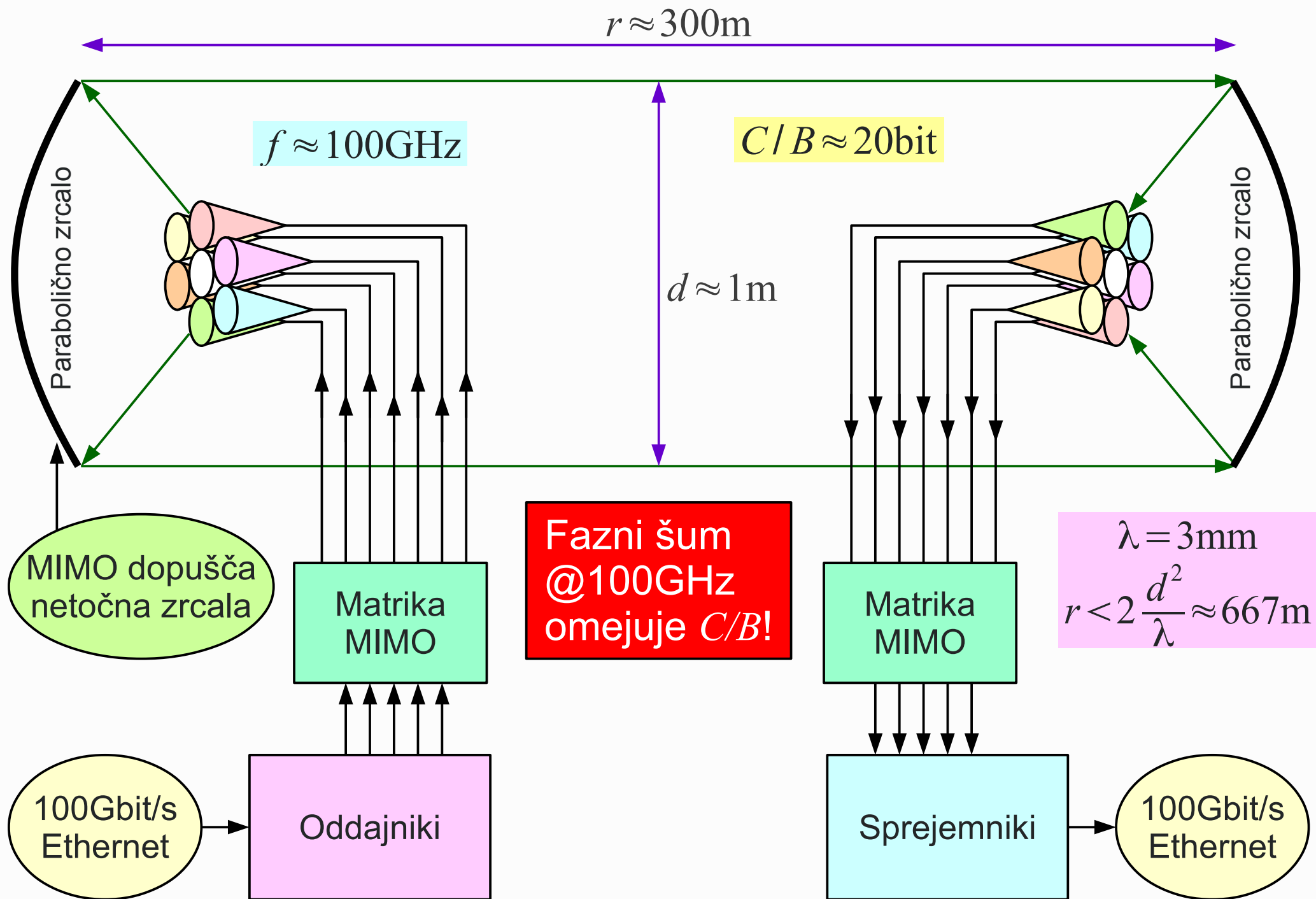
$$\frac{e}{f} = \frac{d}{r}$$

Točki razločimo, ko je razlika poti večja od $\lambda/2$:

$$l_2 - l_1 = \sqrt{f^2 + (d/2 + e)^2} - \sqrt{f^2 + (d/2 - e)^2} \approx \frac{de}{f} > \frac{\lambda}{2}$$

Pogoj $r < \frac{2d^2}{\lambda}$

Geometrijska optika



Visokozmogljiva zveza na kratko razdaljo