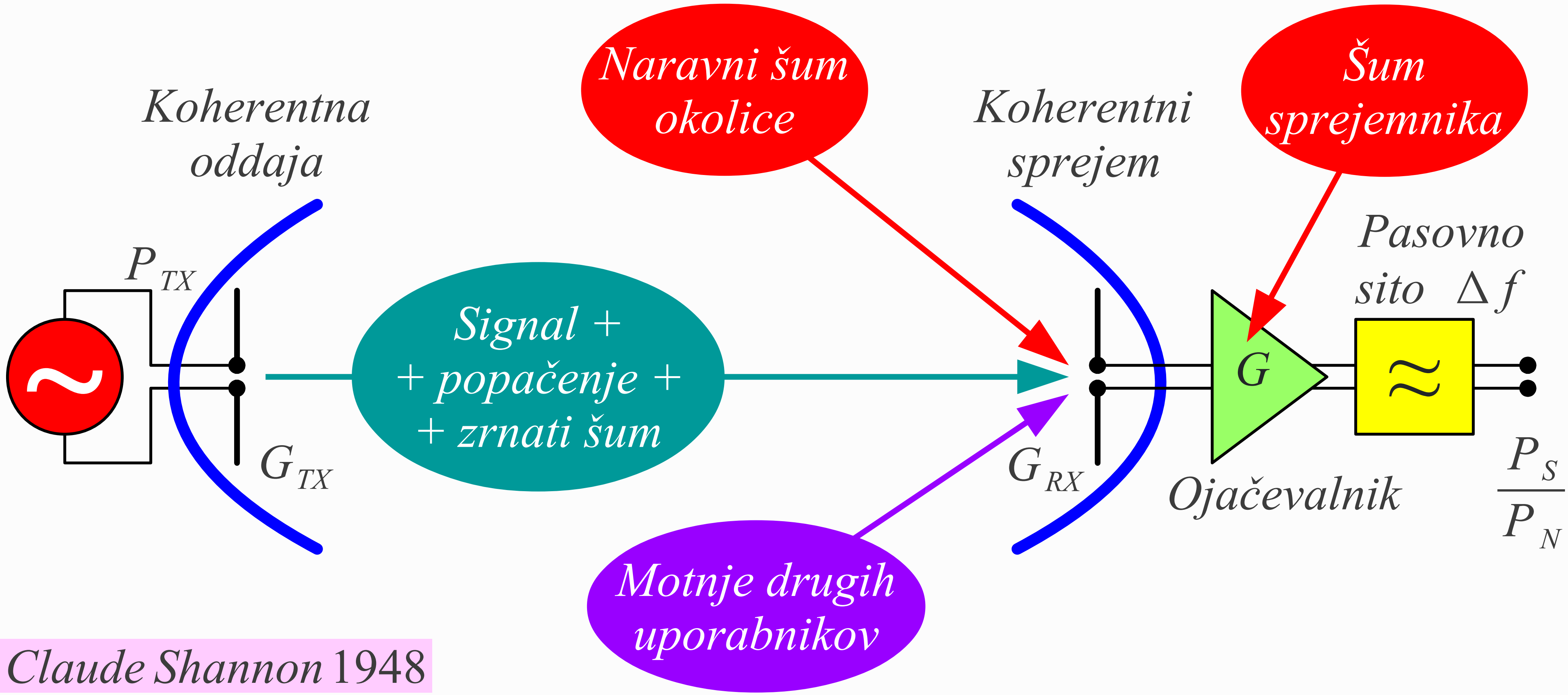
Šum



Claude Shannon 1948

$$C = \Delta f \log_2\left(1 + \frac{P_S}{P_N}\right) = \Delta f \log_2\left(1 + \frac{P_S}{P_{popačenja} + P_{zrnati} + P_{okolice} + P_{sprejemnika} + P_{motenj}}\right)$$

$\Delta f\,[\mathrm{Hz}] = B \equiv$ *pasovna širina*

$P_S\,[\mathrm{W}] \equiv$ *moč signala*

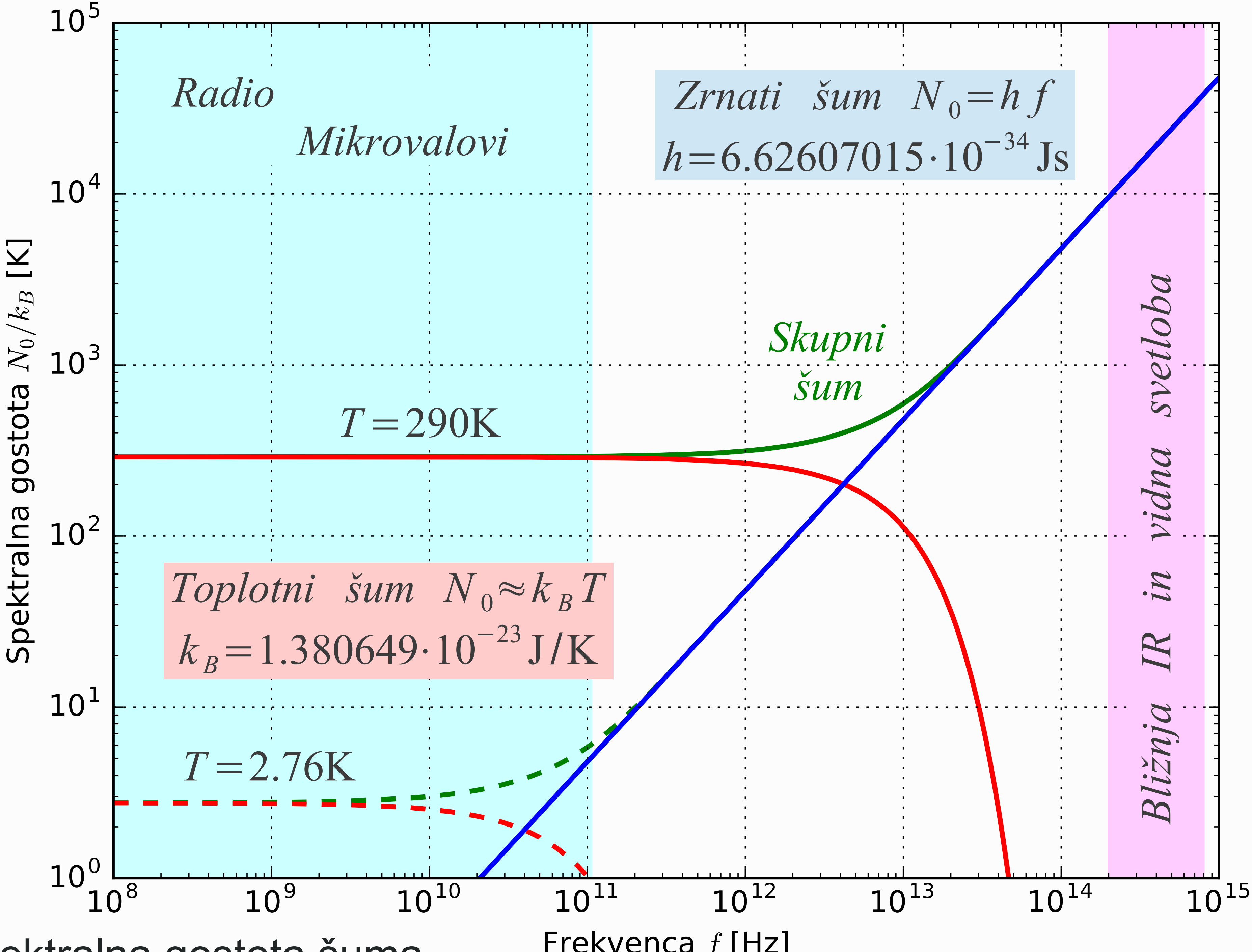
$C\,[\mathrm{bit/s}] \equiv$ *zmogljivost radijske zveze*

$P_N\,[\mathrm{W}] = \Delta f \cdot N_0 \equiv$ *moč šuma*

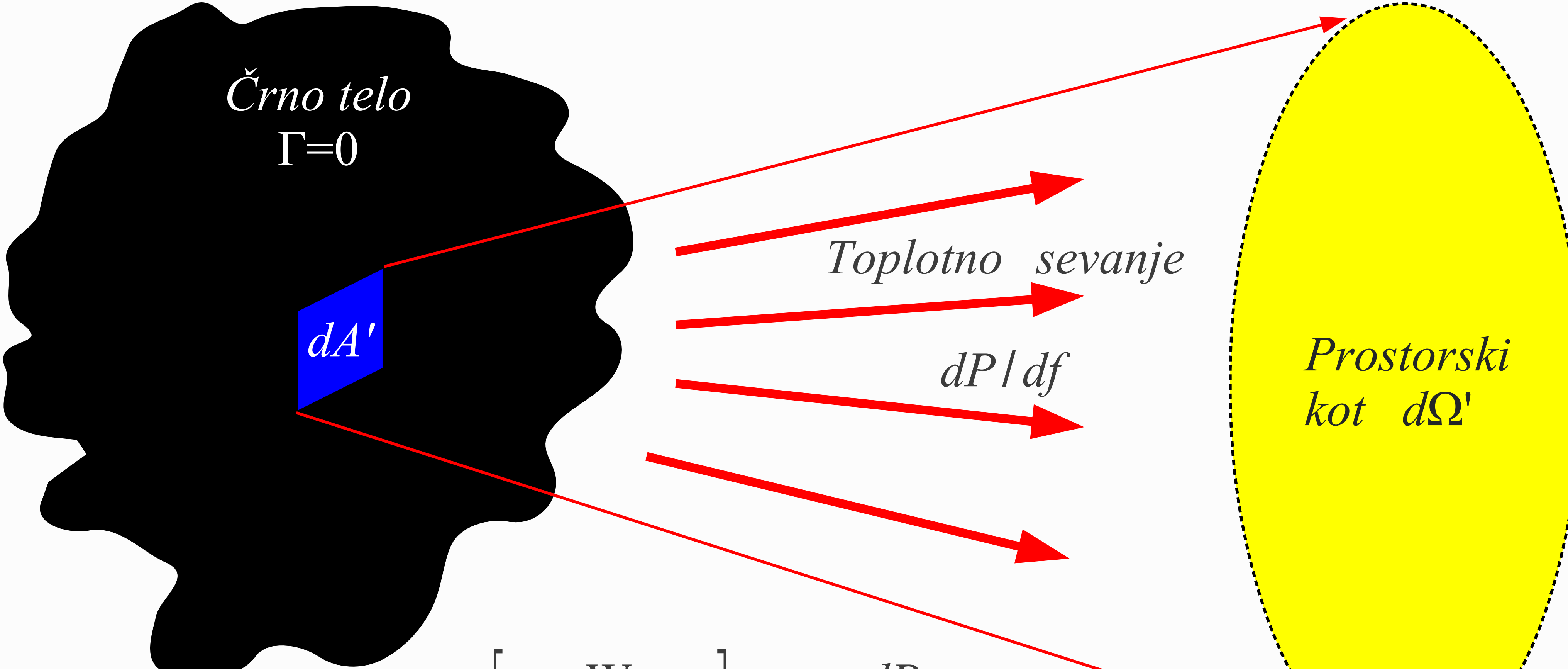
$N_0\,[\mathrm{W/Hz} = \mathrm{J}] \equiv$ *spektralna gostota šuma*

Zmogljivost radijske zveze

$P_N = \Delta f \, N_0 \quad \Delta f \equiv pasovna \; širina \quad N_0 \equiv spektralna \; gostota \; šuma$



Spektralna gostota šuma



Spektralna svetlost $B_f\left[\dfrac{\mathrm{W}}{\mathrm{Hz\,m^2\,srd}}\right]=\dfrac{dP}{df\,dA'\,d\Omega'}$

Planckov zakon $B_f(f,T)=\dfrac{2hf^3}{c_0^2}\cdot\dfrac{1}{e^{\frac{hf}{k_B T}}-1}$

Prazen prostor ϵ_0,μ_0

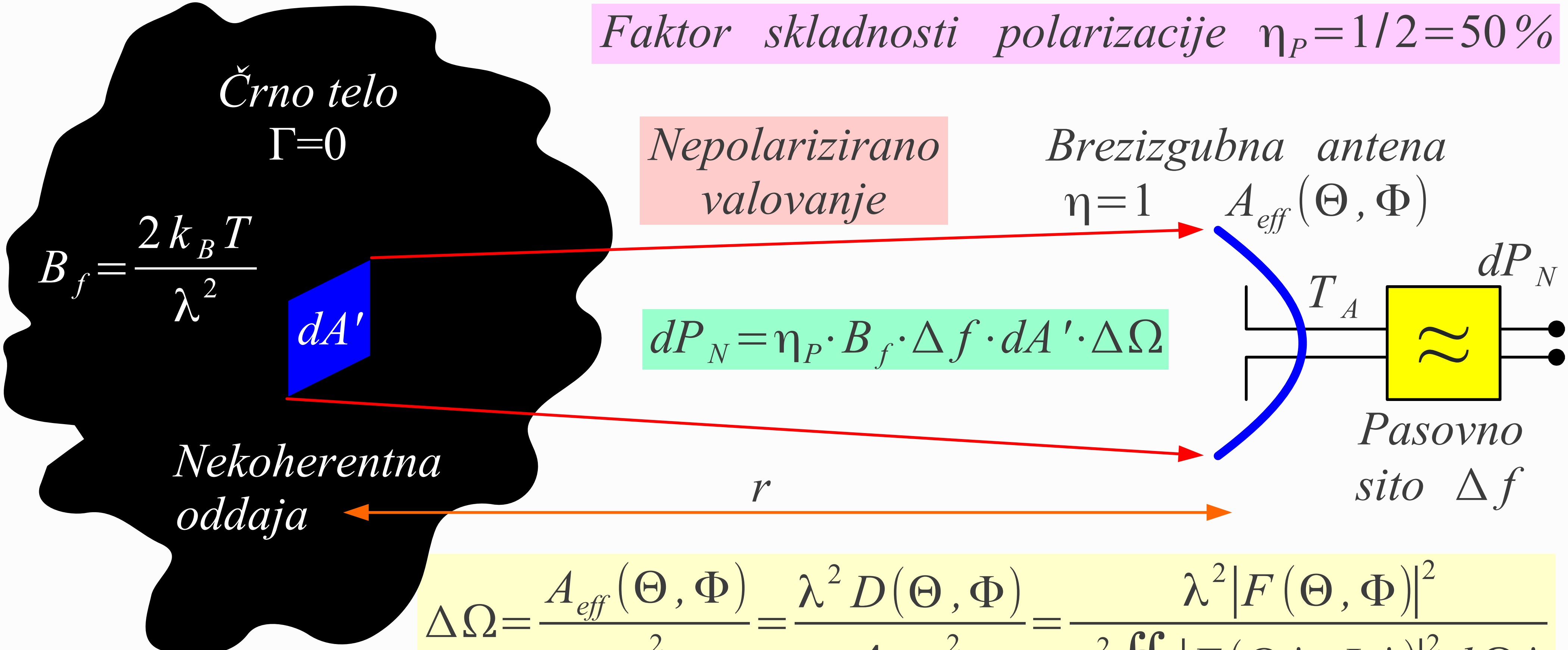
$c_0=299792458\,\mathrm{m/s}\approx 3\cdot 10^8\,\mathrm{m/s}$

$hf \ll k_B T \;\rightarrow$ *Rayleigh−Jeansov približek* $B_f(f,T)\approx\dfrac{2k_B T f^2}{c_0^2}=\dfrac{2k_B T}{\lambda^2}$

Toplotno sevanje črnega telesa

Sprejeta moč toplotnega šuma

Faktor skladnosti polarizacije $\eta_P = 1/2 = 50\,\%$



$$B_f = \frac{2 k_B T}{\lambda^2}$$

$$dP_N = \eta_P \cdot B_f \cdot \Delta f \cdot dA' \cdot \Delta\Omega$$

$$\Delta\Omega = \frac{A_{eff}(\Theta,\Phi)}{r^2} = \frac{\lambda^2 D(\Theta,\Phi)}{4\pi r^2} = \frac{\lambda^2 |F(\Theta,\Phi)|^2}{r^2 \oiint_{4\pi} |F(\Theta^*,\Phi^*)|^2 d\Omega^*}$$

$$P_N = \oiint_{A'} \frac{1}{2} \cdot B_f \cdot \Delta f \cdot dA' \cdot \Delta\Omega$$

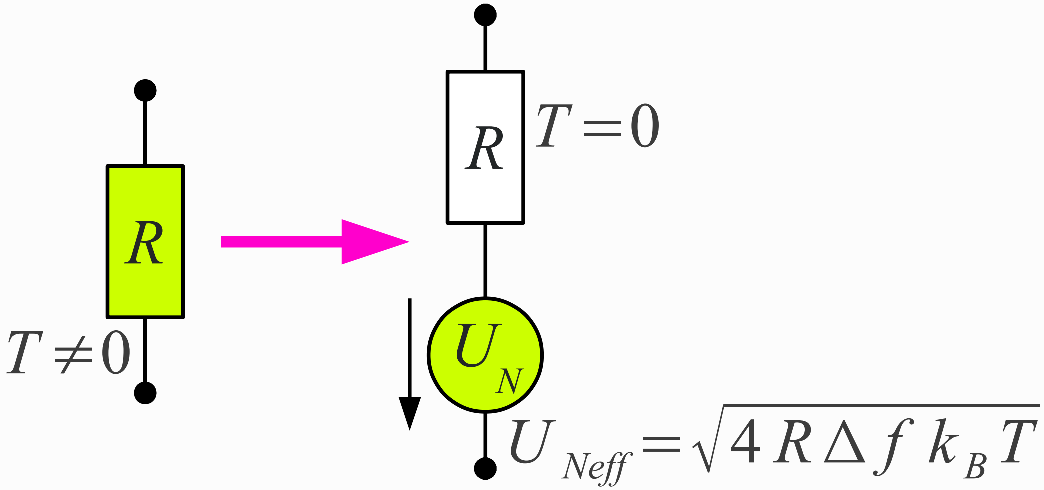
$$dA' = r^2 d\Omega$$

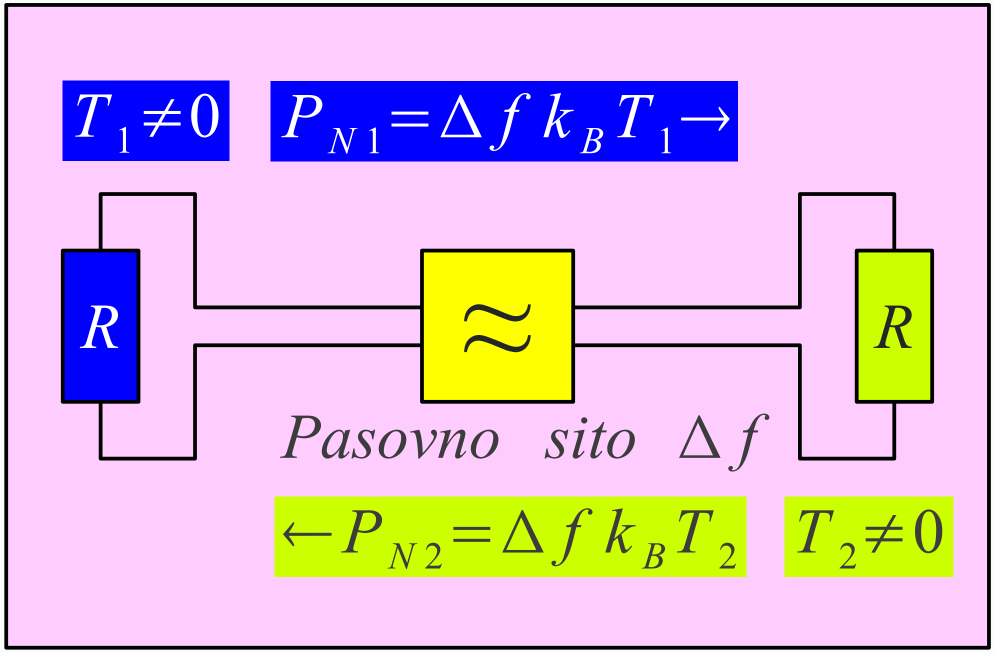
$$P_N = \Delta f\, k_B \frac{\oiint_{4\pi} T(\Theta,\Phi) |F(\Theta,\Phi)|^2 d\Omega}{\oiint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

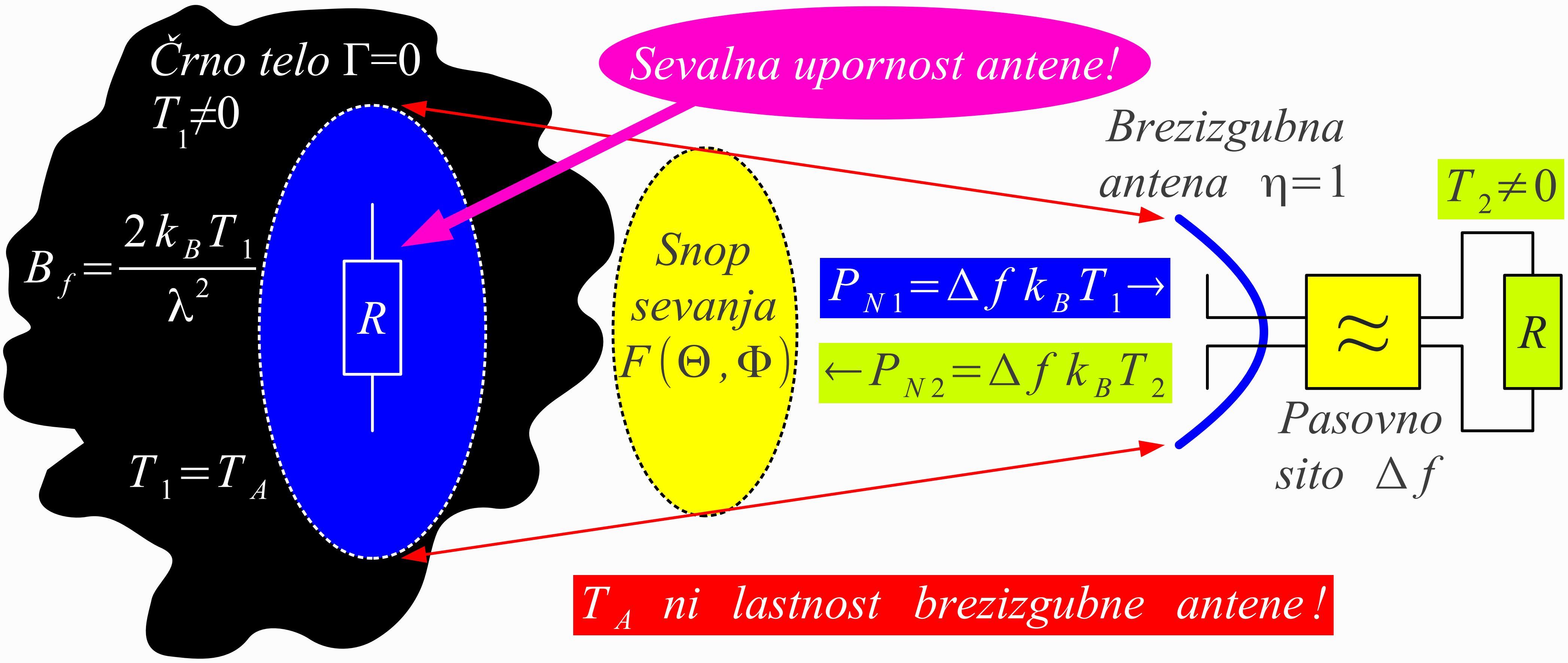
$$P_N = \Delta f\, N_0 = \Delta f\, k_B T_A$$

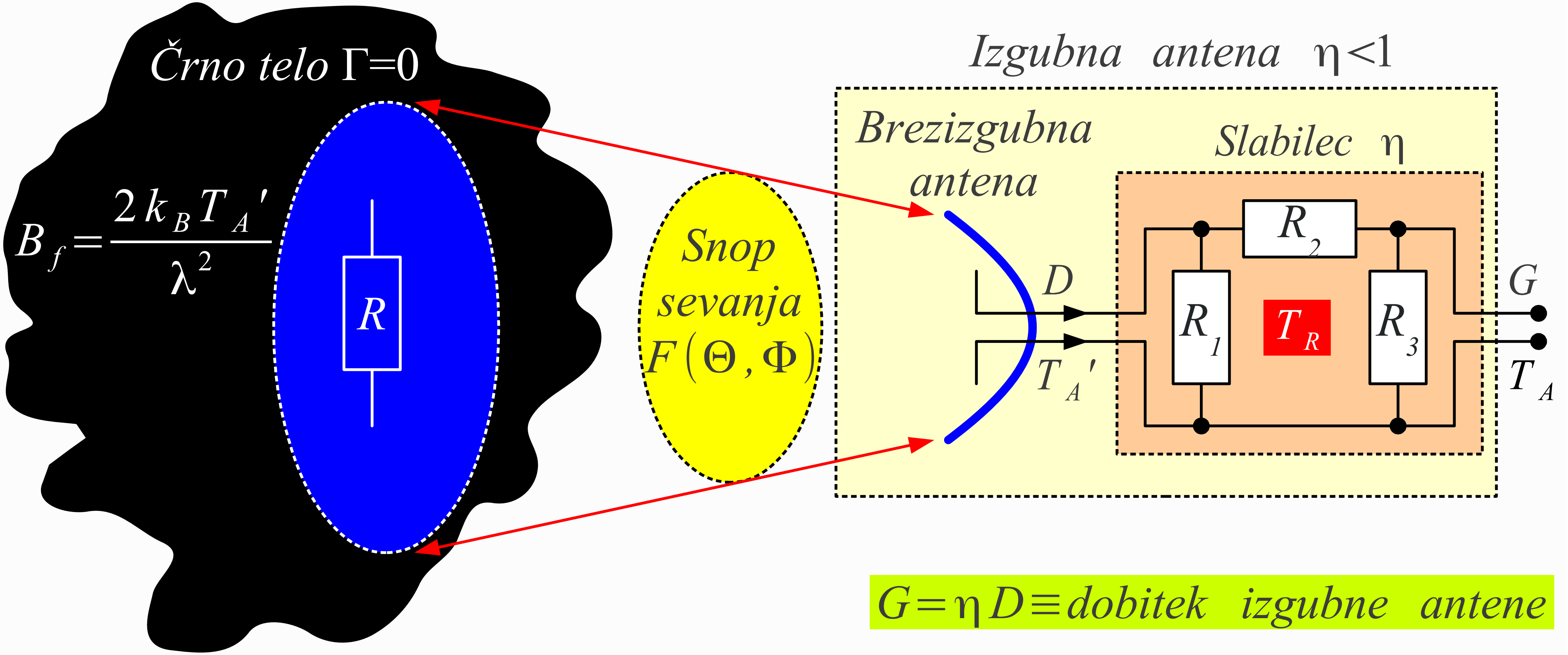
$$T_A = \frac{\oiint_{4\pi} T(\Theta,\Phi) |F(\Theta,\Phi)|^2 d\Omega}{\oiint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

Toplotno ravnovesje









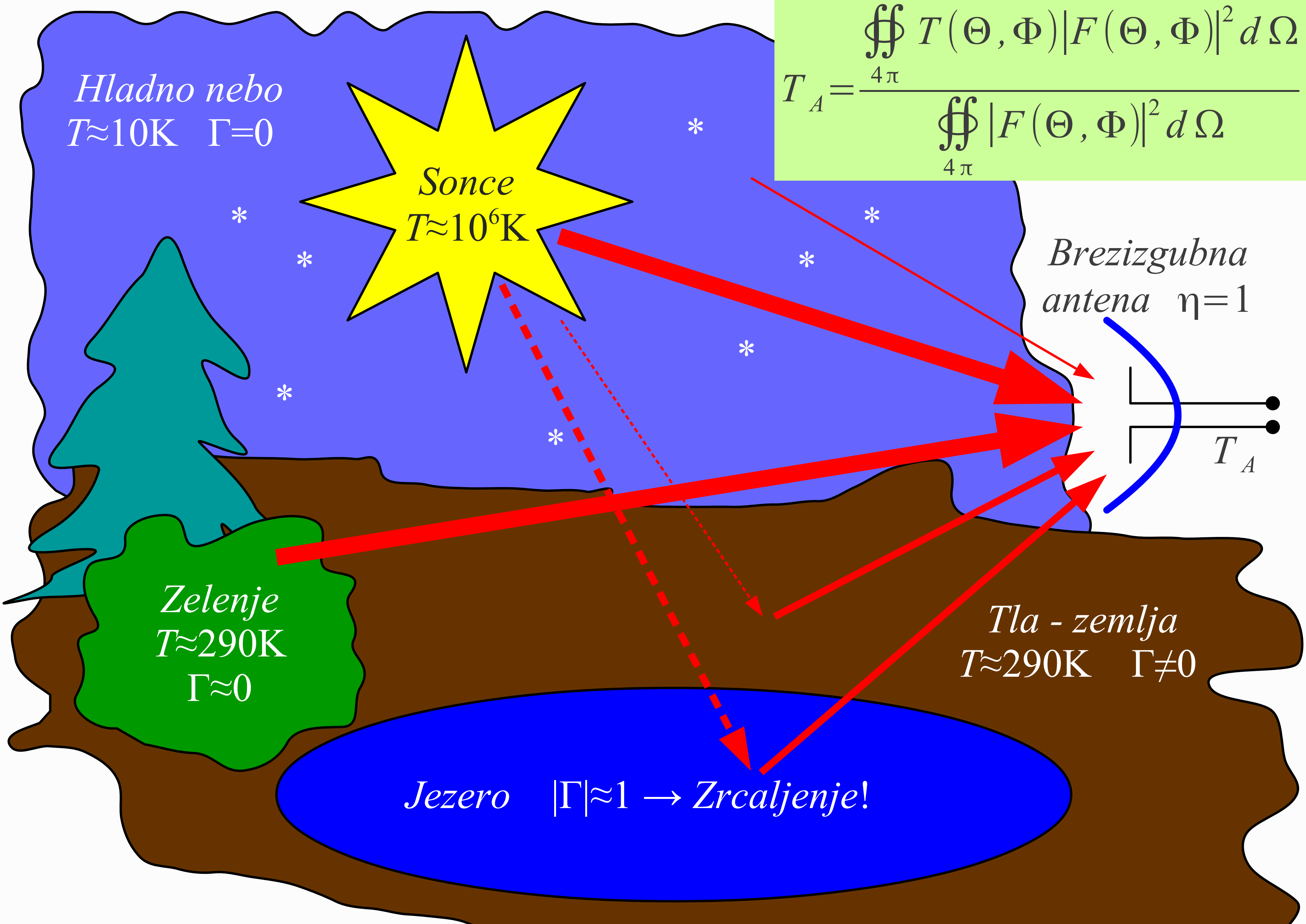
$G = \eta D \equiv$ *dobitek izgubne antene*

$T_A = \eta T_A' + (1-\eta) T_R \equiv$ *šumna temperatura izgubne antene*

$T_R \approx 290\mathrm{K} \equiv$ *temperatura slabilca*

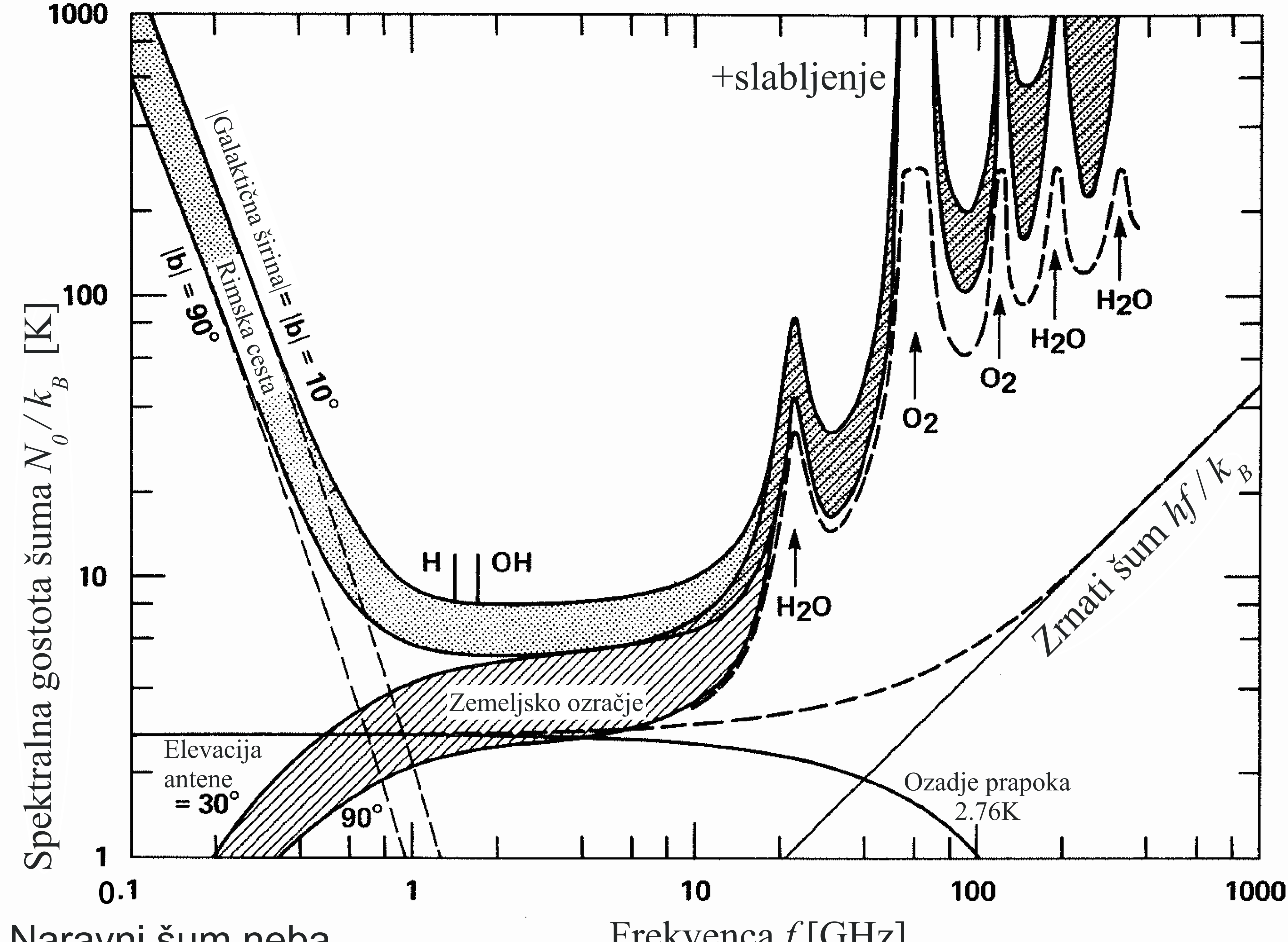
$$T_A = \eta \left[\frac{\oiint_{4\pi} T(\Theta,\Phi) |F(\Theta,\Phi)|^2 d\Omega}{\oiint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega} \right] + (1-\eta) T_R$$

Dobitek in šumna temperatura izgubne antene



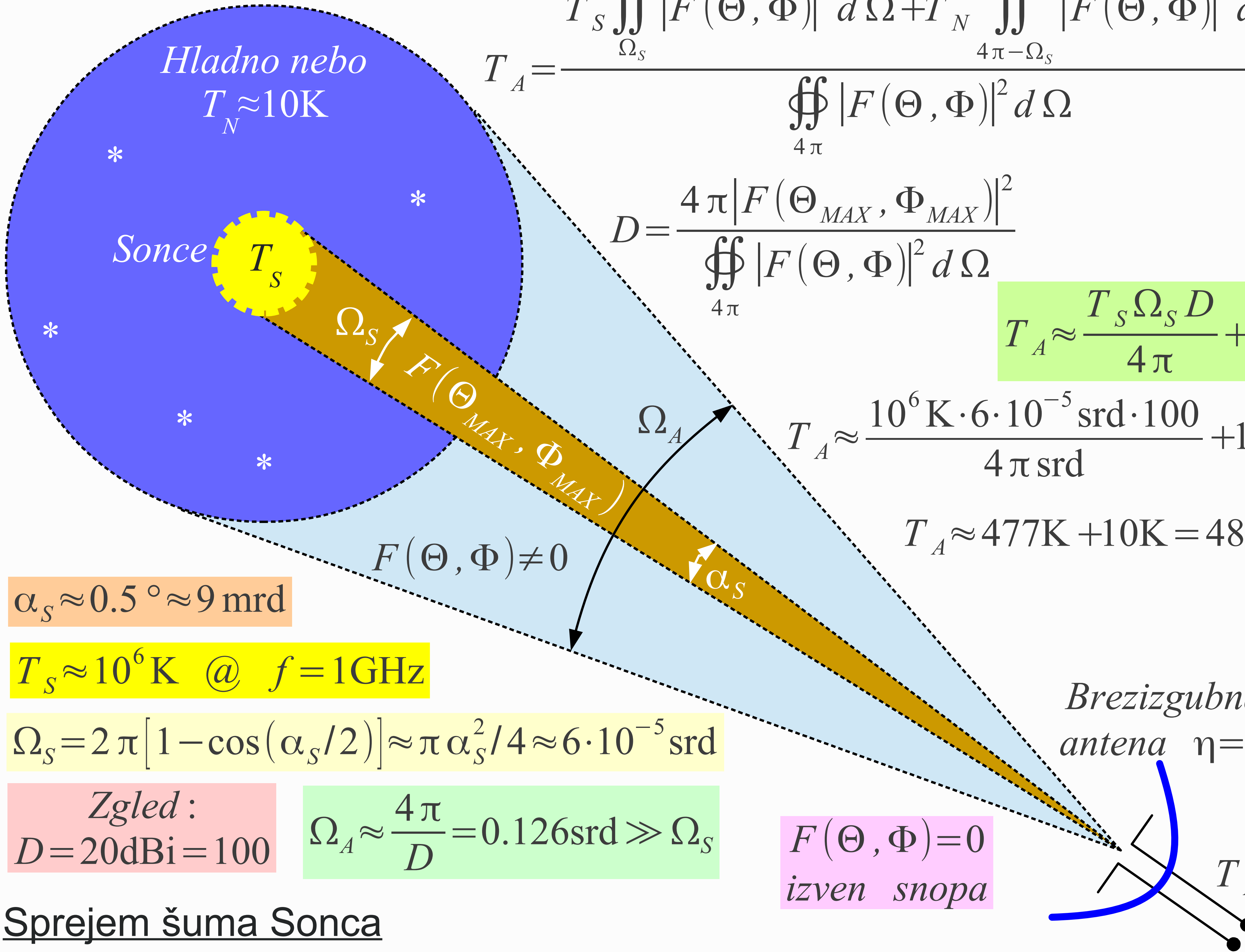
$$T_A = \frac{\oiint_{4\pi} T(\Theta,\Phi)\left|F(\Theta,\Phi)\right|^2 d\Omega}{\oiint_{4\pi} \left|F(\Theta,\Phi)\right|^2 d\Omega}$$

Naravni izvori šuma



Naravni šum neba

Sprejem šuma Sonca



$$T_A = \frac{T_S \iint_{\Omega_S} |F(\Theta,\Phi)|^2 d\Omega + T_N \iint_{4\pi-\Omega_S} |F(\Theta,\Phi)|^2 d\Omega}{\oiint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

$$D = \frac{4\pi |F(\Theta_{MAX}, \Phi_{MAX})|^2}{\oiint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

$$T_A \approx \frac{T_S \Omega_S D}{4\pi} + T_N$$

$$T_A \approx \frac{10^6\,\text{K} \cdot 6 \cdot 10^{-5}\,\text{srd} \cdot 100}{4\pi\,\text{srd}} + 10\text{K}$$

$$T_A \approx 477\text{K} + 10\text{K} = 487\text{K}$$

$\alpha_S \approx 0.5° \approx 9\,\text{mrd}$

$T_S \approx 10^6\,\text{K} \quad @ \quad f = 1\text{GHz}$

$\Omega_S = 2\pi[1 - \cos(\alpha_S/2)] \approx \pi\alpha_S^2/4 \approx 6 \cdot 10^{-5}\,\text{srd}$

Zgled:
$D = 20\text{dBi} = 100$

$\Omega_A \approx \frac{4\pi}{D} = 0.126\text{srd} \gg \Omega_S$

$F(\Theta, \Phi) = 0$
izven snopa

$S_f\,[\mathrm{W/m^2/Hz}] \equiv$ *spektralna gostota pretoka moči*

$$\Delta P_N = \frac{1}{2} S_f \, \Delta f \, A_{eff}$$

$$S_f = 2 k_B \frac{T_Z \Omega_Z}{\lambda^2} = B_f \Omega_Z$$

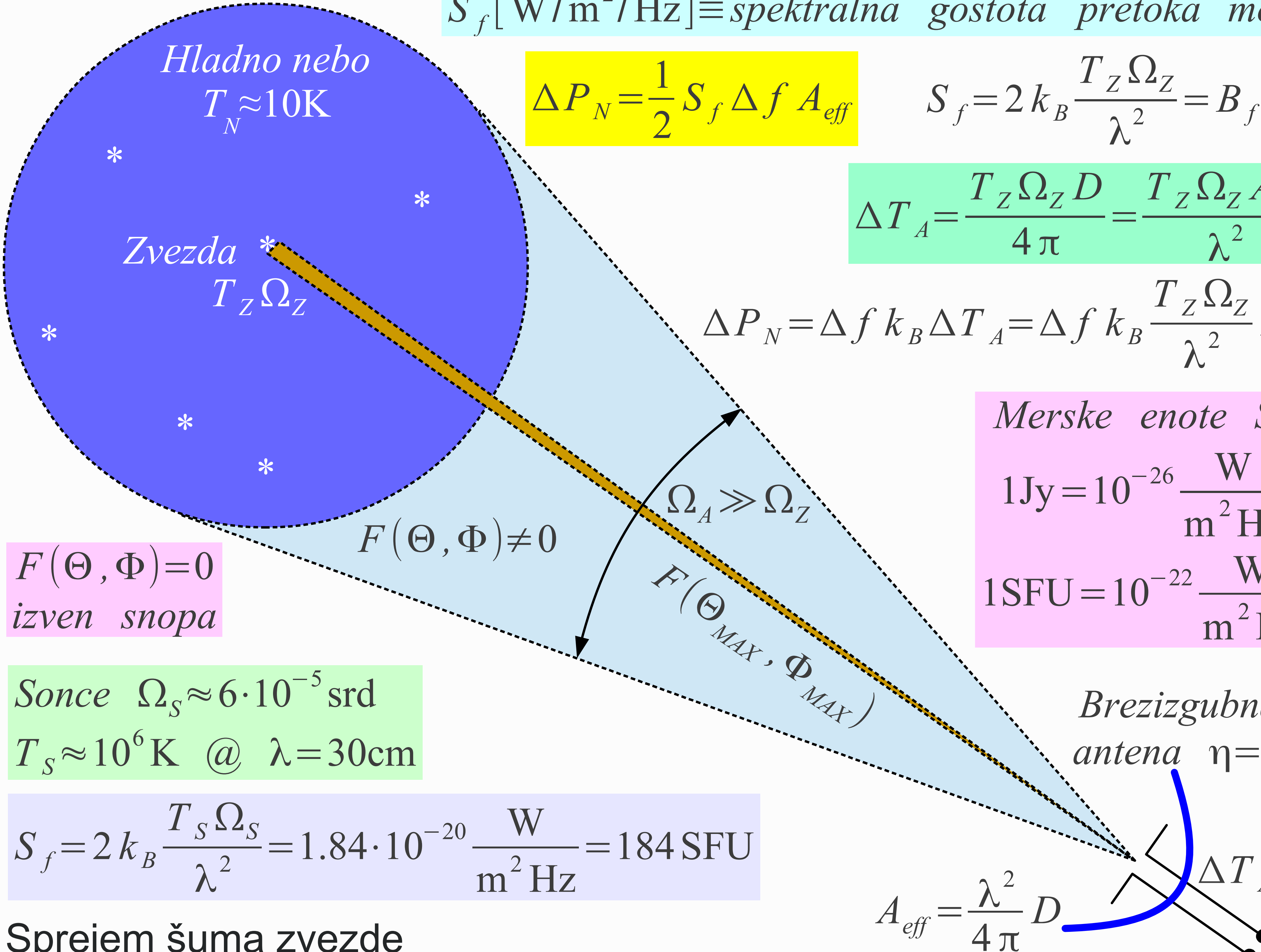
$$\Delta T_A = \frac{T_Z \Omega_Z D}{4\pi} = \frac{T_Z \Omega_Z A_{eff}}{\lambda^2}$$

$$\Delta P_N = \Delta f \, k_B \Delta T_A = \Delta f \, k_B \frac{T_Z \Omega_Z}{\lambda^2} A_{eff}$$

Merske enote S_f

$$1\mathrm{Jy} = 10^{-26} \frac{\mathrm{W}}{\mathrm{m^2 Hz}}$$

$$1\mathrm{SFU} = 10^{-22} \frac{\mathrm{W}}{\mathrm{m^2 Hz}}$$



$F(\Theta, \Phi) = 0$ *izven snopa*

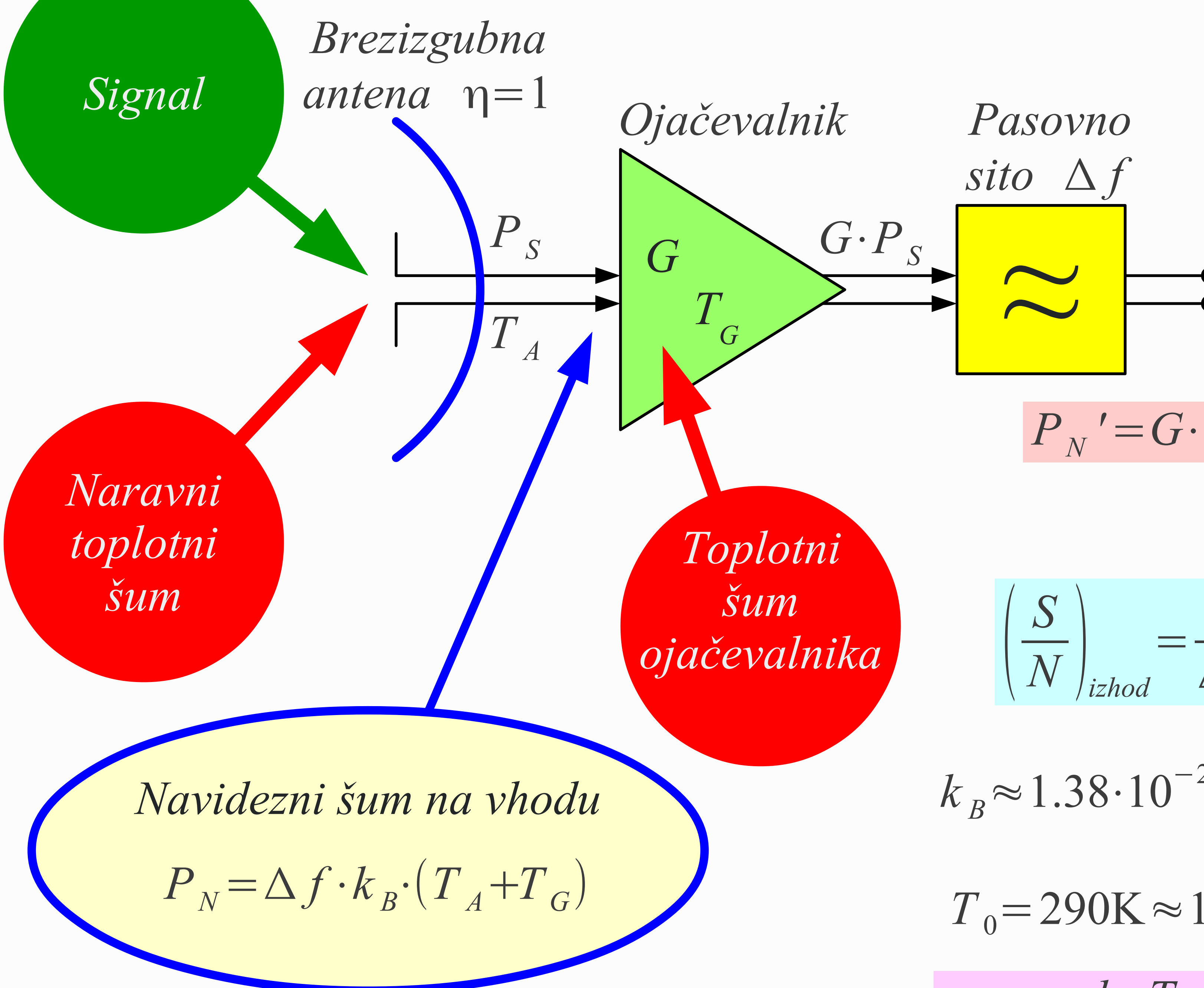
Sonce $\Omega_S \approx 6 \cdot 10^{-5}\,\mathrm{srd}$

$T_S \approx 10^6\,\mathrm{K}$ @ $\lambda = 30\,\mathrm{cm}$

$$S_f = 2 k_B \frac{T_S \Omega_S}{\lambda^2} = 1.84 \cdot 10^{-20} \frac{\mathrm{W}}{\mathrm{m^2 Hz}} = 184\,\mathrm{SFU}$$

Sprejem šuma zvezde

$T_S \equiv temperatura\ \ ojačevalnika\ \ preračunana\ \ na\ \ vhod\,!$



$$P_S' = G \cdot P_S$$

$$\left(\frac{S}{N}\right)_{izhod} = \frac{P_S'}{P_N'}$$

$$P_N' = G \cdot \Delta f \cdot k_B \cdot (T_A + T_G)$$

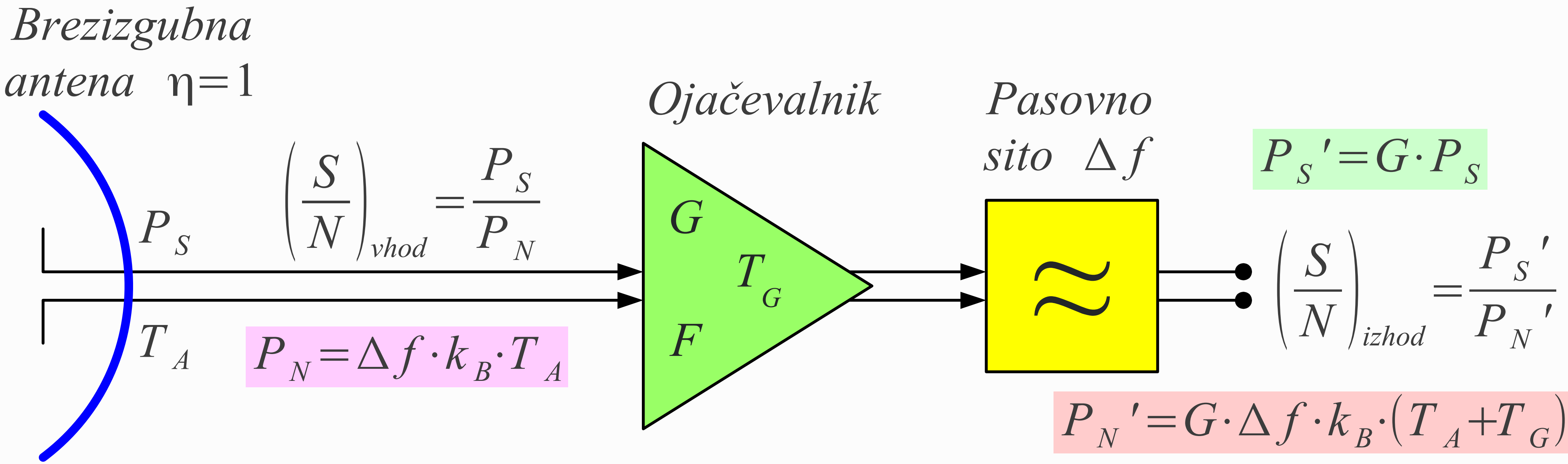
$$\left(\frac{S}{N}\right)_{izhod} = \frac{P_S}{\Delta f \cdot k_B \cdot (T_A + T_G)}$$

$$k_B \approx 1.38 \cdot 10^{-23}\ \mathrm{J/K}$$

$$T_0 = 290\mathrm{K} \approx 17\,^\circ\mathrm{C}$$

$$10 \log_{10} \frac{k_B T_0}{1\mathrm{mJ}} \approx -174\ \mathrm{dBm/Hz}$$

Razmerje signal/šum sprejemnika



Nesmiselna definicija šumnega števila:

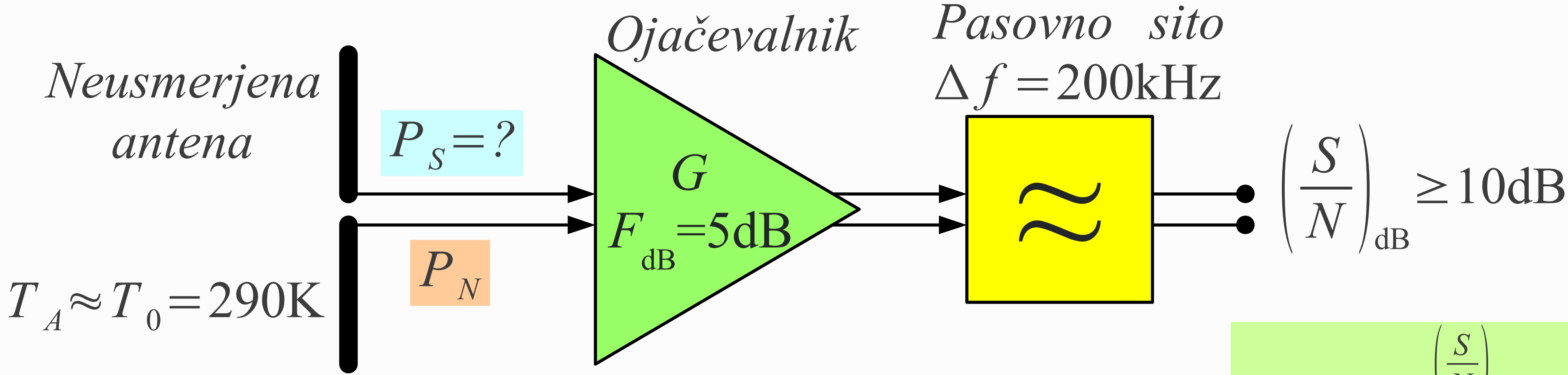
$$F=\frac{\left(\frac{S}{N}\right)_{vhod}}{\left(\frac{S}{N}\right)_{izhod}}=\frac{\frac{P_S}{\Delta f\, k_B T_A}}{\frac{G\,P_S}{G\,\Delta f\, k_B(T_A+T_G)}}=\frac{T_A+T_G}{T_A}=1+\frac{T_G}{T_A}$$

Lastnost ojačevalnika ne more biti funkcija T_A!

Smiselna definicija $F=1+\frac{T_G}{T_0}$ @ $T_0=290\mathrm{K}\approx 17\,^\circ\mathrm{C}$ $\longleftrightarrow$ $T_G=T_0(F-1)$

Logaritemske enote $F_{\mathrm{dB}}=10\log_{10}F=10\log_{10}\left(1+\frac{T_G}{T_0}\right)$ $\longleftrightarrow$ $T_G=T_0\left(10^{\frac{F_{\mathrm{dB}}}{10}}-1\right)$

Šumno število ojačevalnika



$$T_G = T_0 \cdot \left(10^{\frac{F_{\text{dB}}}{10}} - 1\right) = 290\text{K} \cdot (3.162 - 1) = 627\text{K}$$

$$\left(\frac{S}{N}\right) = 10^{\frac{\left(\frac{S}{N}\right)_{\text{dB}}}{10}} \geq 10$$

$$P_N = \Delta f \cdot k_B \cdot (T_A + T_G) = 200\text{kHz} \cdot 1.38 \cdot 10^{-23}\,\text{J/K} \cdot (290\text{K} + 627\text{K}) = 2.53 \cdot 10^{-15}\,\text{W}$$

$$P_S = P_N \cdot \left(\frac{S}{N}\right) = P_N \cdot 10 = 2.53 \cdot 10^{-14}\,\text{W}$$

$$P_{S\,\text{dBm}} = 10 \log_{10} \frac{P_S}{1\text{mW}} = -106\text{dBm}$$

Poenostavljen izračun izključno v primeru $T_A \approx T_0 = 290\text{K}$

$$P_{S\,\text{dBm}} \approx (S/N)_{\text{dB}} + (\Delta f)_{\text{dB}\cdot\text{Hz}} + (k_B T_0)_{\text{dBm/Hz}} + F_{\text{dB}}$$

$$(k_B T_0)_{\text{dBm/Hz}} = -174\text{dBm/Hz}$$

$$(\Delta f)_{\text{dB}\cdot\text{Hz}} = 10 \log_{10}\left(\frac{\Delta f}{1\text{Hz}}\right) = 53\text{dB}\cdot\text{Hz}$$

$$P_{S\,\text{dBm}} \approx 10\text{dB} + 53\text{dB}\cdot\text{Hz} - 174\text{dBm/Hz} + 5\text{dB} = -106\text{dBm}$$

Občutljivost GSM telefona

Dva različna sprejemnika #1 in #2:

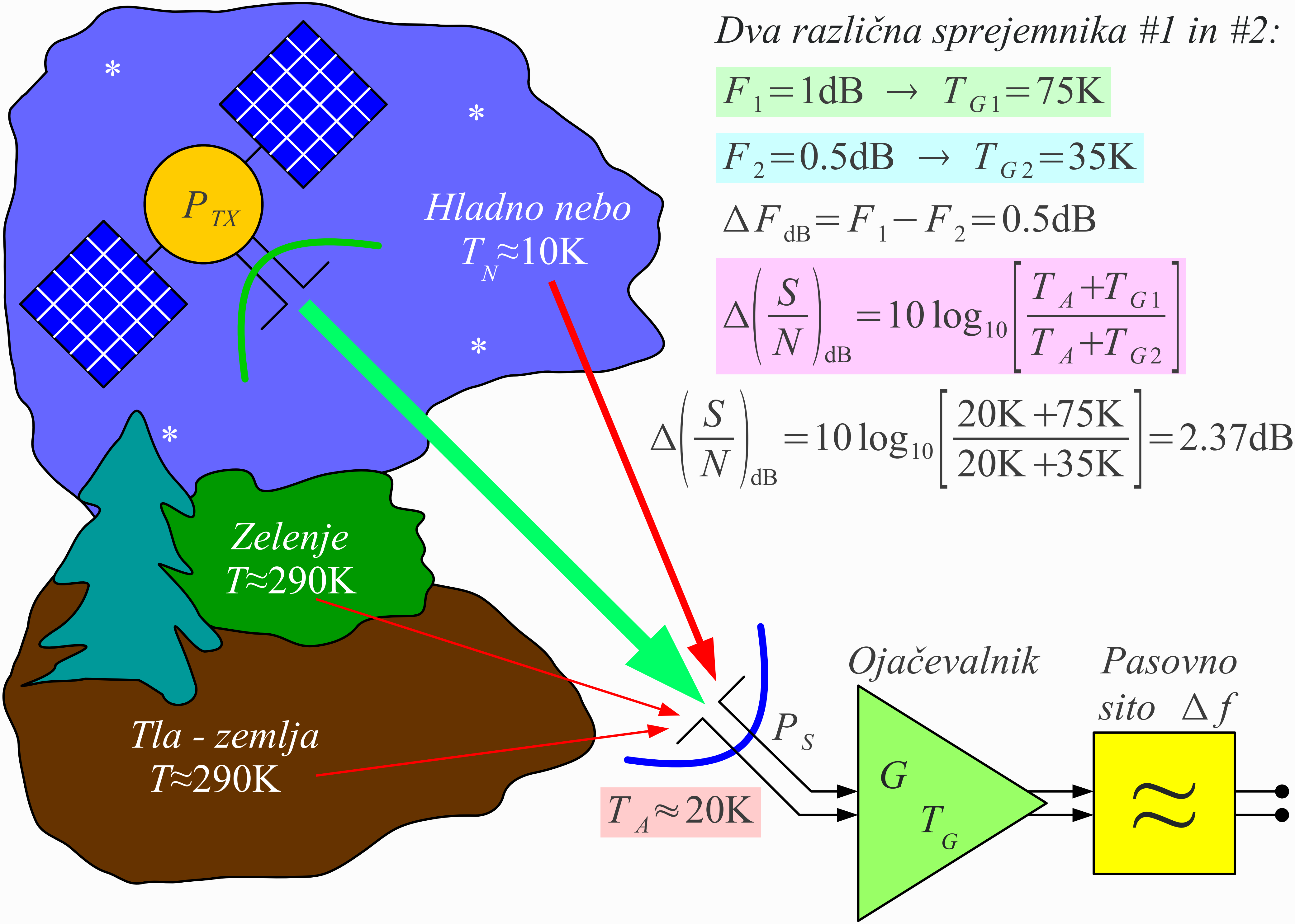
$$F_1 = 1\text{dB} \rightarrow T_{G1} = 75\text{K}$$

$$F_2 = 0.5\text{dB} \rightarrow T_{G2} = 35\text{K}$$

$$\Delta F_{\text{dB}} = F_1 - F_2 = 0.5\text{dB}$$

$$\Delta\left(\frac{S}{N}\right)_{\text{dB}} = 10\log_{10}\left[\frac{T_A + T_{G1}}{T_A + T_{G2}}\right]$$

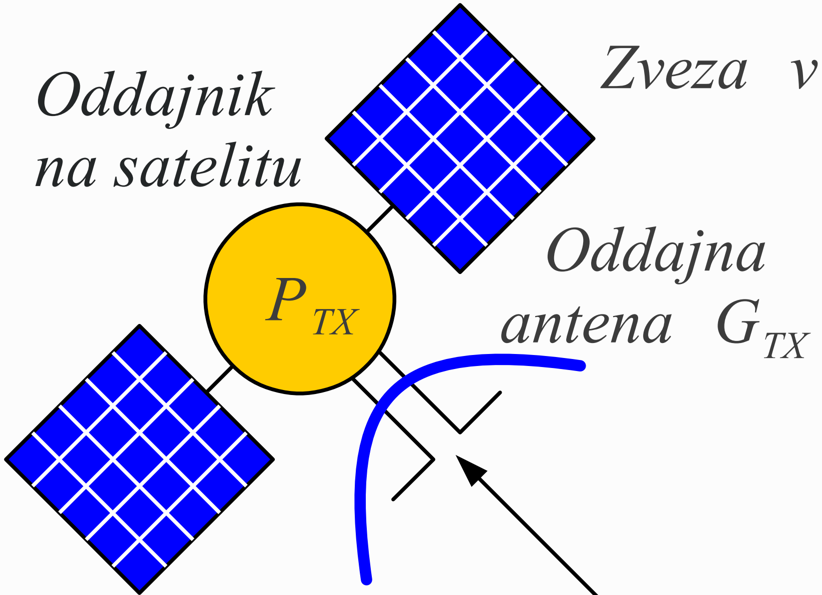
$$\Delta\left(\frac{S}{N}\right)_{\text{dB}} = 10\log_{10}\left[\frac{20\text{K} + 75\text{K}}{20\text{K} + 35\text{K}}\right] = 2.37\text{dB}$$



Spremembi F in S/N pri satelitski TV

Razmerje G/T sprejemne postaje

Zveza v praznem prostoru $P_S = P_{TX} \cdot G_{TX} \cdot G_{RX} \cdot \left(\frac{\lambda}{4\pi r} \right)^2$



$$\left(\frac{S}{N} \right)_{izhod} = P_{TX} \cdot G_{TX} \cdot \frac{1}{\Delta f \cdot k_B} \cdot \left(\frac{\lambda}{4\pi r} \right)^2 \cdot \frac{G_{RX}}{(T_A + T_G)}$$

Oddajnik: $P_{TX} \cdot G_{TX}$

Sistem: $\frac{1}{\Delta f \cdot k_B} \cdot \left(\frac{\lambda}{4\pi r} \right)^2$

Sprejemnik: $\frac{G_{RX}}{(T_A + T_G)}$

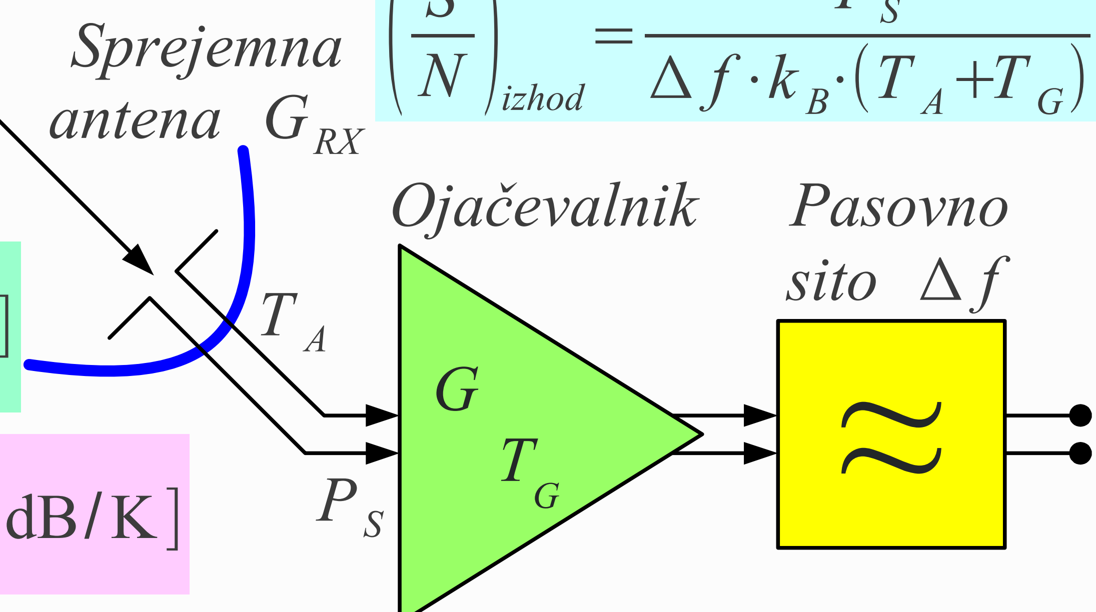
$$\left(\frac{S}{N} \right)_{izhod} = \frac{P_S}{\Delta f \cdot k_B \cdot (T_A + T_G)}$$

Sprejemna postaja

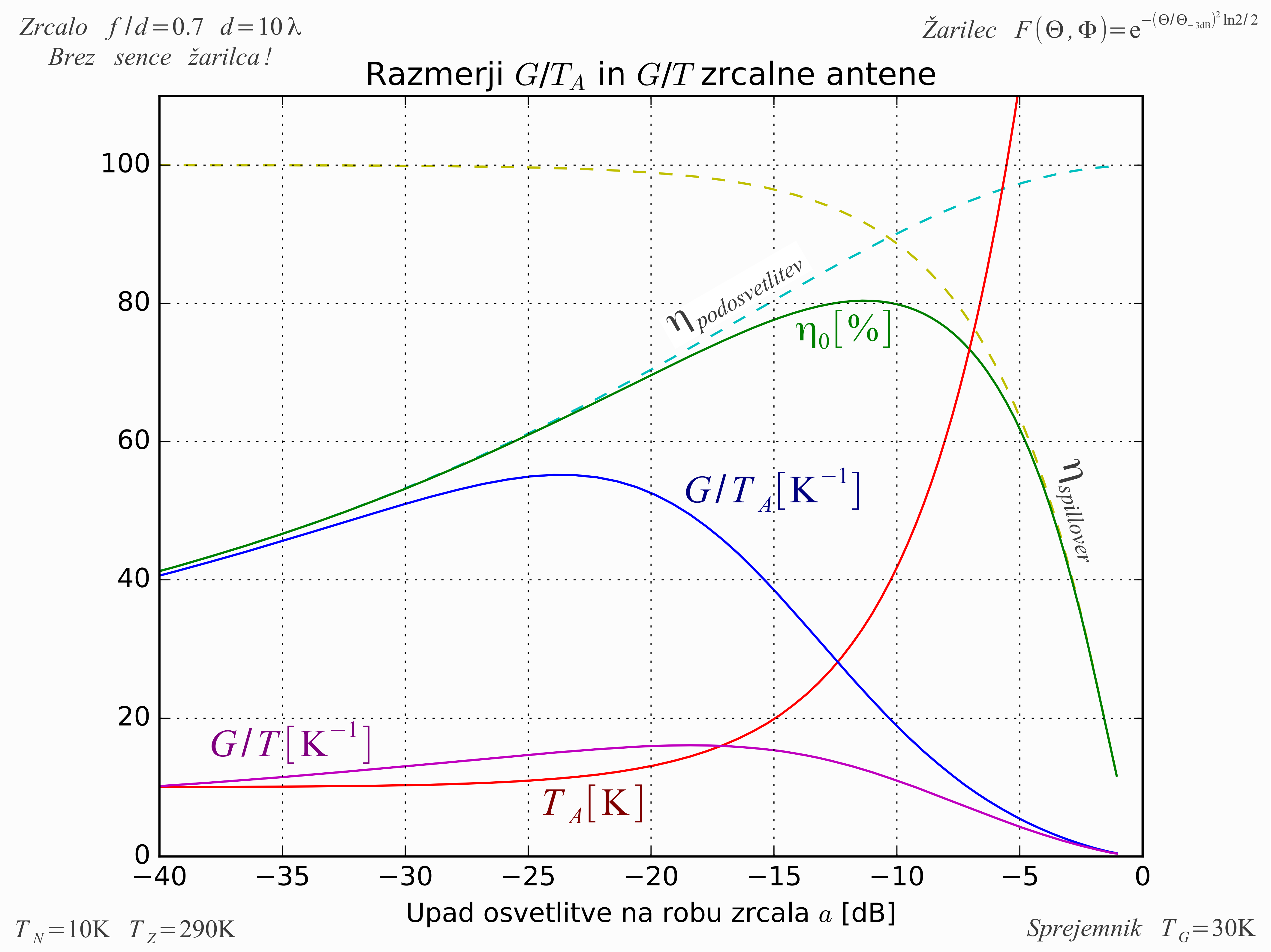
$$(G/T) = \frac{G_{RX}}{(T_A + T_G)} \quad [\mathrm{K}^{-1}]$$

$$(G/T)_{\mathrm{dB/K}} = 10 \log_{10} \frac{G_{RX} \cdot 1\mathrm{K}}{(T_A + T_G)} \quad [\mathrm{dB/K}]$$

$$(G/T)_{\mathrm{dB/K}} = G_{RX\,\mathrm{dB}} - 10 \log_{10} \frac{T_A + T_G}{1\mathrm{K}} \quad [\mathrm{dB/K}]$$



Zemeljska sprejemna postaja



Zrcalo f / d = 0.7 d = 10 λ
Brez sence žarilca !
Žarilec F(Θ, Φ) = e^−(Θ/Θ−3dB)² ln2/2
Razmerji G/T_A in G/T zrcalne antene
100
80
60
40
20
0
−40
−35
−30
−25
−20
−15
−10
−5
0
η podosvetlitev
η0 [%]
G/T_A [K^−1]
η spillover
G/T [K^−1]
T_A [K]
Upad osvetlitve na robu zrcala a [dB]
T_N = 10K T_Z = 290K
Sprejemnik T_G = 30K