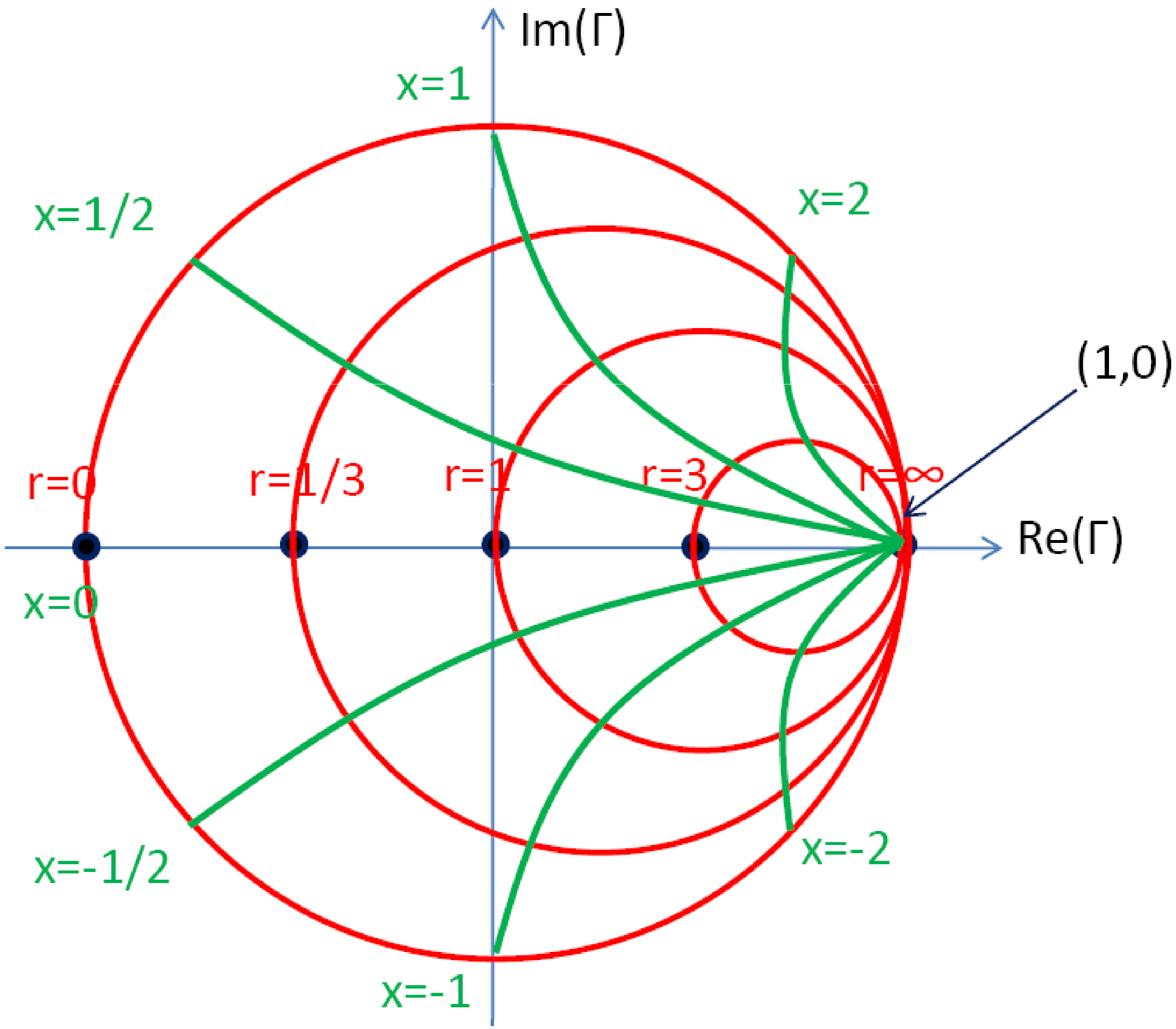
Smithov diagram



Mobitel d.d., izobraževanje

9. 10. 2009, predavanje 24

Prof. dr. Jožko Budin

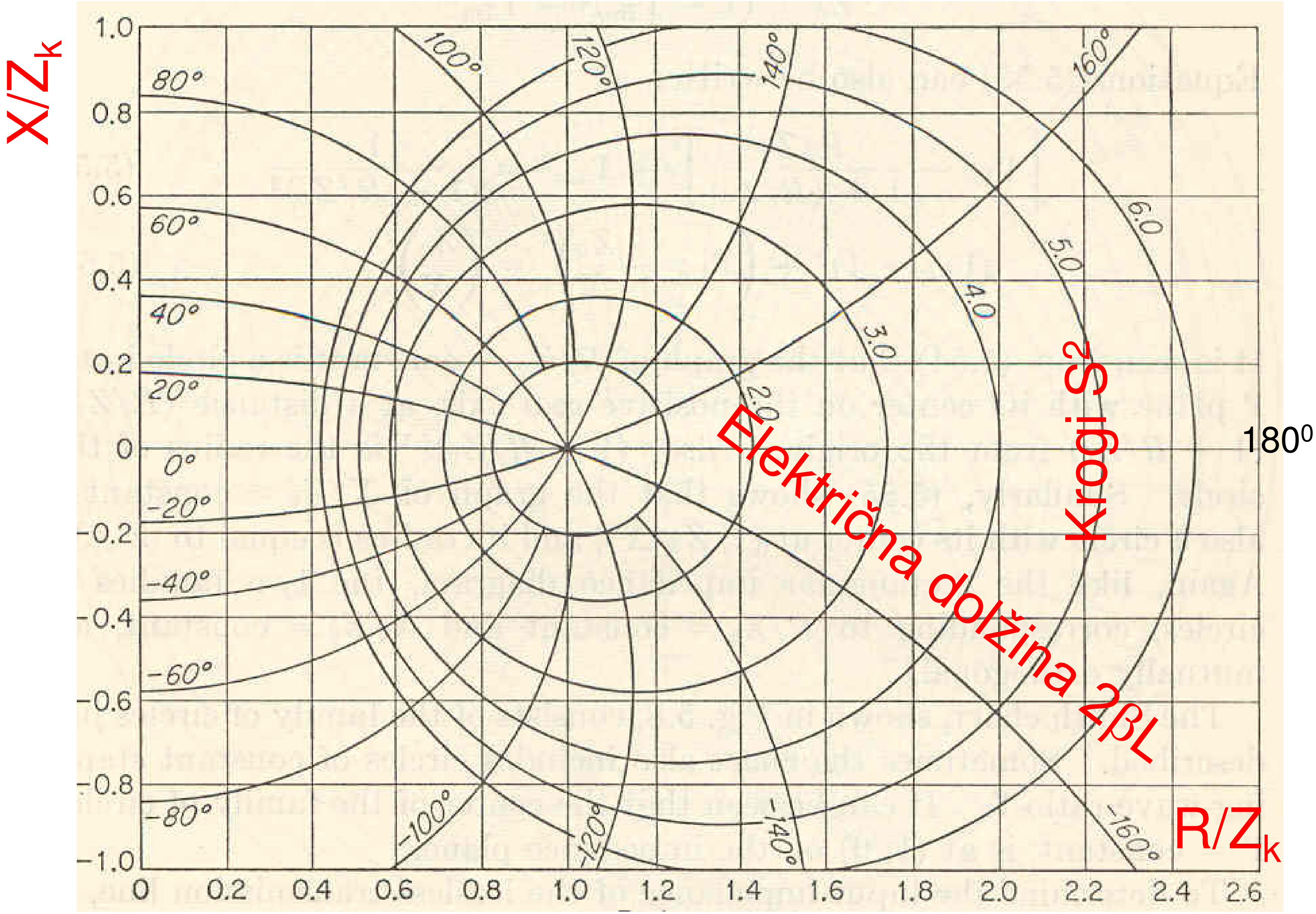
Vsebina

1. Osnove in zamisel diagrama
2. Bilinearna transformacija
3. Polje odbojnosti, impedance R, X, admitance G, B,

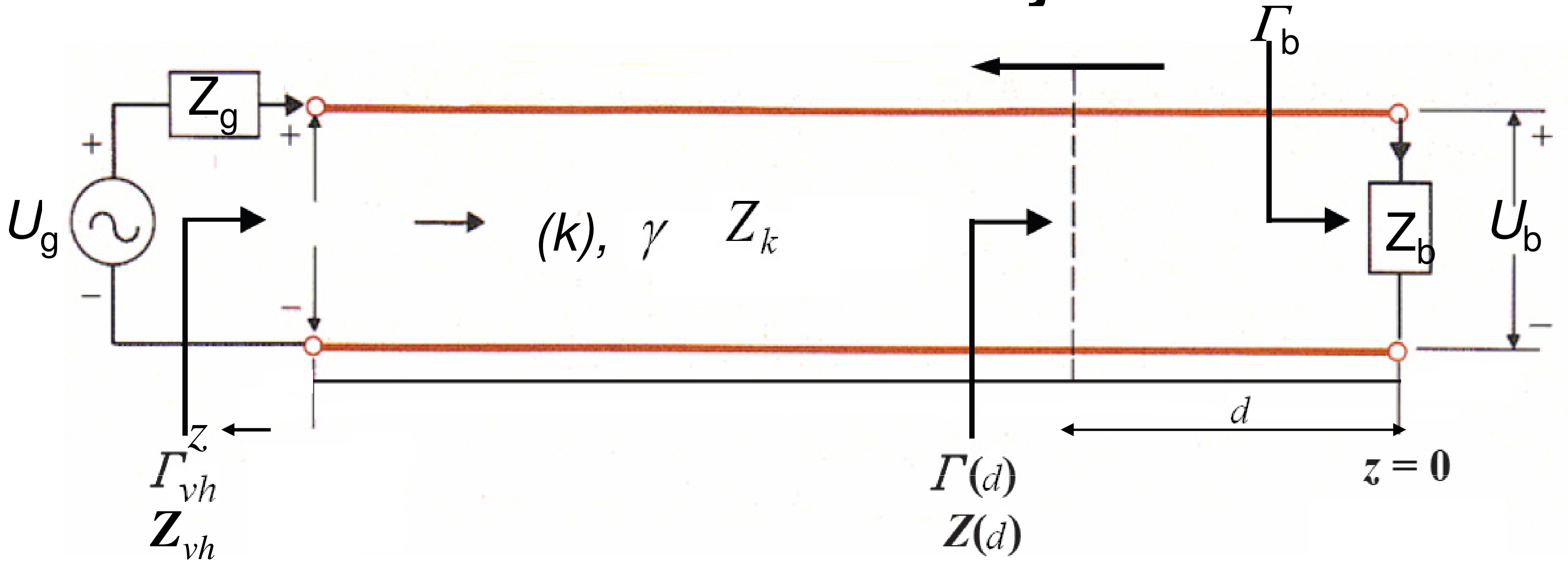
   impeanca |Z|, Θ
4. Značilnosti diagrama
5. Impedančno – admitančni diagram
6. Operacije v Smithovem diagramu
7. Diagrami za praktično uporabo

Pravokotni impedančni diagram

Predhodnik Smithovega diagrama



Prenosna linija



Odbojnost na razdalji d od konca linije brez izgub ($\gamma = k$):

$$\Gamma(d) = \Gamma = \Gamma_b e^{-j2kd}$$

$$|\Gamma| = |\Gamma_b|$$

Napetostni in tokovni potujoči valovi na liniji z izgubami

- Linija z izgubami $\gamma = \alpha + j\,\beta$
- Vpadni val se širi v smeri osi z, odbiti val v smeri osi -z

$$V(z) = V^{+} e^{-\gamma z} + V^{-} e^{\gamma z}$$

Vpadni val — Odbiti val

$$I(z) = \frac{1}{Z_0}\left(V^{+} e^{-\gamma z} - V^{-} e^{\gamma z}\right)$$

Vpadni val — Odbiti val

Odbiti val toka je v protifazi z odbitim valom napetosti

Transformacija Γ - Z

Odbojnost v odvisnosti od impedance

$$\Gamma(\ell) = \frac{Z(\ell) - Z_0}{Z(\ell) + Z_0}$$

Impedanca v odvisnosti od odbojnosti

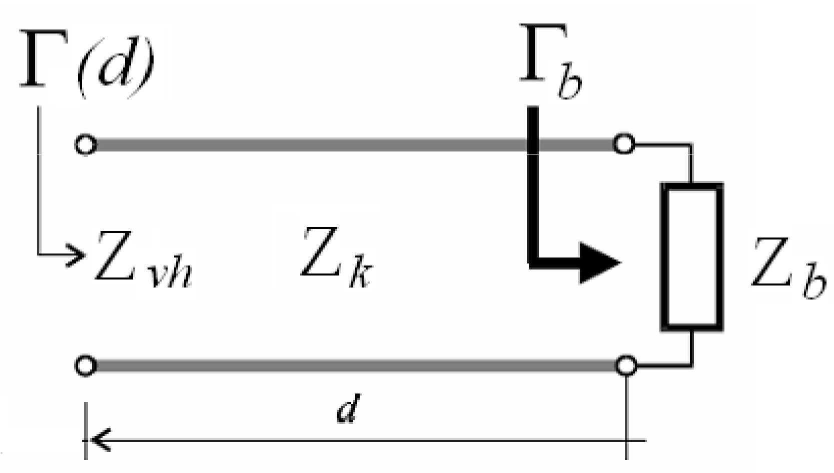
$$Z(\ell) = Z_0 \frac{1 + \Gamma(\ell)}{1 - \Gamma(\ell)}$$

Normirana impedanca

$$z = \frac{Z(\ell)}{Z_0}$$

Vhodna impedanca, vhodna admitanca

(Normirana) vhodna impedanca ali admitanca sta dani z odbojnostjo na vhodu linije. Le-ta je enaka odbojnosti bremena, zasukani za kot -2βd (proti generatorju).



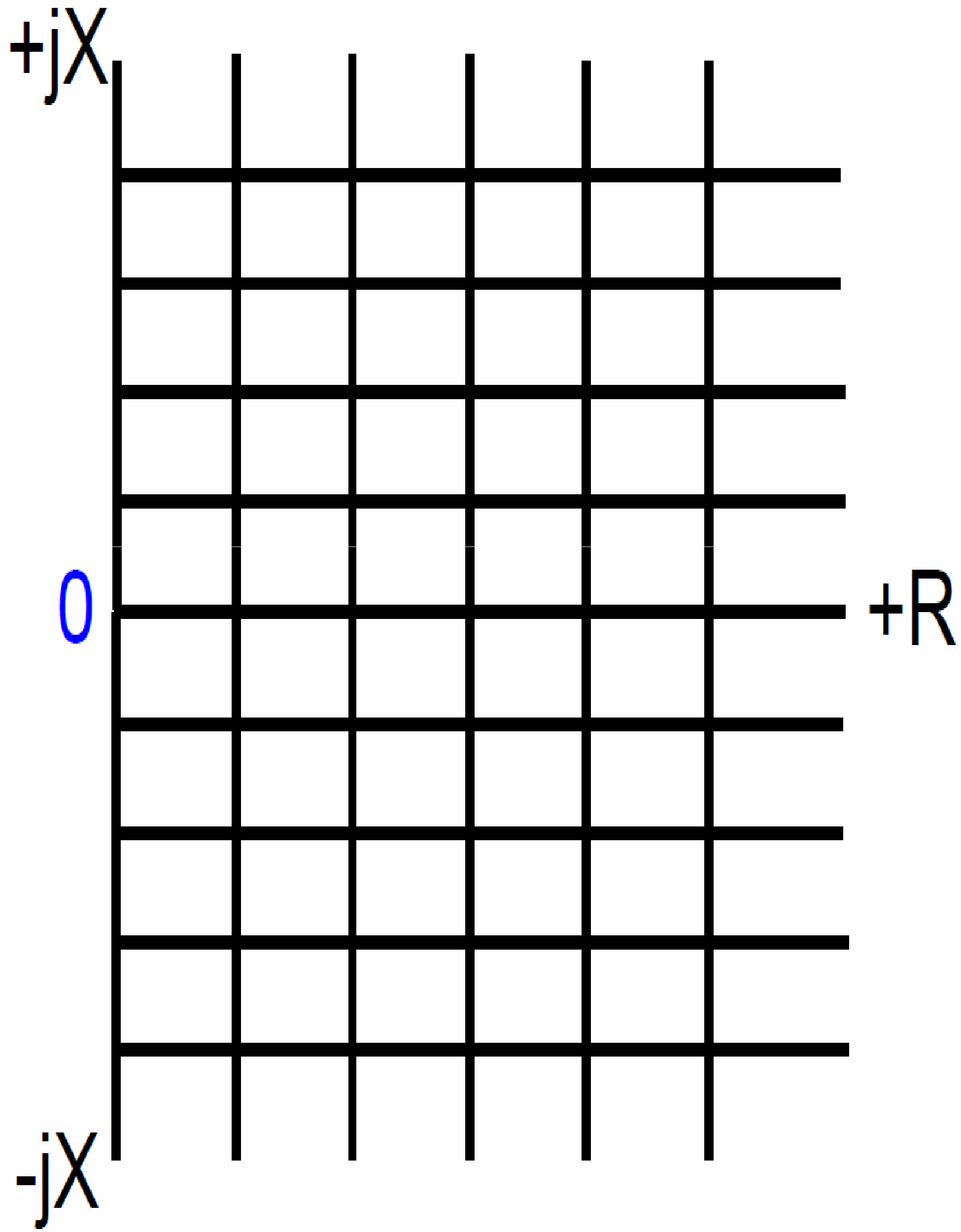
Vhodna impedanca in vhodna odbojnost na dolžini d od bremena

$$z_{vh} = \frac{Z_{vh}}{Z_k} = r + jx = \frac{1+\Gamma(d)}{1\ \ \Gamma(d)}$$

$$\Gamma(d) = \Gamma(0)\ e^{-j\beta d}$$

$$y_{vh} = \frac{Y_{vh}}{Y_k} = \frac{1}{z_{vh}} = \frac{1\ \ \Gamma(d)}{1+\Gamma(d)}$$

Pravokotna koordinatna mreža impedance



Impedanca pasivnih elementov:

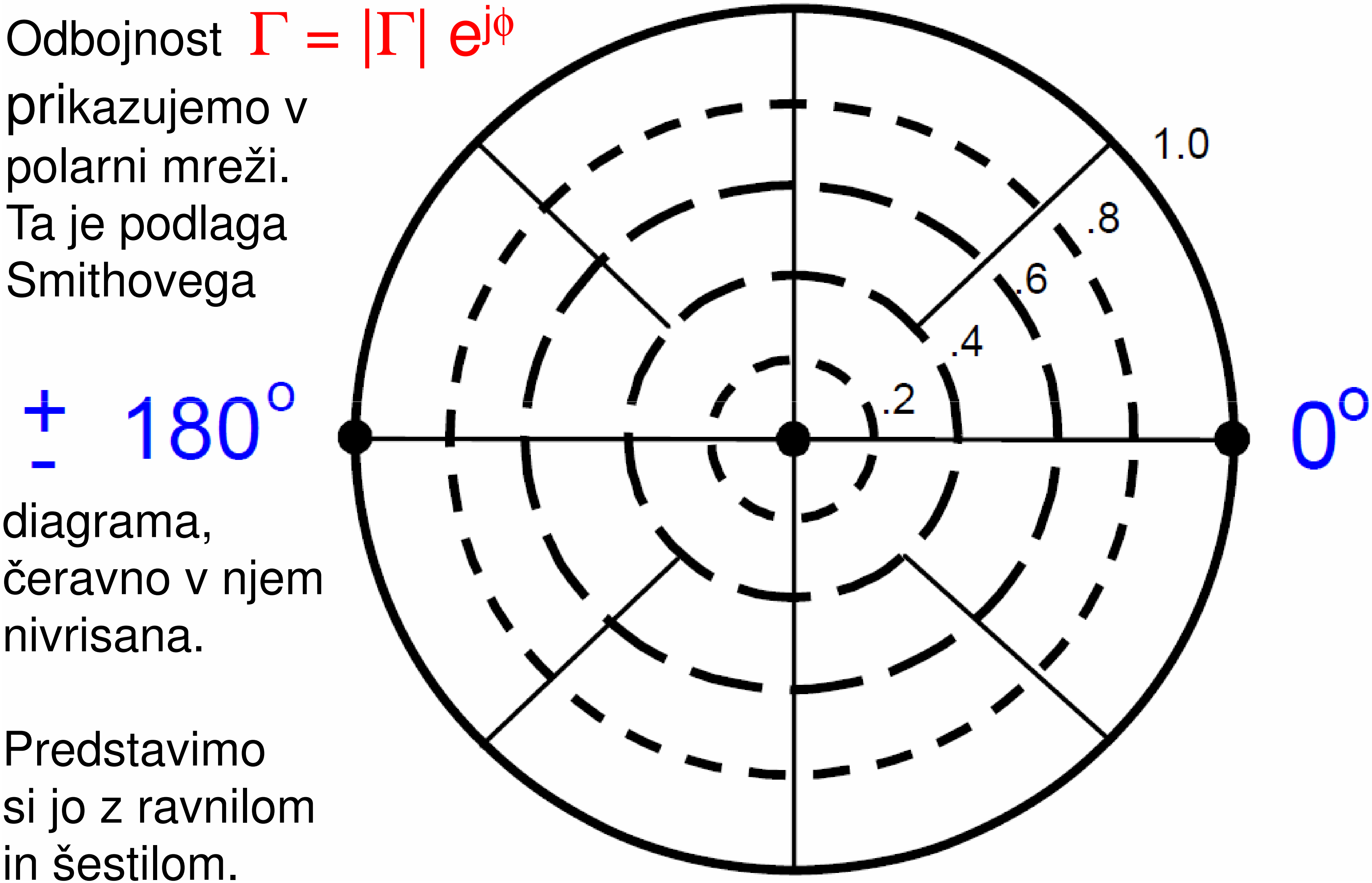
R > 0, desna polovica kompleksne ravnine

Reaktanca pasivnih elementov:

jX in -jX

Polarna mreža odbojnosti

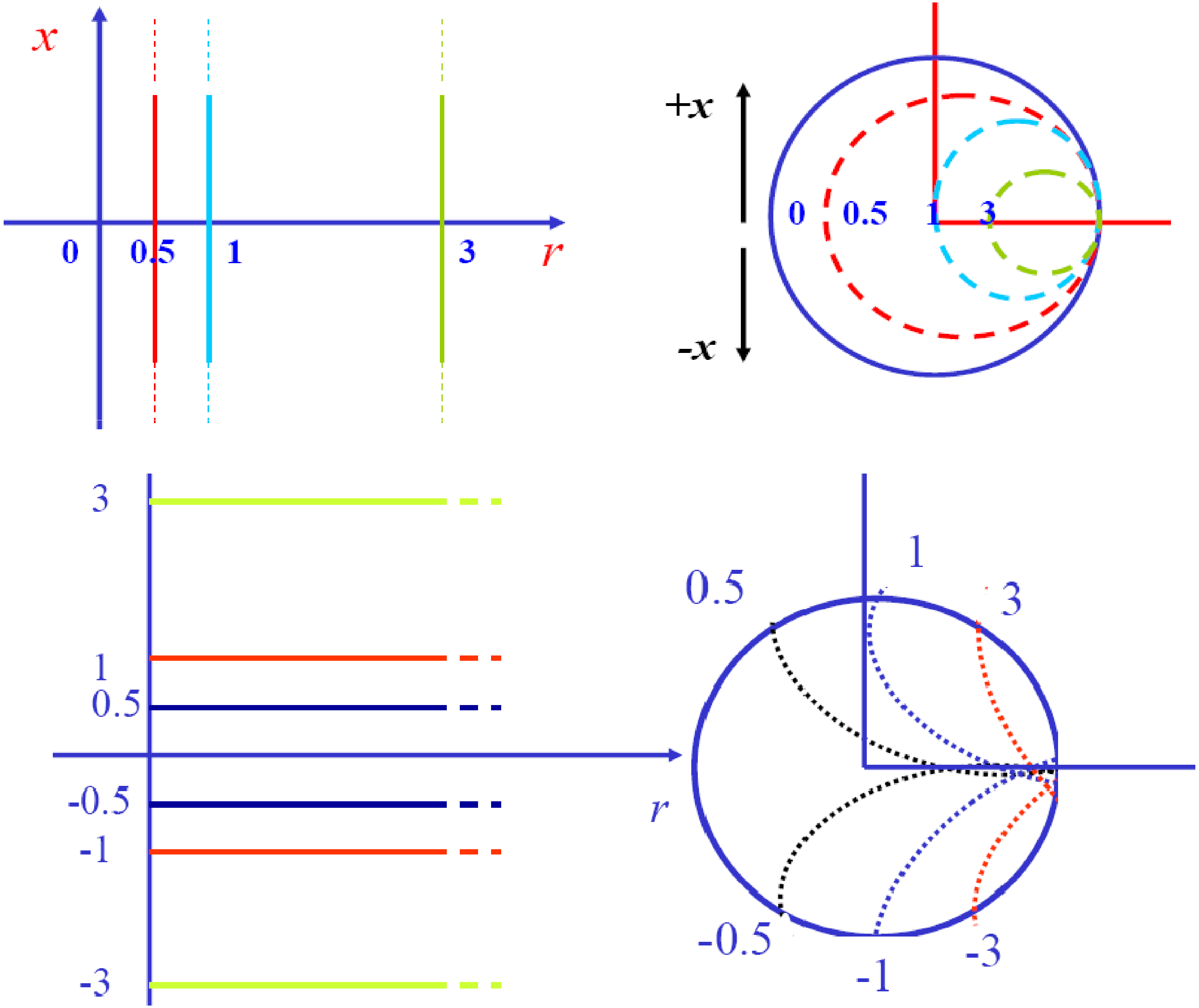
Odbojnost $\Gamma = |\Gamma|\, e^{j\phi}$ prikazujemo v polarni mreži. Ta je podlaga Smithovega



diagrama, čeravno v njem nivrisana.

Predstavimo si jo z ravnilom in šestilom.

Preslikave

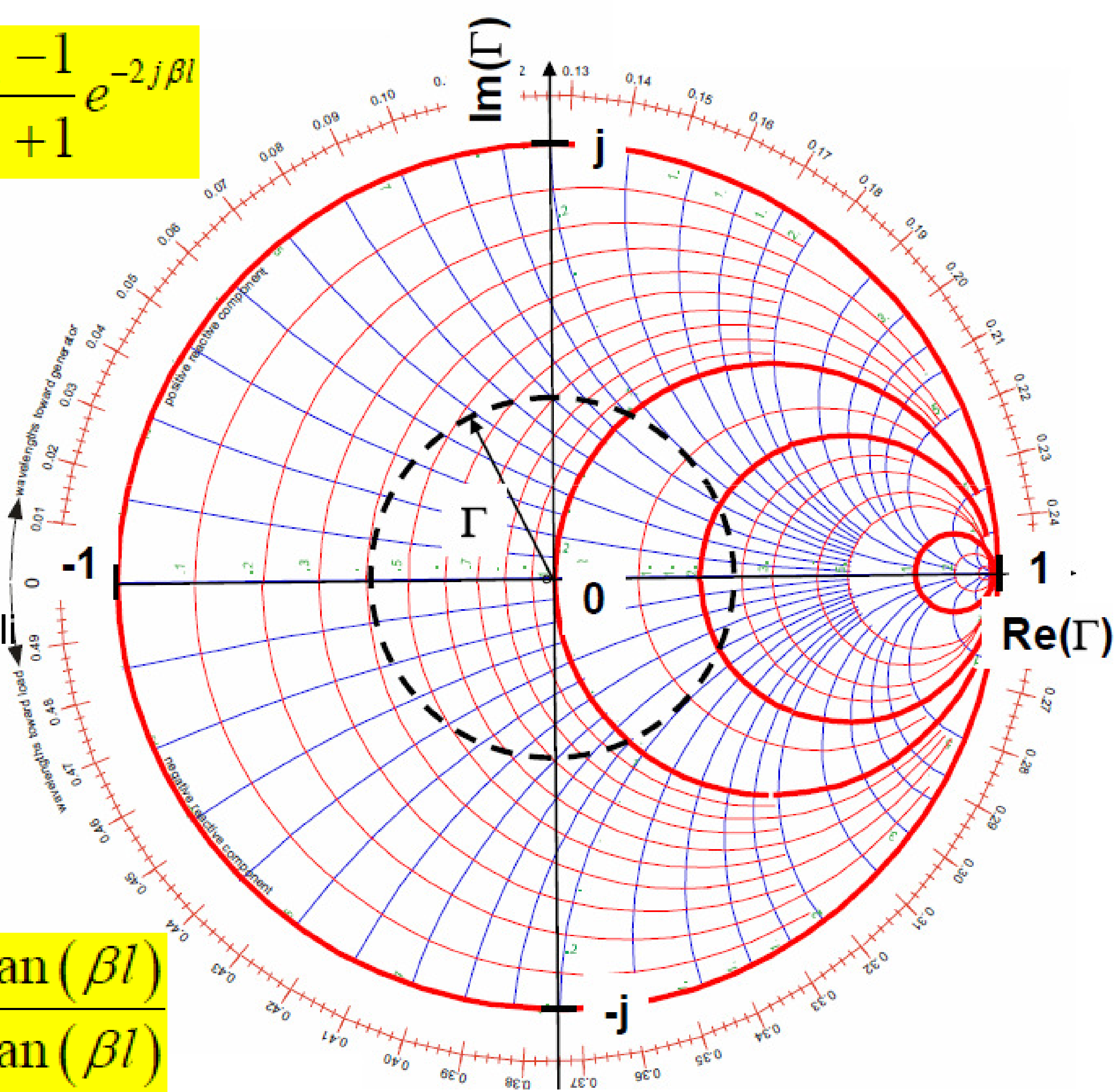


Prehod na Smithov diagram

$$\Gamma(l) = \Gamma_L e^{-2j\beta l} = \frac{Z_{nL} - 1}{Z_{nL} + 1} e^{-2j\beta l}$$

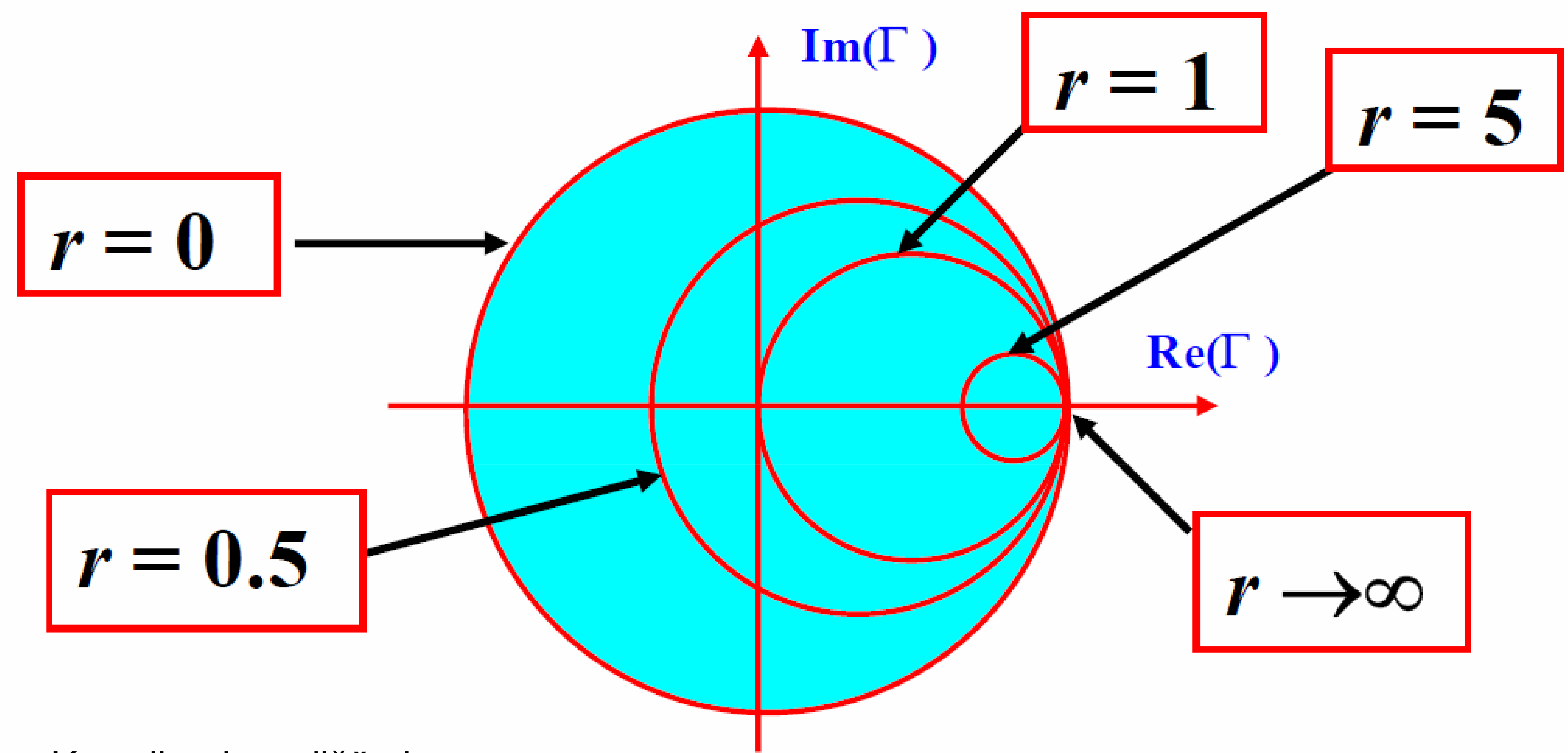
Pomembnost Smithovega diagrama za:

- Nazorno predstavo o problemu in njegovem reševanju
- Prikazovanje računanih ali merilnih rezultatov
- Numerično računanje (impedančni kalkulator)



$$Z(l) = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)}$$

Predstavitev krogov r = konst.



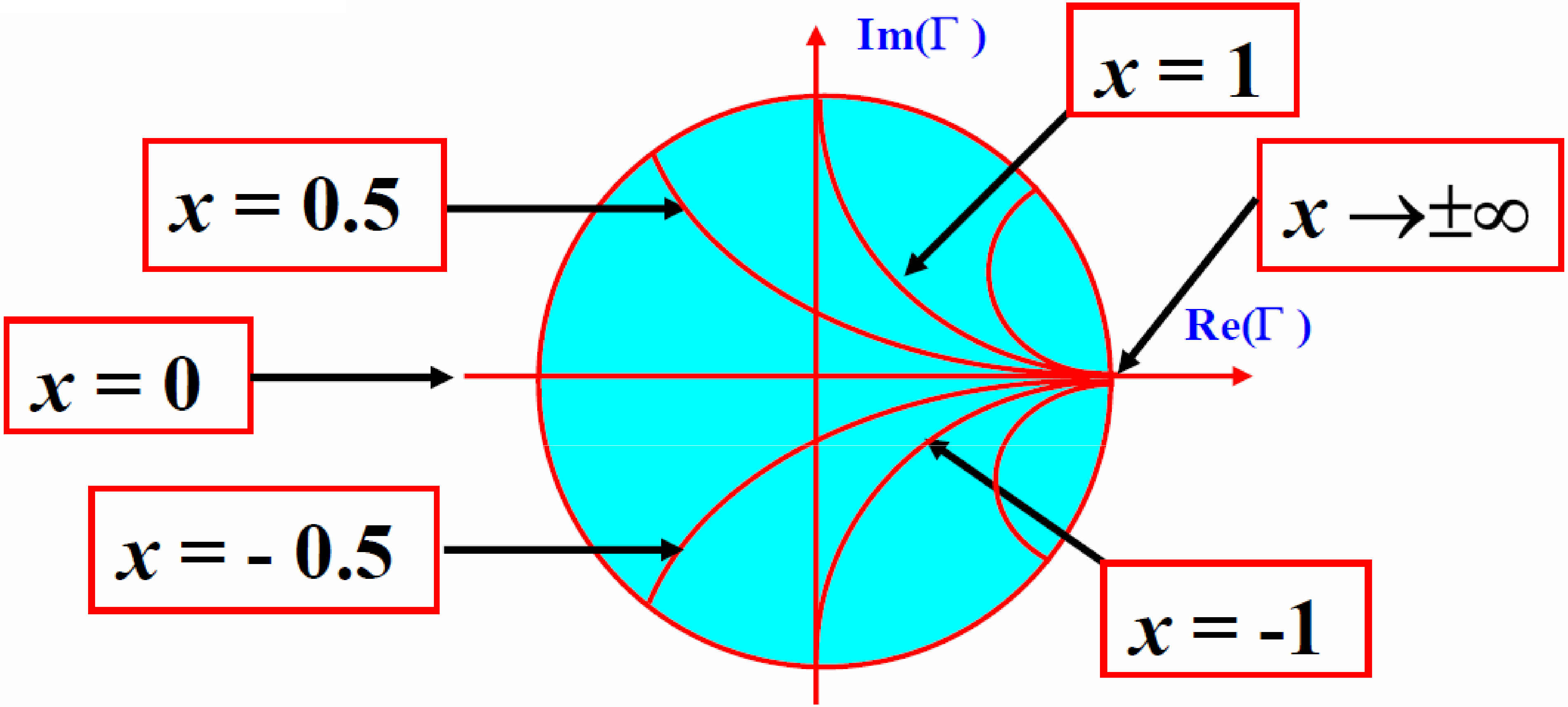
Koordinati središča krogov:

$$\frac{r}{1+r}, 0$$

Polmer krogov:

$$\frac{1}{1+r}$$

Predstavitev krogov x = konst.



Koordinati središča krogov:

$$1,\ \frac{1}{x}$$

Polmer krogov:

$$\frac{1}{x}$$

Realne in imaginarne komponente

$$\Gamma = \frac{z-1}{z+1} = \Gamma_{re} + j\Gamma_{im}$$

$$z = r + jx = \frac{1+\Gamma}{1-\Gamma} = \frac{(1+\Gamma_{re}) + j\Gamma_{im}}{(1-\Gamma_{re}) - j\Gamma_{im}}$$

$$r = \frac{1-\Gamma_{re}{}^2 - \Gamma_{im}{}^2}{(1-\Gamma_{re})^2 + \Gamma_{im}{}^2} \qquad x = \frac{2\Gamma_{im}}{(1-\Gamma_{re})^2 + \Gamma_{im}{}^2}$$

Enačba krogov r = konst

$$r = \frac{1 - \mathrm{Re}^2(\Gamma) - \mathrm{Im}^2(\Gamma)}{(1 - \mathrm{Re}(\Gamma))^2 + \mathrm{Im}^2(\Gamma)}$$

Ravnina $\Gamma = \mathrm{Re}(\Gamma) + \mathrm{Im}(\Gamma)$

$$r(\mathrm{Re}(\Gamma) - 1)^2 + \left(\mathrm{Re}^2(\Gamma) - 1\right) + r\,\mathrm{Im}^2(\Gamma) + \mathrm{Im}^2(\Gamma) + \frac{1}{1+r} - \frac{1}{1+r} = 0$$

$$\left[r(\mathrm{Re}(\Gamma) - 1)^2 + \left(\mathrm{Re}^2(\Gamma) - 1\right) + \frac{1}{1+r} \right] + (1+r)\,\mathrm{Im}^2(\Gamma) = \frac{1}{1+r}$$

$$(1+r)\left[\mathrm{Re}^2(\Gamma) - 2\,\mathrm{Re}(\Gamma)\frac{r}{1+r} + \frac{r^2}{(1+r)^2} \right] + (1+r)\,\mathrm{Im}^2(\Gamma) = \frac{1}{1+r}$$

$$\Rightarrow \qquad \left[\mathrm{Re}(\Gamma) - \frac{r}{1+r} \right]^2 + \mathrm{Im}^2(\Gamma) = \left(\frac{1}{1+r} \right)^2$$

Enačbe krogov x = konst.

$$x = \frac{2\,\mathrm{Im}(\Gamma)}{(1-\mathrm{Re}(\Gamma))^2 + \mathrm{Im}^2(\Gamma)}$$

Ravnina $\Gamma = \mathrm{Re}(\Gamma) + \mathrm{Im}(\Gamma)$

$$x^2\left[(1-\mathrm{Re}(\Gamma))^2 + \mathrm{Im}^2(\Gamma)\right] - 2x\,\mathrm{Im}(\Gamma) + 1 - 1 = 0$$

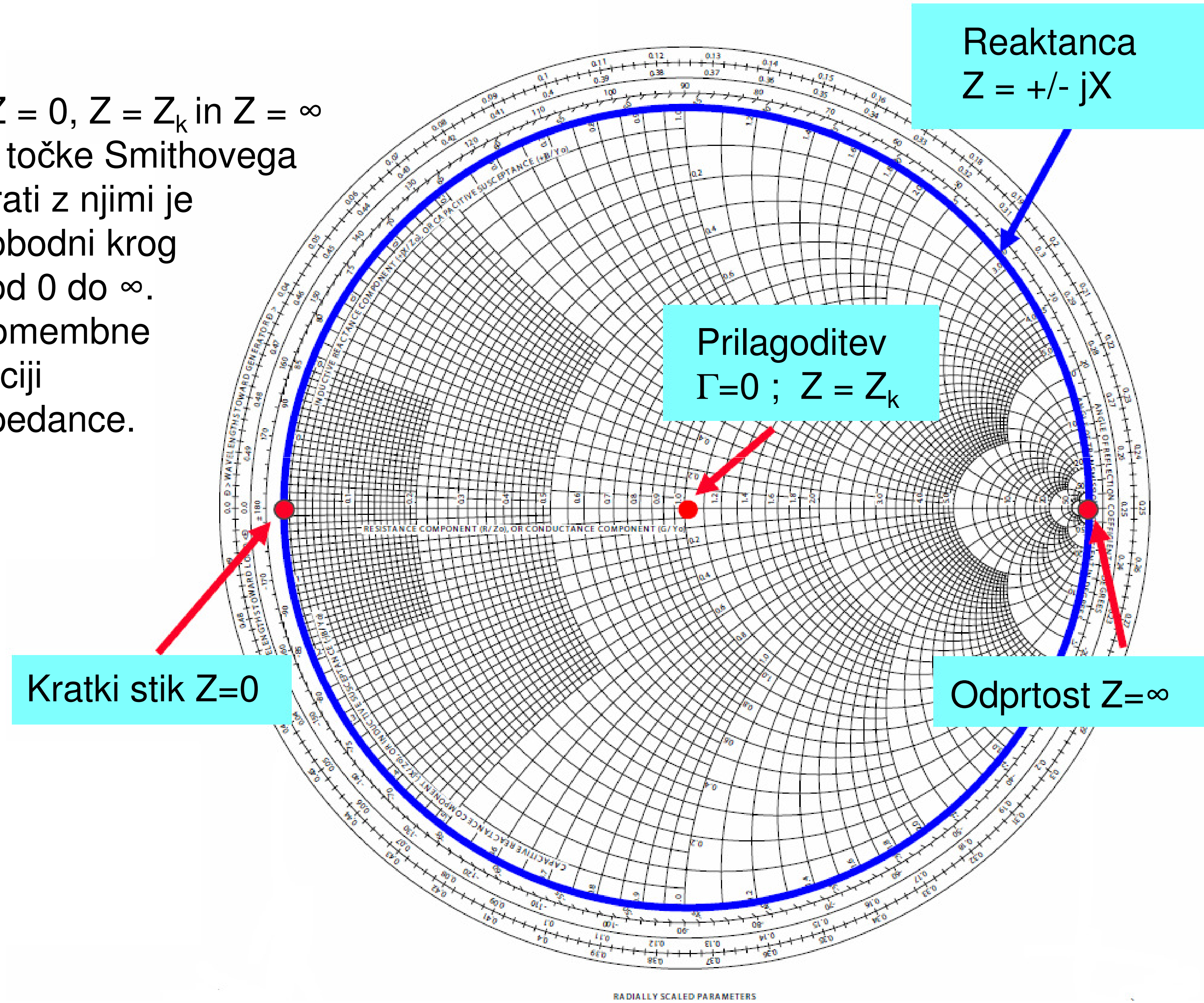
$$\left[(1-\mathrm{Re}(\Gamma))^2 + \mathrm{Im}^2(\Gamma)\right] - \frac{2}{x}\mathrm{Im}(\Gamma) + \frac{1}{x^2} = \frac{1}{x^2}$$

$$(1-\mathrm{Re}(\Gamma))^2 + \left[\mathrm{Im}^2(\Gamma) - \frac{2}{x}\mathrm{Im}(\Gamma) + \frac{1}{x^2}\right] = \frac{1}{x^2}$$

$$\Rightarrow \qquad (\mathrm{Re}(\Gamma)-1)^2 + \left[\mathrm{Im}(\Gamma) - \frac{1}{x}\right]^2 = \frac{1}{x^2}$$

Značilne točke diagrama

Impedance $Z = 0$, $Z = Z_k$ in $Z = \infty$ so tri značilne točke Smithovega diagrama. Hkrati z njimi je značilen tudi obodni krog reaktance jX od 0 do ∞. Te točke so pomembne tudi pri kalibraciji merilnikov impedance.



Primeri toček v diagramu

Vaja (normirana impedanca $z = Z/Z_k$):

z1 = 2 + j

z2 = 1.5 -j2

z3 = j4

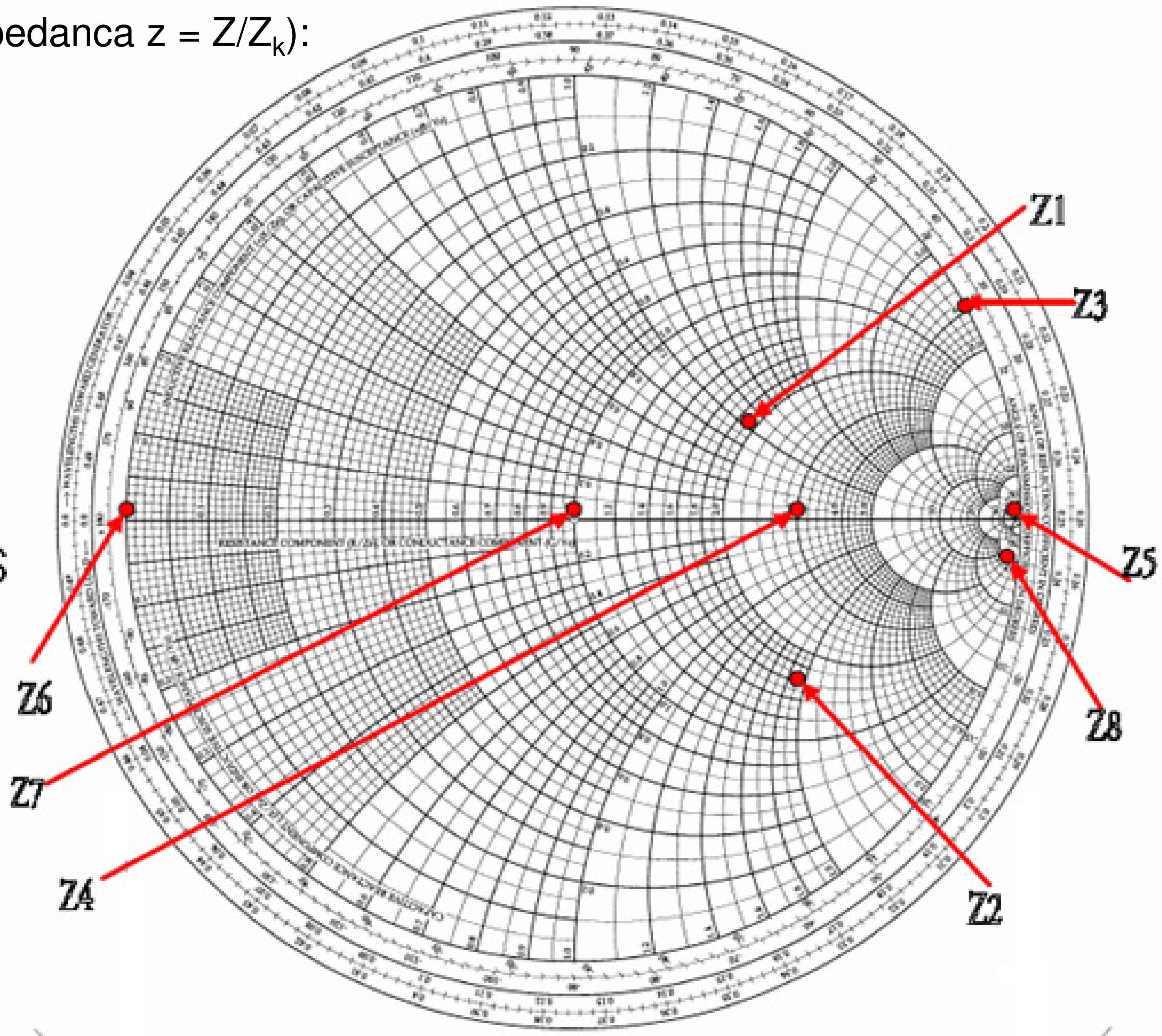
z4 = 3

z5 =

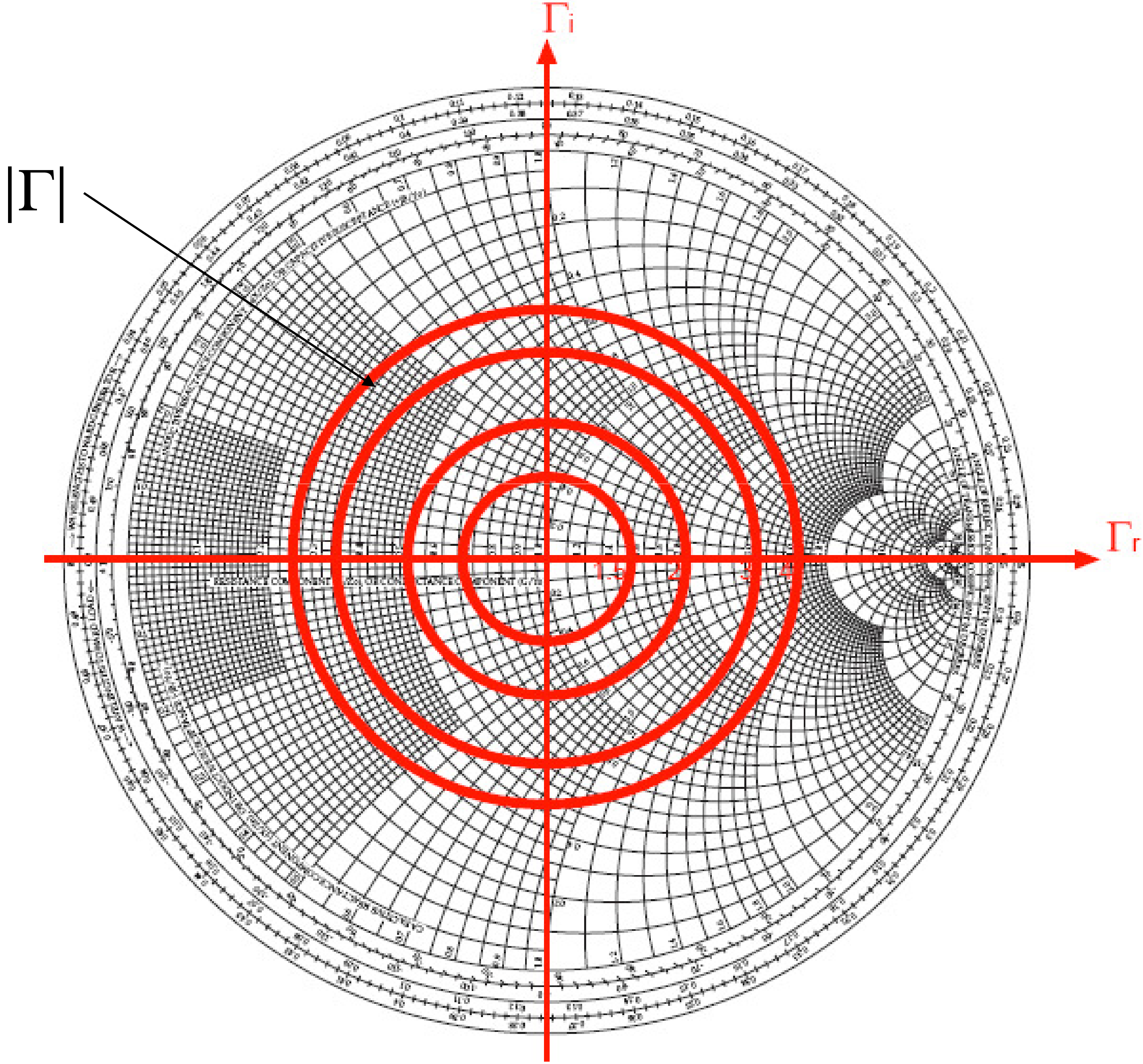
z6 = 0

z7 = 1

z8 = 3.68 -j18S



Upodobitev odbojnosti $|\Gamma|$

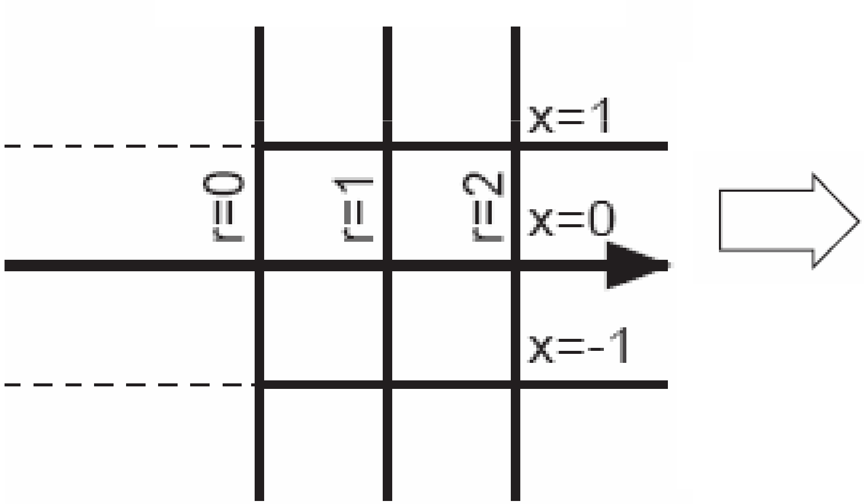


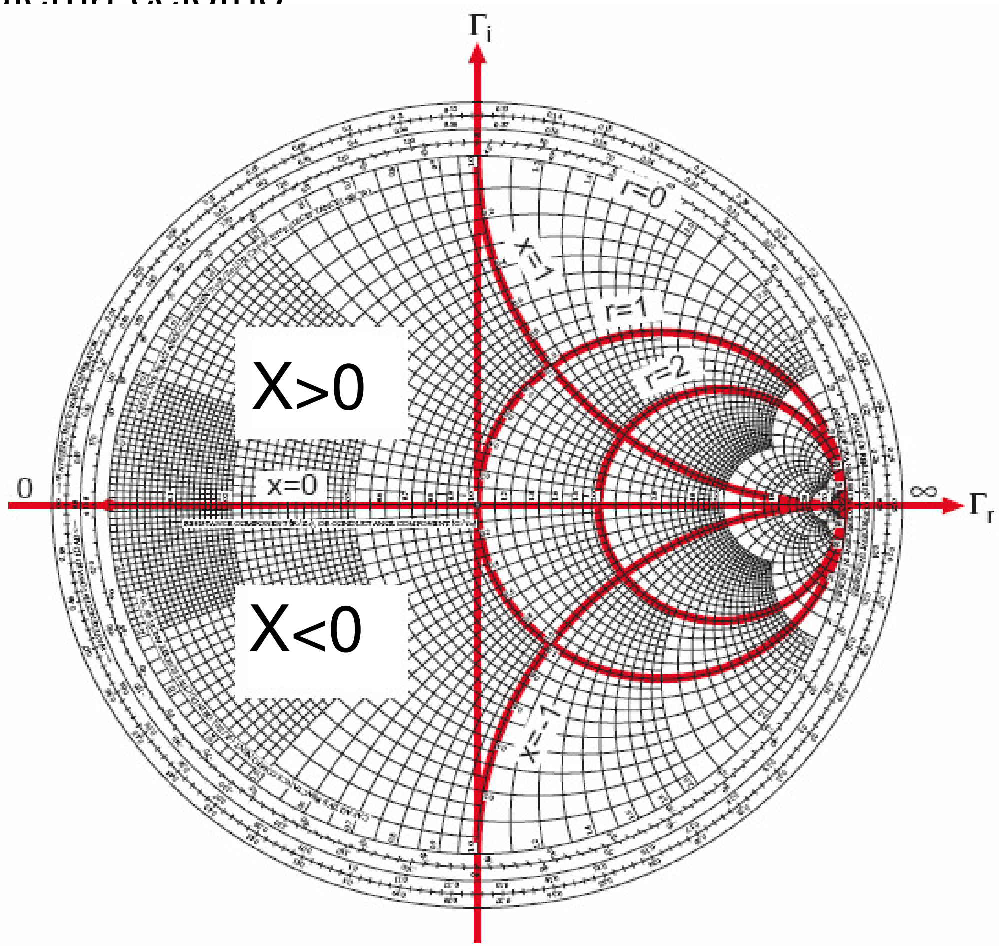
Upodobitev impedance

• Desna polovica kompleksne ravnine, ki predstavlja normirano impedanco z = r + jx pri r>0 in x >/<0, se upodobi v znotraj kroga polmera 1, ki zajema celotno področje impedance pasivnih elemento

• Krogi konstantnega r imajo središče na realni osi

Bilinearna transformacija

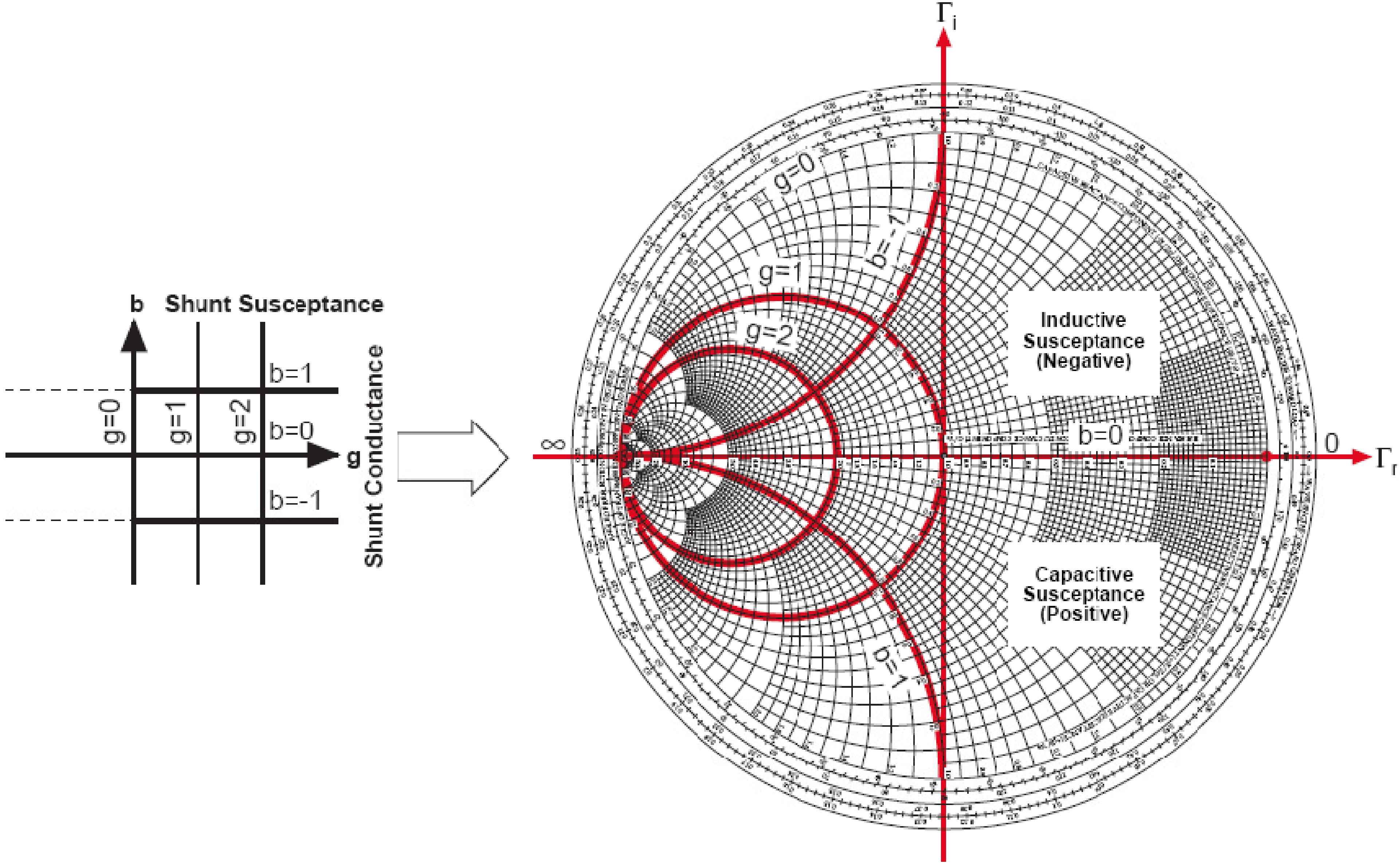




• Leva polovica kompleksne ravnine, ki predstavlja normirano impedanco pri r<0 in x>/<0 aktivnih elementov se preslika

• Krogi konstantnega x imajo središče na imag. osi

Upodobitev admitance

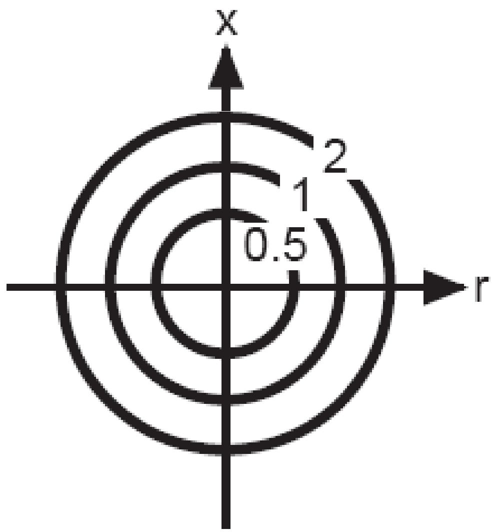


Impedančni |Z| – θ diagram

Pravokotna koordinatna mreža

$z = r + jx$

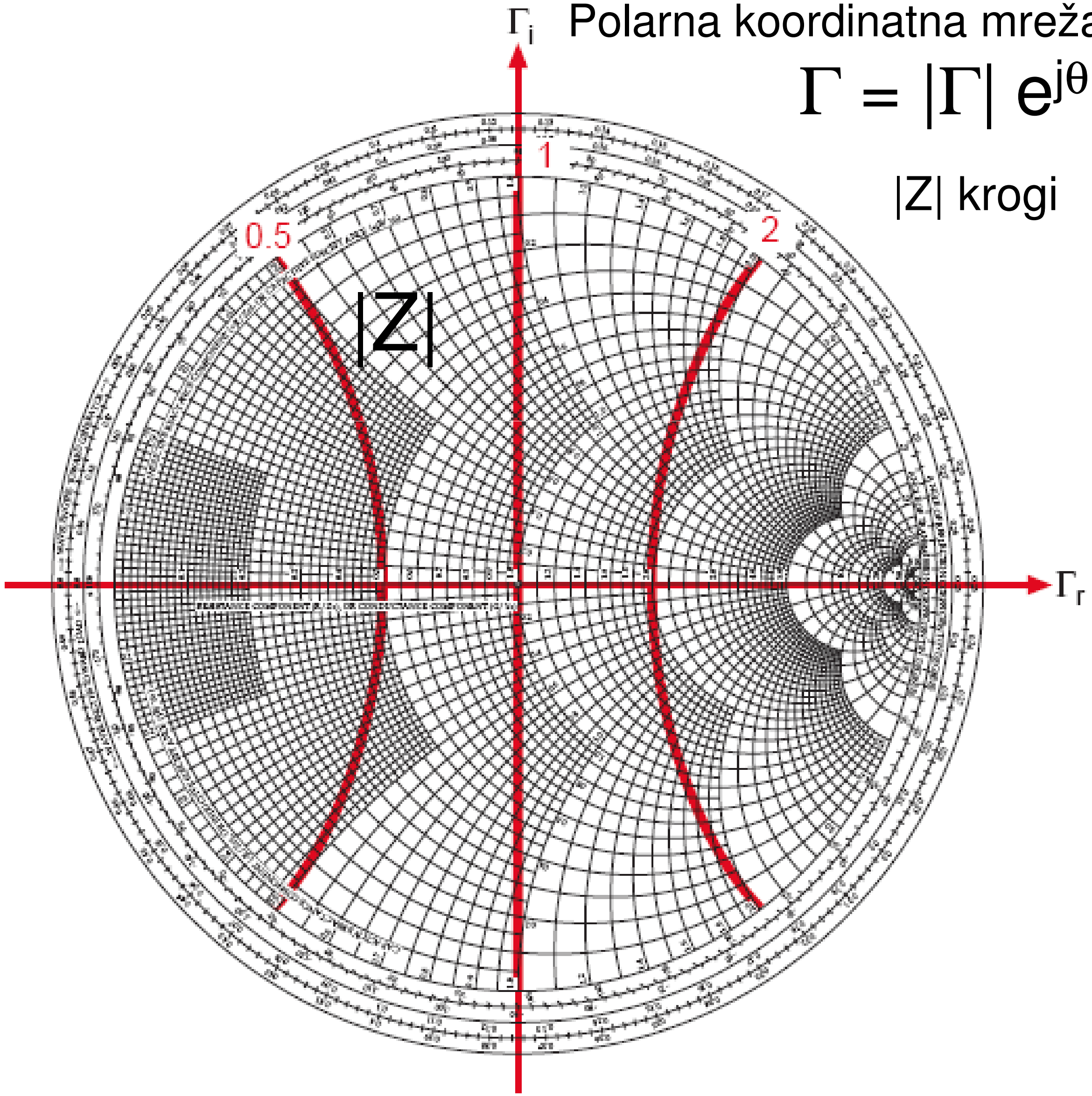
|z| koncentrični krogi



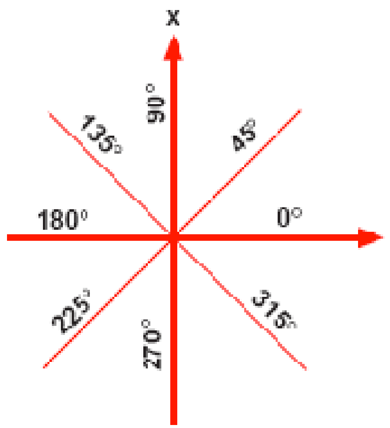
Polarna koordinatna mreža

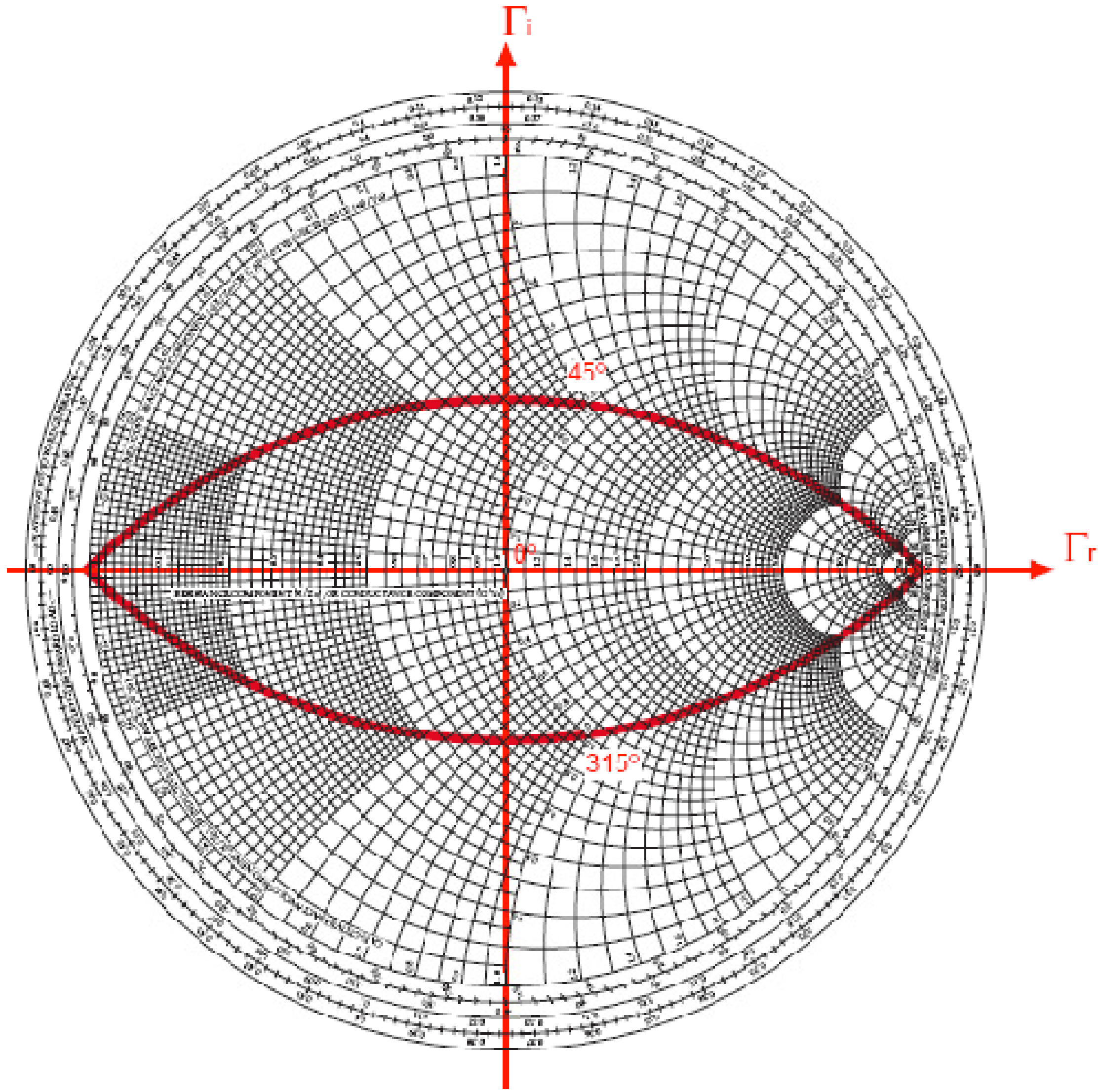
$\Gamma = |\Gamma| e^{j\theta}$

|Z| krogi



Impedančni |Z| – Θ diagram





Relacije z-Γ

$$\frac{1+\Gamma}{1-\Gamma} = \frac{1+\dfrac{\left[\left(r+j\cdot x\right)-1\right]}{\left[\left(r+j\cdot x\right)+1\right]}}{1-\dfrac{\left[\left(r+j\cdot x\right)-1\right]}{\left[\left(r+j\cdot x\right)+1\right]}} \cdot \frac{\left[\left(r+j\cdot x\right)+1\right]}{\left[\left(r+j\cdot x\right)+1\right]} = \frac{\left[\left(r+j\cdot x\right)+1\right]+\left[\left(r+j\cdot x\right)-1\right]}{\left[\left(r+j\cdot x\right)+1\right]-\left[\left(r+j\cdot x\right)-1\right]} = \frac{2r+j\cdot 2x}{2}$$

$$\frac{1+\Gamma}{1-\Gamma} = r+j\cdot x = \frac{Z_L}{Z_0}$$

$$\Gamma = u + j\cdot v$$

$$r+j\cdot x = \frac{1+\Gamma}{1-\Gamma} = \frac{1+\left(u+j\cdot v\right)}{1-\left(u+j\cdot v\right)} = \frac{\left(1+u\right)+j\cdot v}{\left(1-u\right)-j\cdot v}\cdot\frac{\left(1-u\right)+j\cdot v}{\left(1-u\right)+j\cdot v} = \frac{1-u^2-v^2+2\cdot j\cdot v}{\left(1-u\right)^2+v^2}$$

$$r + j\cdot x = \frac{1-\left(u^2+v^2\right)}{\left(1-u\right)^2+v^2} + j\cdot\frac{2\cdot v}{\left(1-u\right)^2+v^2}$$

Relacije z-Γ

$$r + j \cdot x = \frac{1 - (u^2 + v^2)}{(1-u)^2 + v^2} + j \cdot \frac{2 \cdot v}{(1-u)^2 + v^2}$$

$$\Gamma = u + j \cdot v$$

$$r = \frac{1 - (u^2 + v^2)}{(1-u)^2 + v^2}$$

$$r \cdot \left[(1-u)^2 + v^2\right] = 1 - (u^2 + v^2)$$

$$r \cdot \left[1 - 2u + u^2 + v^2\right] = 1 - (u^2 + v^2)$$

$$u^2 \cdot (r+1) - 2ur + v^2 \cdot (r+1) = 1 - r$$

$$u^2 - 2u\frac{r}{r+1} + v^2 = \frac{1-r}{r+1}$$

$$\left[u^2 - 2u\frac{r}{r+1} + \left(\frac{r}{r+1}\right)^2\right] + v^2 = \frac{1-r}{r+1} + \left(\frac{r}{r+1}\right)^2$$

$$\left[u - \frac{r}{r+1}\right]^2 + v^2 = \frac{(1-r)\cdot(1+r) + r^2}{(r+1)^2}$$

$$\left[u - \frac{r}{r+1}\right]^2 + v^2 = \frac{1 - r^2 + r^2}{(r+1)^2}$$

$$\Rightarrow \quad \left(u - \frac{r}{1+r}\right)^2 + v^2 = \frac{1}{(1+r)^2}$$

Krogi konstantnega r in x

X ... normirana admitanca

$$(u-1)^2+\left(v-\frac{1}{x}\right)^2=\frac{1}{x^2}$$

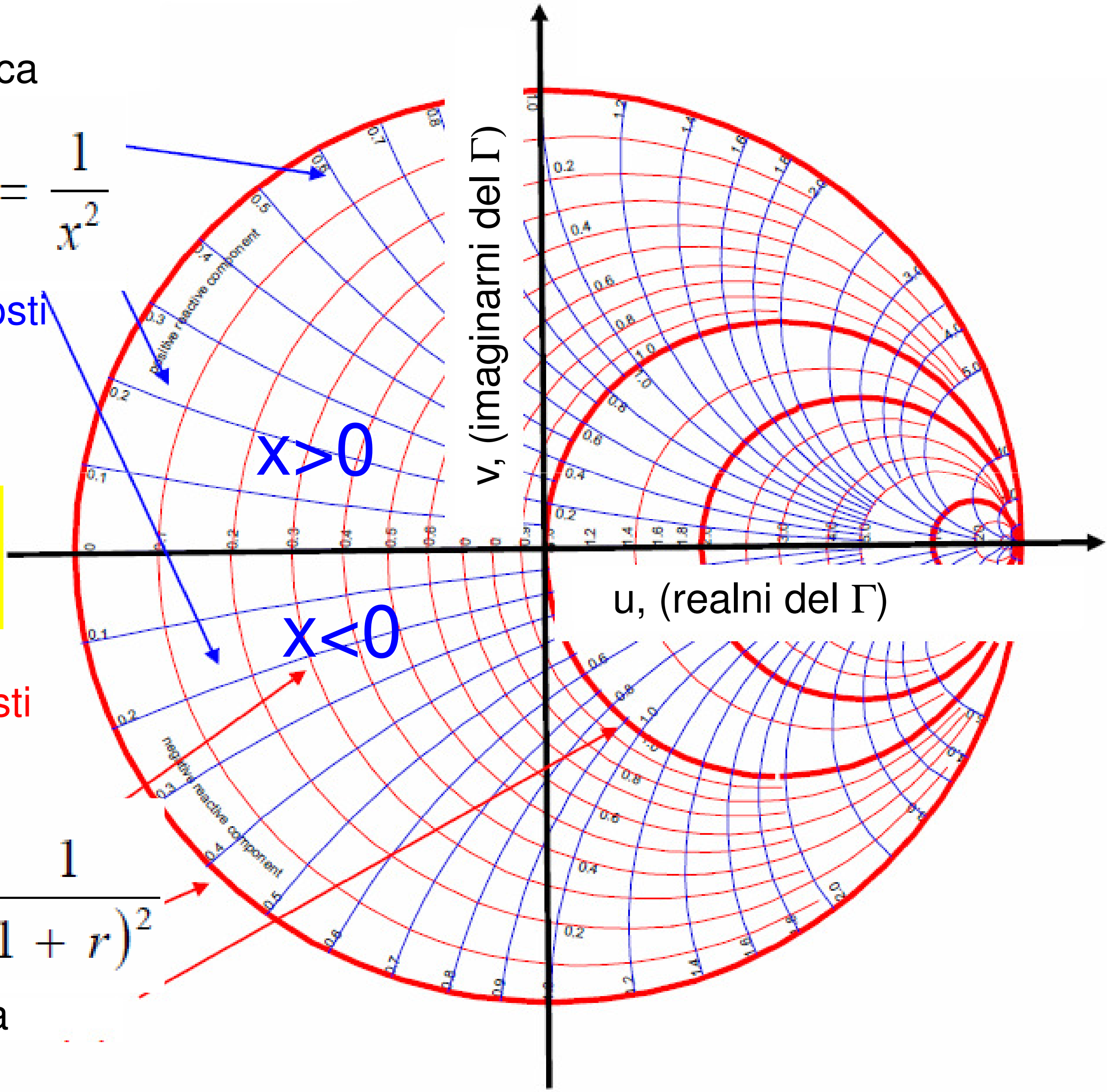
Krogi konstantne vrednosti normirane reaktance x

$$\Gamma_L=\frac{Z_L-Z_o}{Z_o+Z_L}=\frac{(r+jx)-1}{(r+jx)+1}$$

Krogi konstantne vrednosti normirane rezistance r

$$\left(u-\frac{r}{1+r}\right)^2+v^2=\frac{1}{(1+r)^2}$$

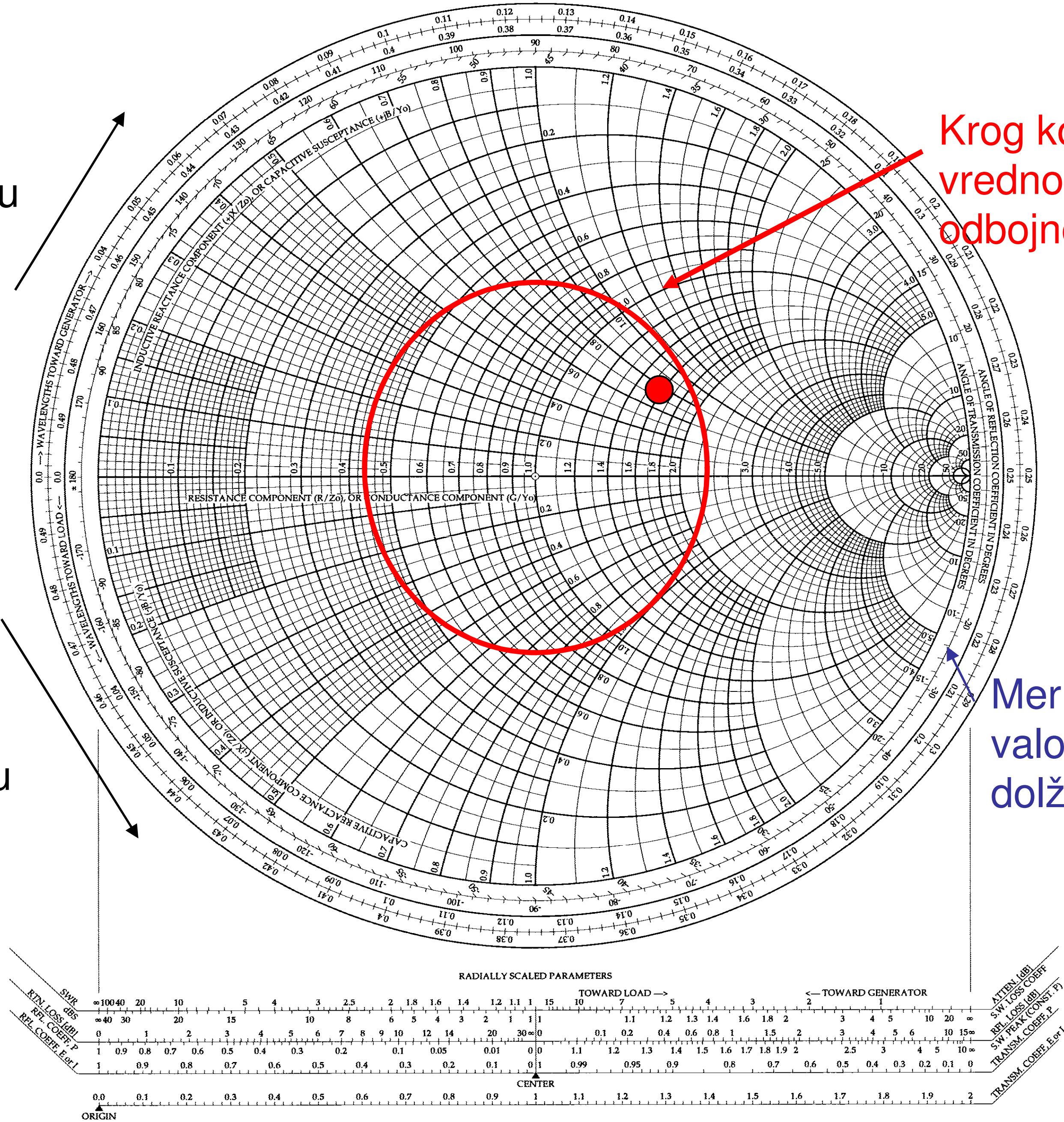
u ... normirana rezistanca



Polni obrat za 360^0 ustreza pomiku na liniji za pol valovne dolžine

Proti generatorju

Proti bremenu

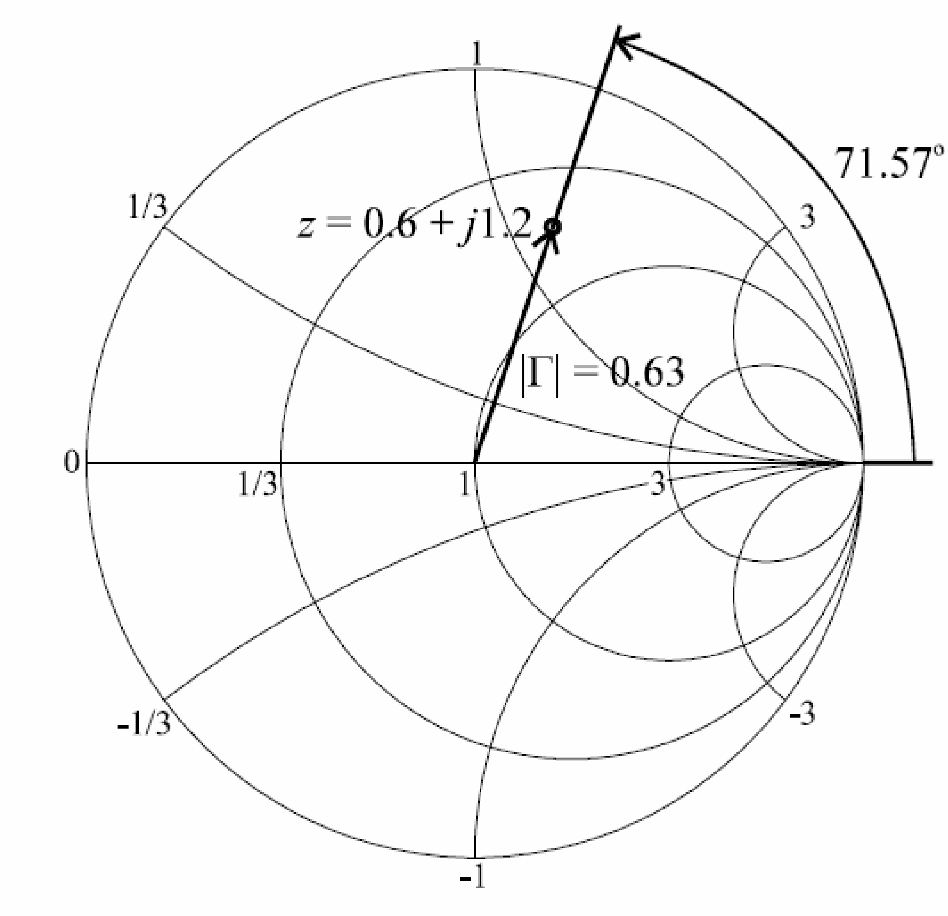


Krog konstantne vrednosti odbojnosti

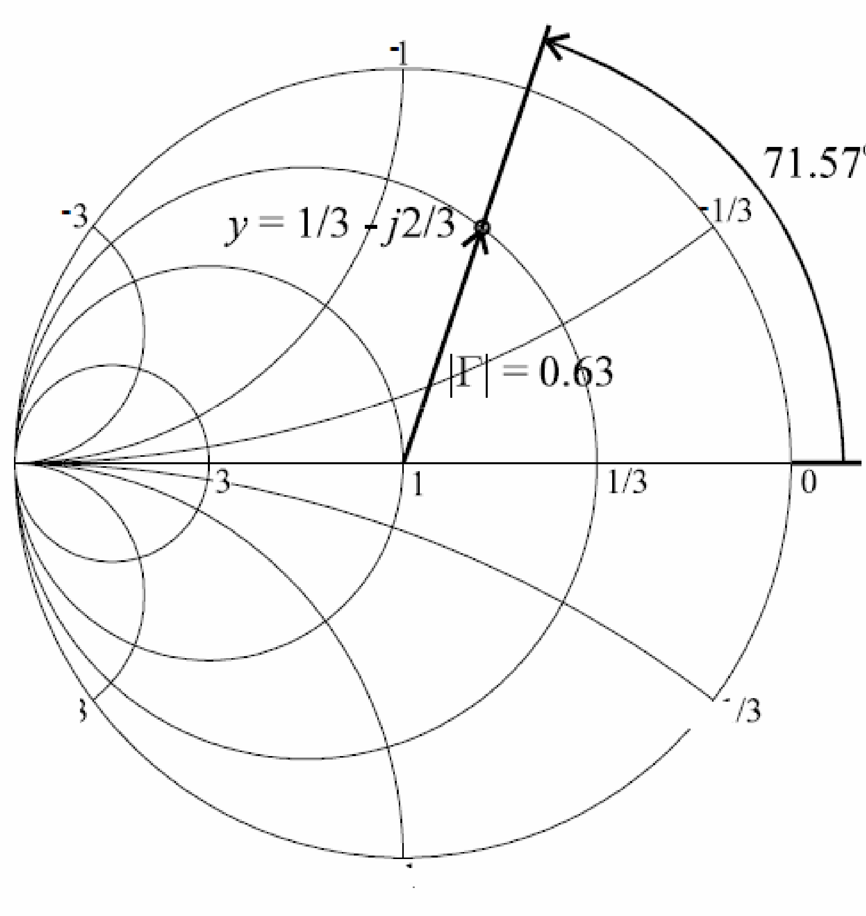
Merilo v valovni dolžini

Impedančni in admitančni diagram

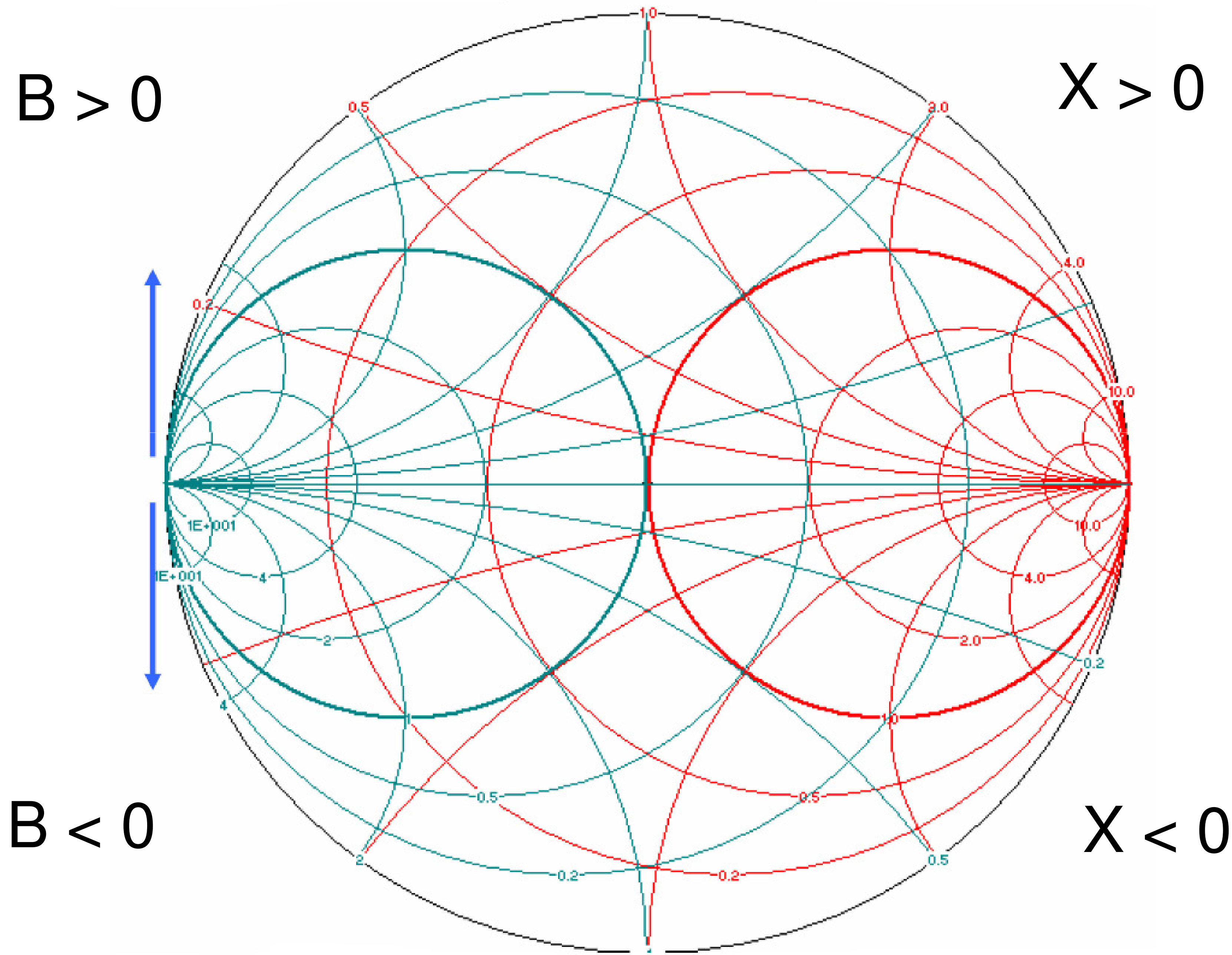
Impedanca



Admitanca



Impedančno-admitančni Smithov diagram



Smithov diagram, različice

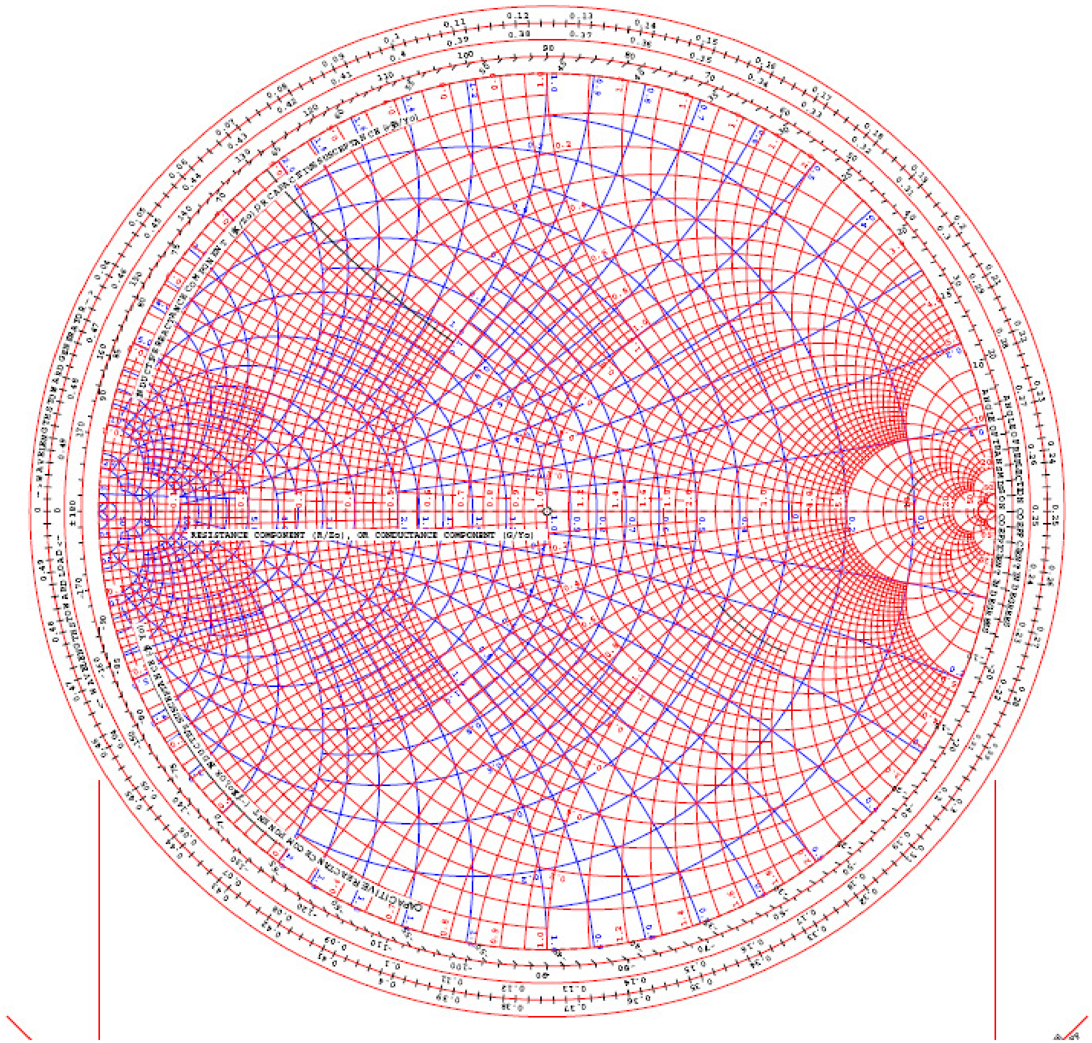
$$z = \frac{1+\Gamma}{1-\Gamma} = \frac{1+|\Gamma|e^{j\theta_\Gamma}}{1-|\Gamma|e^{j\theta_\Gamma}} \qquad \Gamma = \frac{z-1}{z+1}$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad |\Gamma| = \frac{S-1}{S+1}$$

$z = Z/Z_o$...normirana impedanca

Γ.............odbojnost

S.............razmerje stojnega vala



Smithov diagram na krogli ? (Microwave Magazine)

admitančni diagram (G>0) (pasivna vezja)

impedančni (admitančni) diagram (R<0, G<0) (aktivna vezja)

impedančni diagram (R>0) (pasivna vezja)

Operacije v Smithovem diagramu

1, Pomik proti generatorju

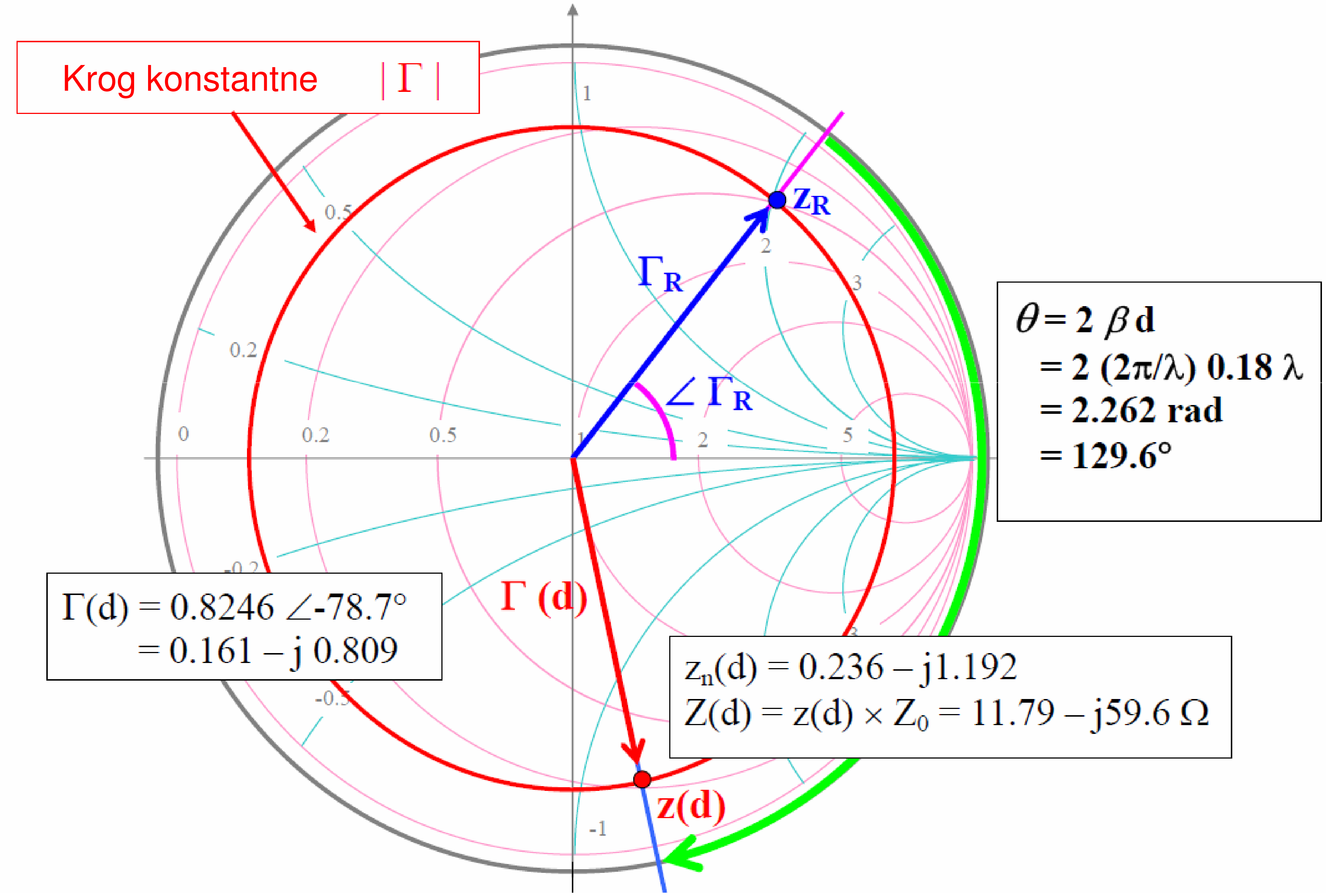
2. Pomik do napetostnega hrbta in vozla

3. Pretvorba impedance v admitanco

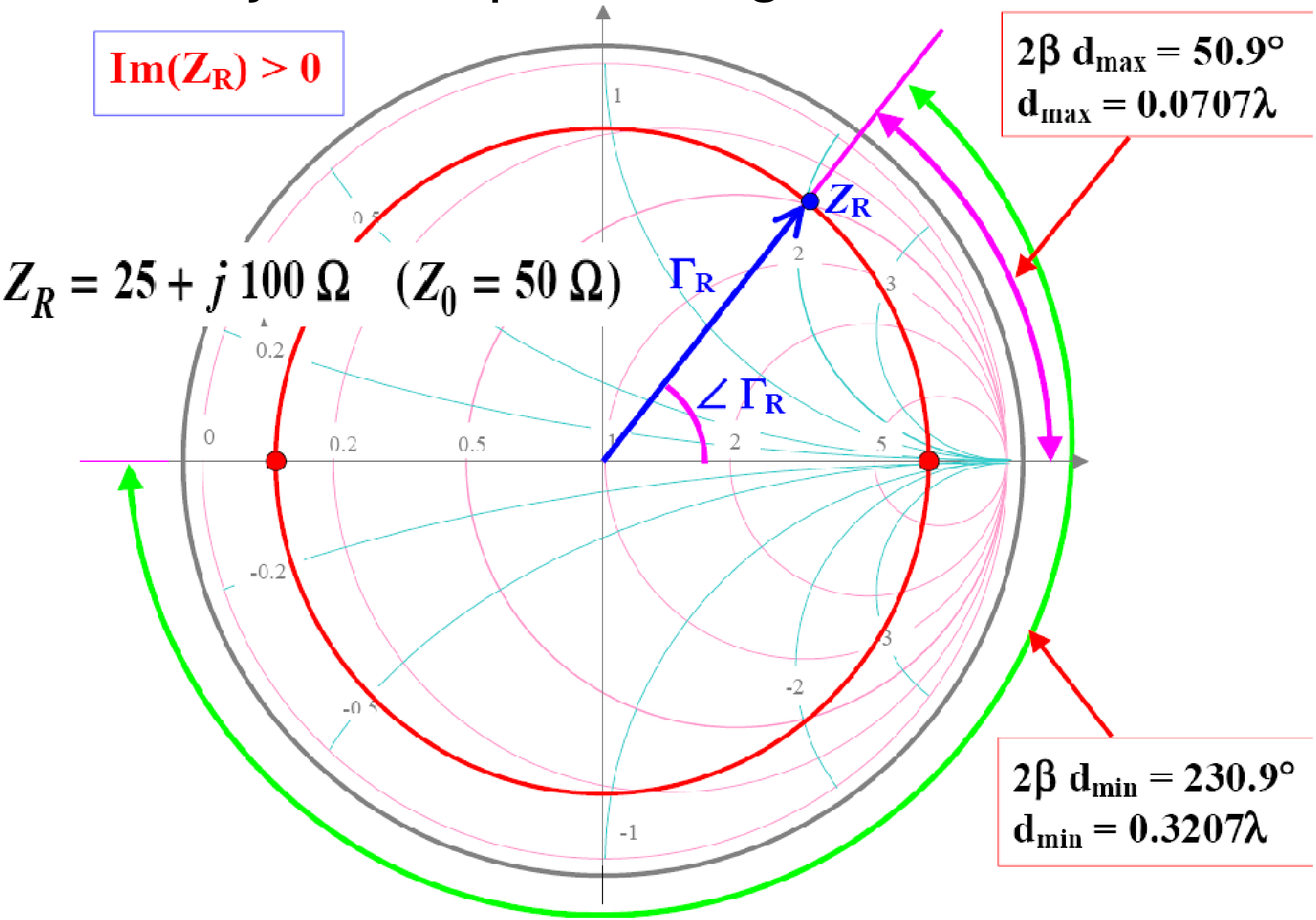
4. Pretvorba admitance v impedanco (admitančni diagram)

5. Postopek prilagajanja

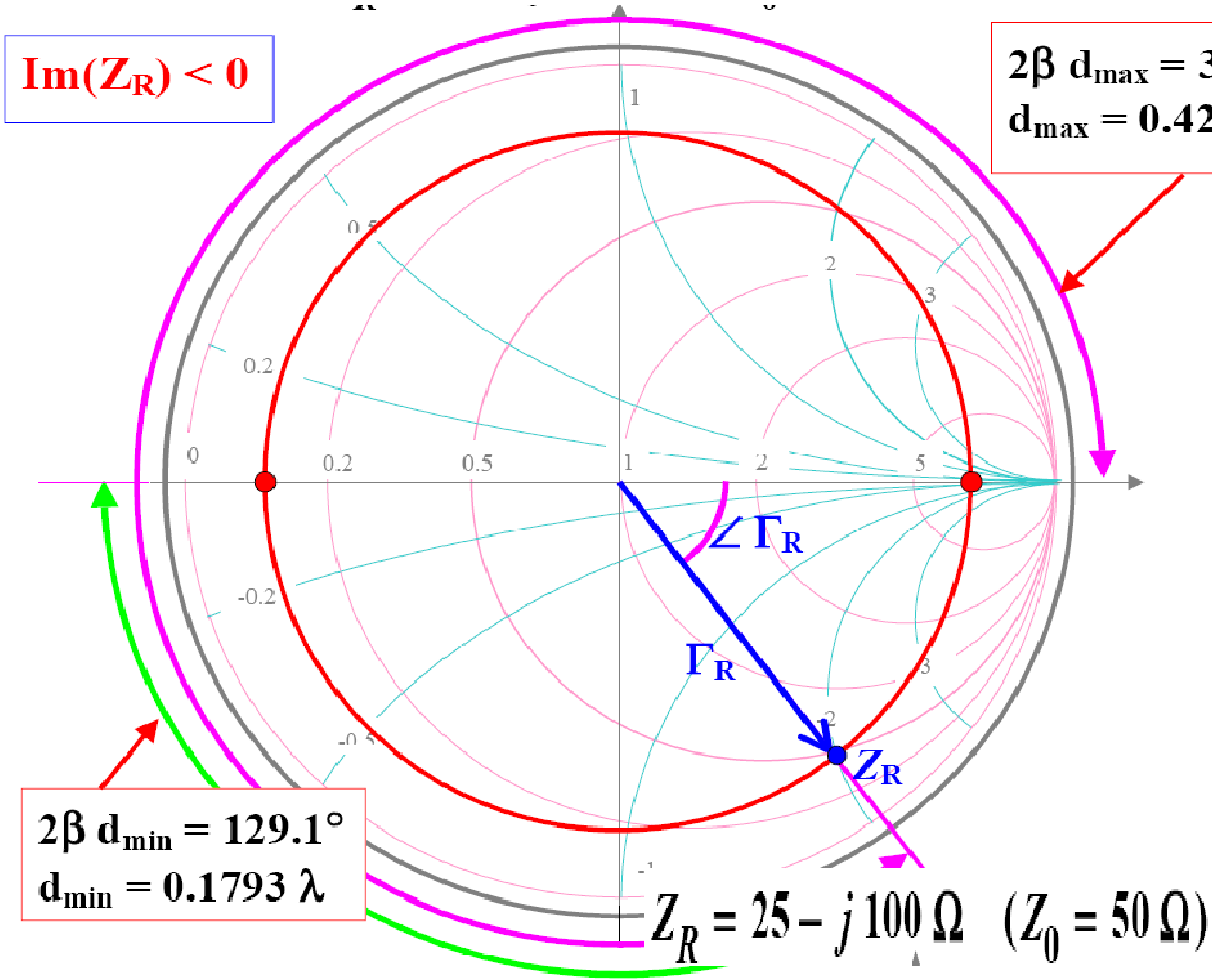
Premik po liniji za dolžino d proti generatorju



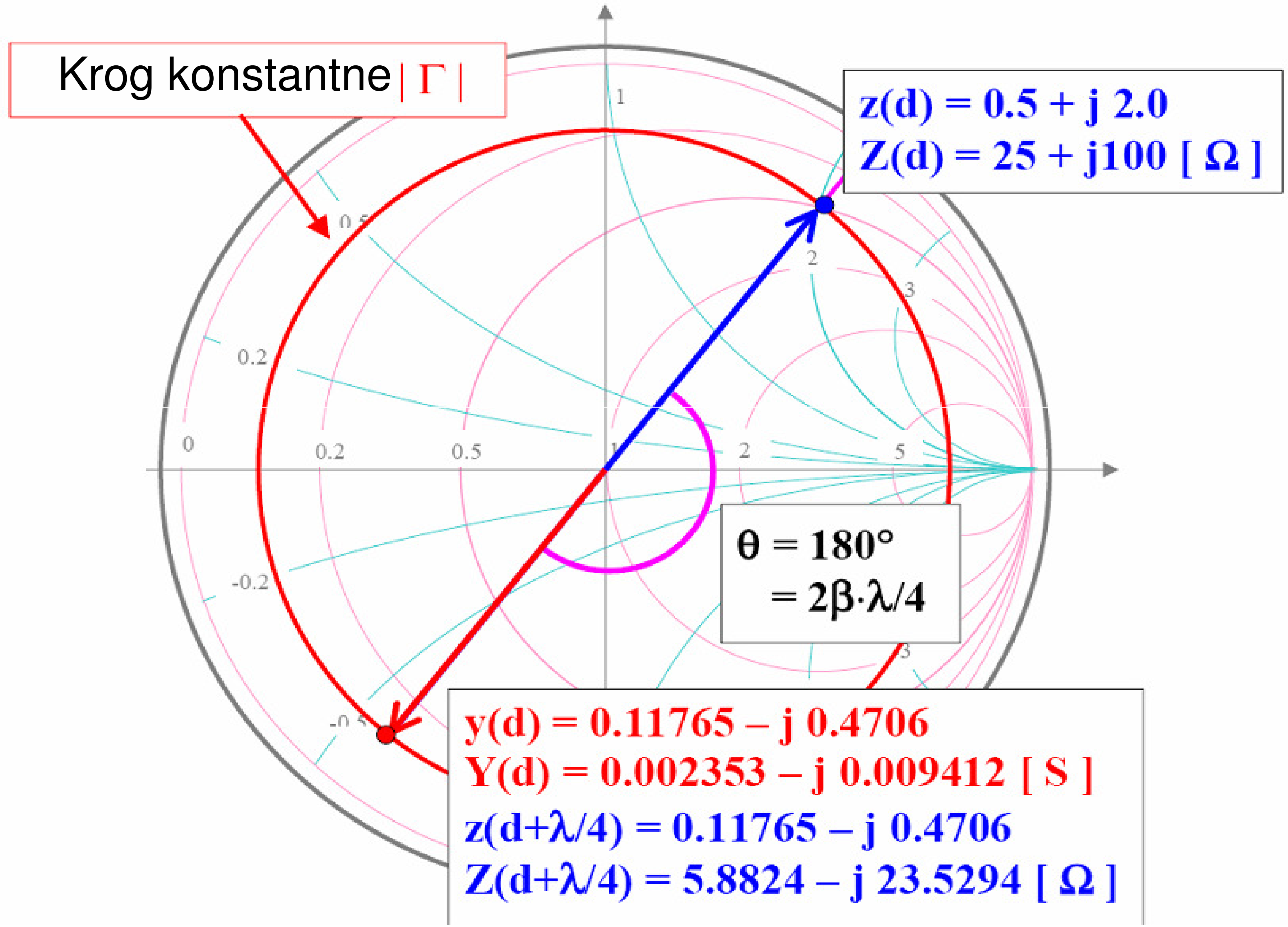
Razdalja do napetostnega hrbta in vozla



Razdalja do napetostnega hrbta in vozla

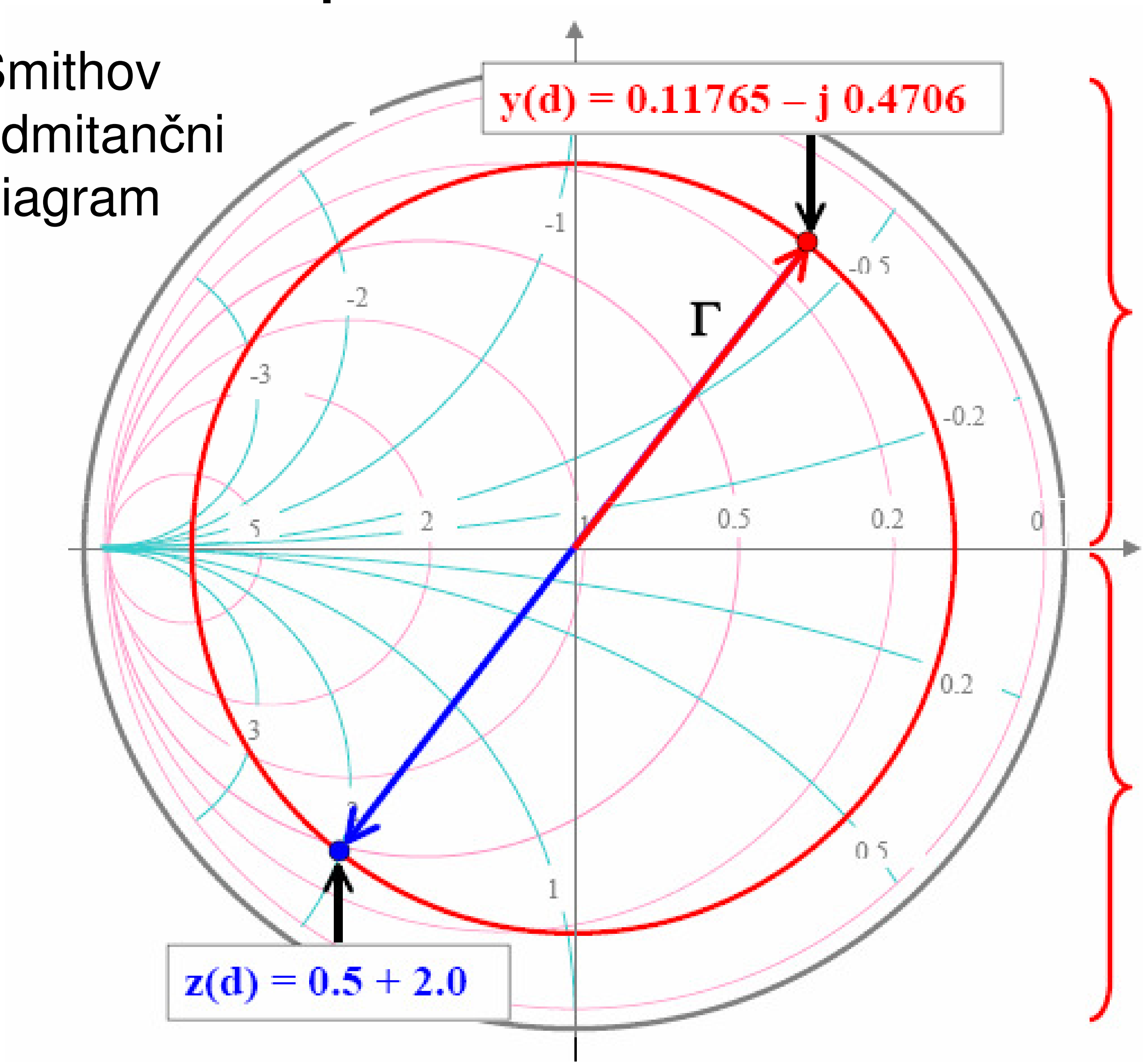


Pretvorba impedance v admitanco



Impedanca v admitanco

Smithov admitančni diagram



Negativna (induktivna) susceptanca

Pozitivna (kapacitivna) susceptanca

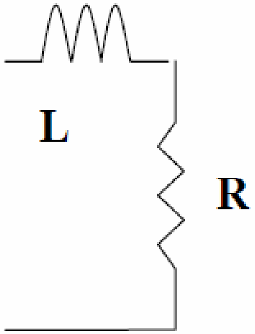
Pretvorba impedance v admitanco

$$z_n = r + j\,x \qquad\qquad y_n = g + j\,b = \frac{1}{r + j\,x}$$

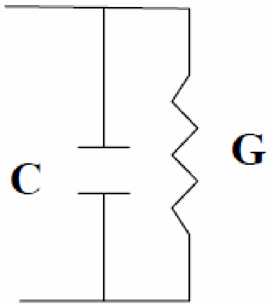
$$y_n = \frac{r - j\,x}{(r + j\,x)(r - j\,x)} = \frac{r - jx}{r^2 + x^2}$$

$$\Rightarrow \qquad g = \frac{r}{r^2 + x^2} \qquad\qquad b = -\frac{x}{r^2 + x^2}$$

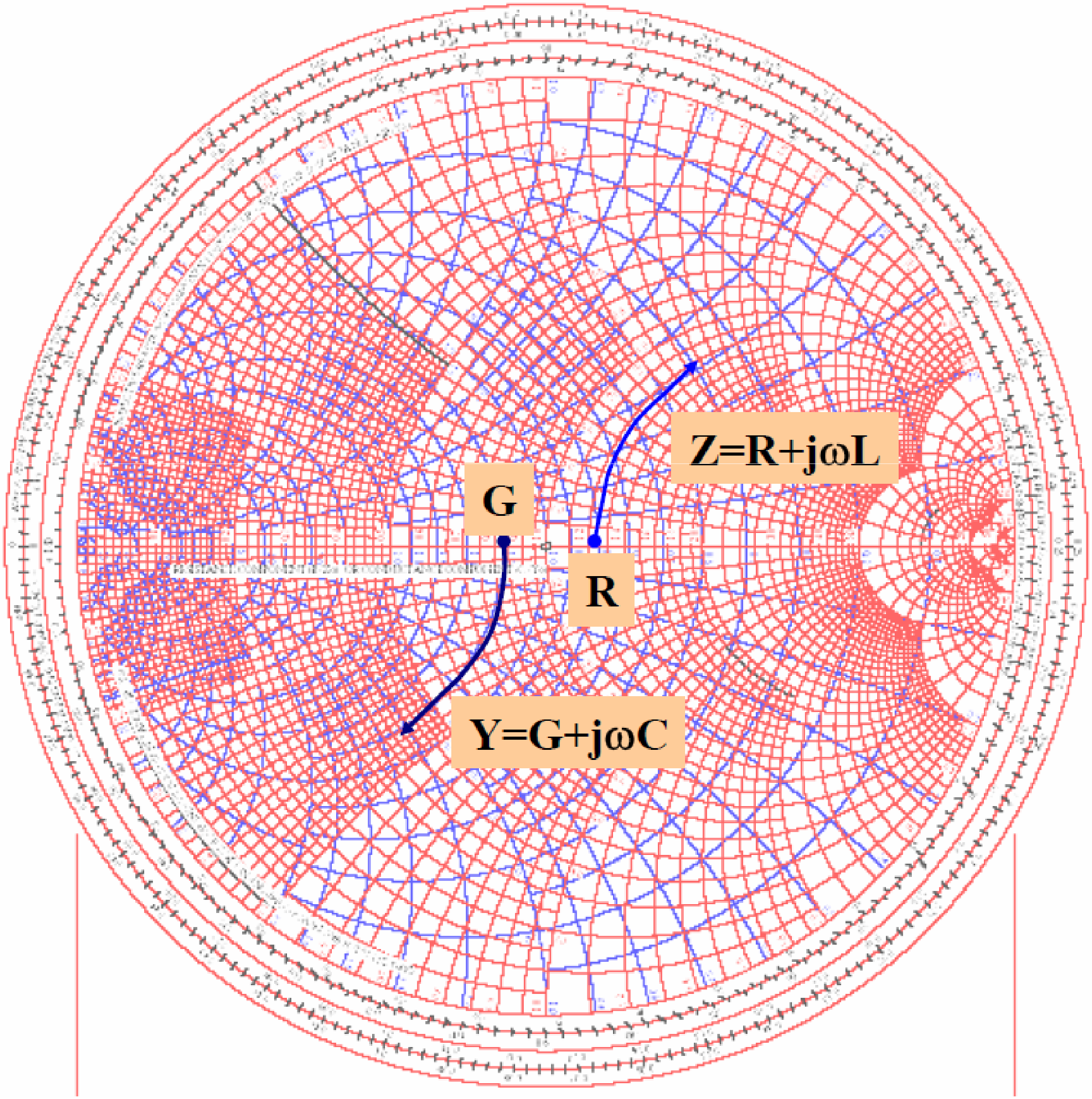
Postopki v Smithovem diagramu 1/8



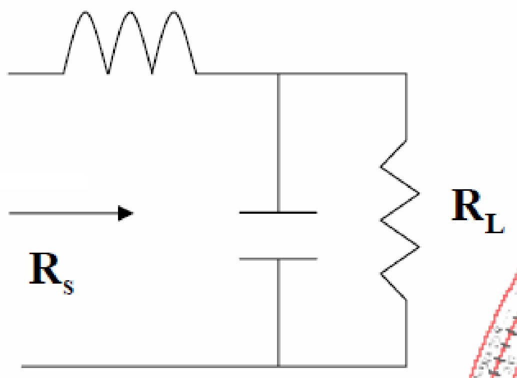
$$Z = R + j\omega L$$



$$Y = G + j\omega C$$

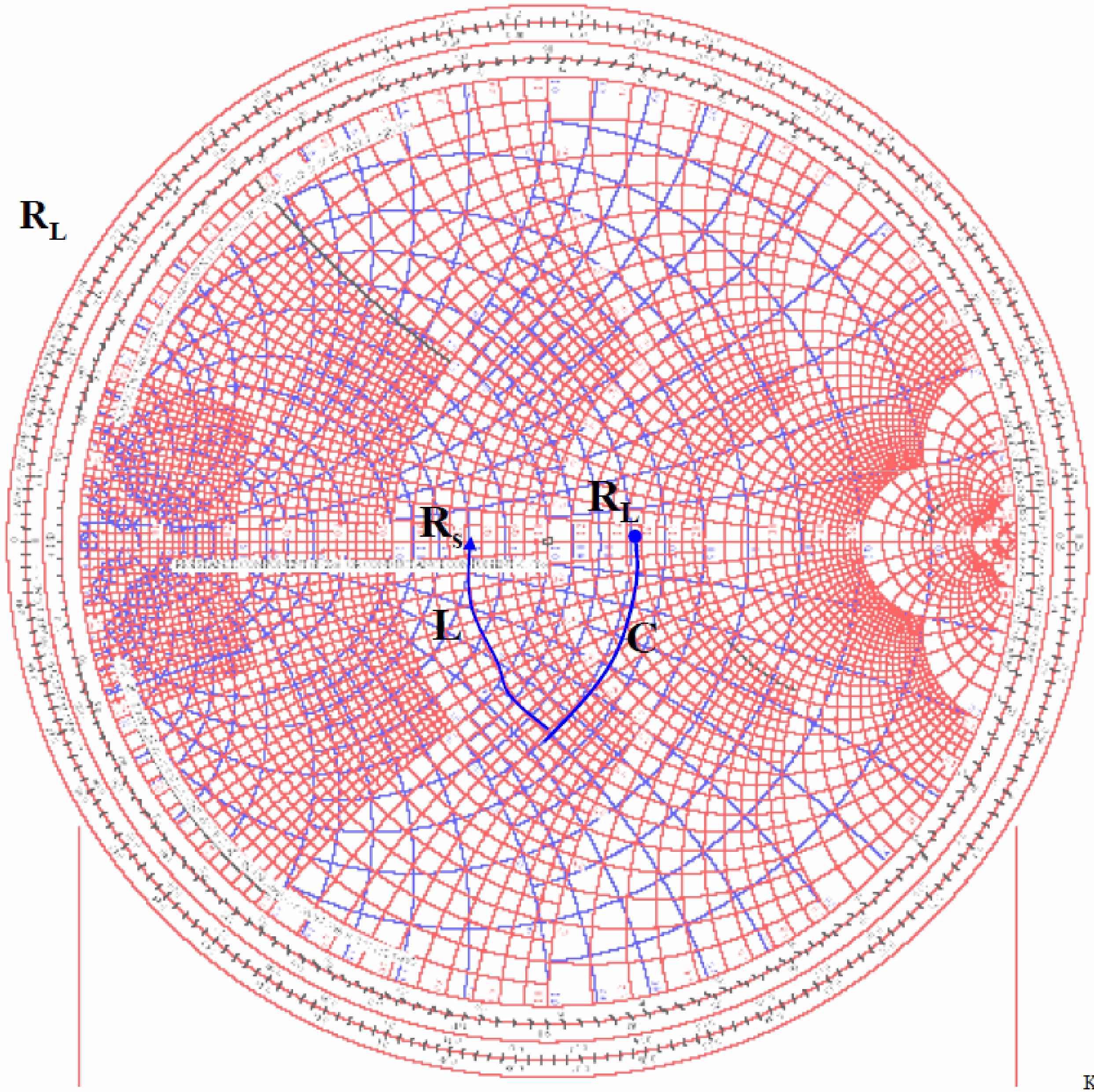


Postopki v Smithovem diagramu 2/8

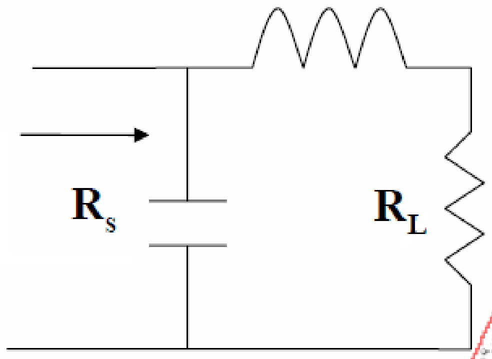


$(R_L > R_s)$

Prilagodilno vezje

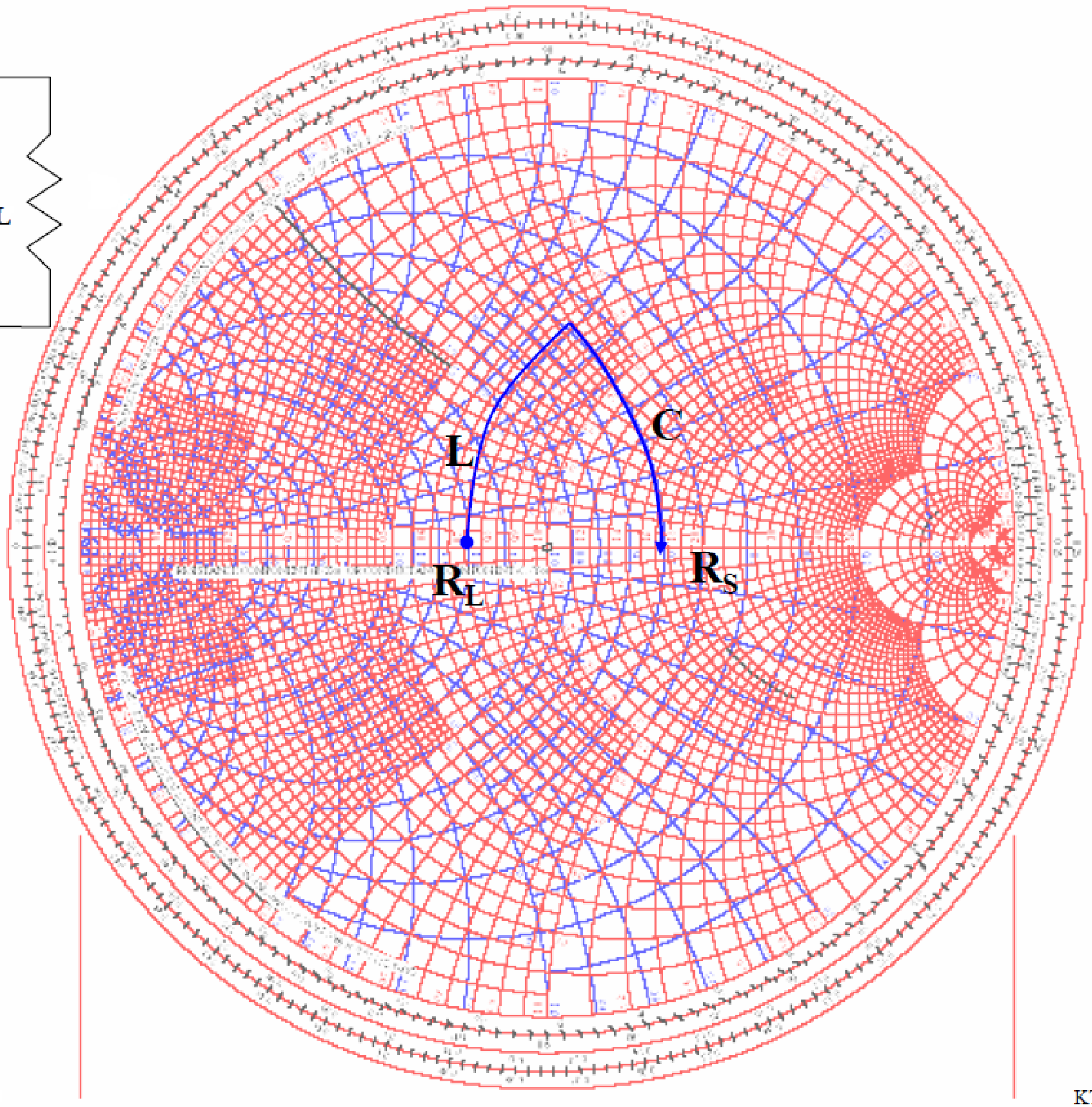


Postopki v Smithovem diagramu 3/8



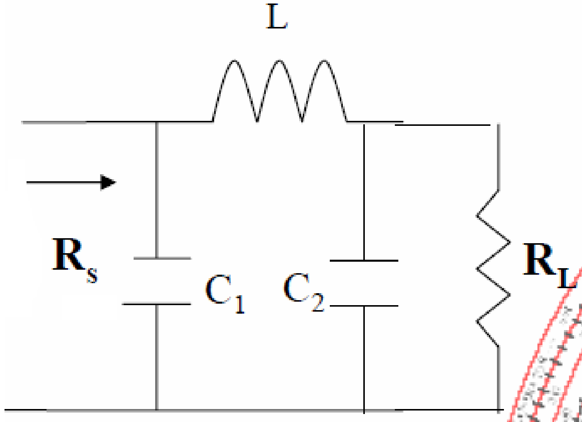
$(R_L < R_s)$

Prilagodilno vezje



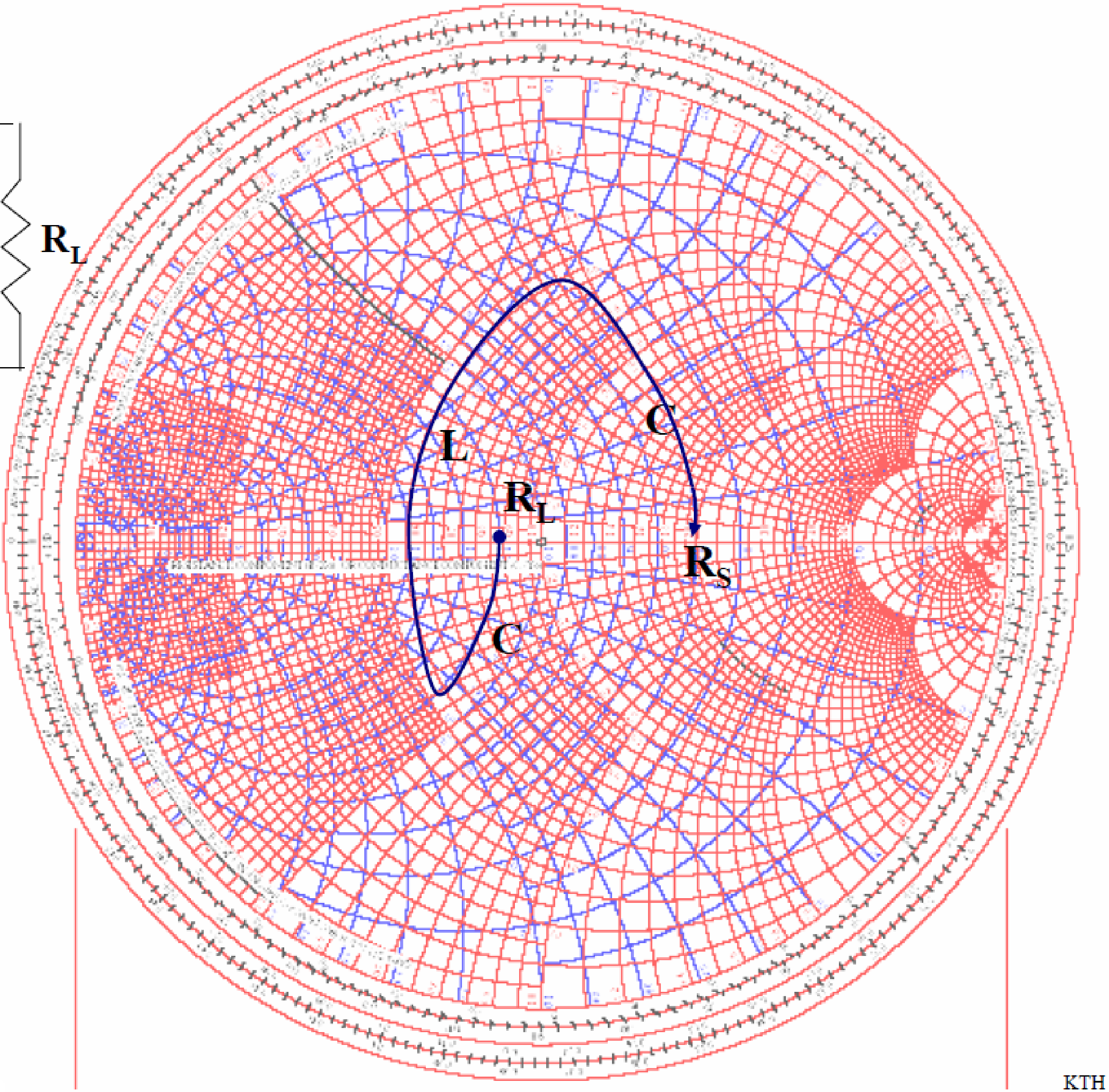
KTH

Postopki v Smithovem diagramu 4/8

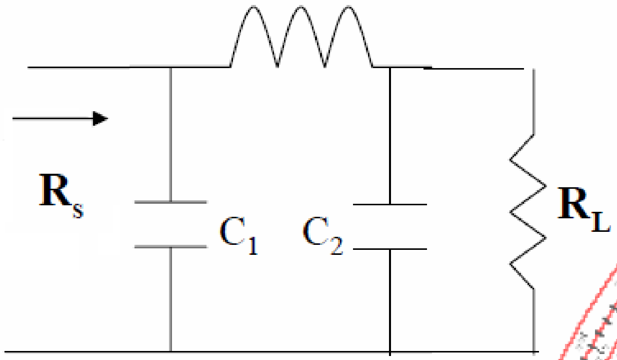


$(R_L < R_s)$

Prilagodilno vezje

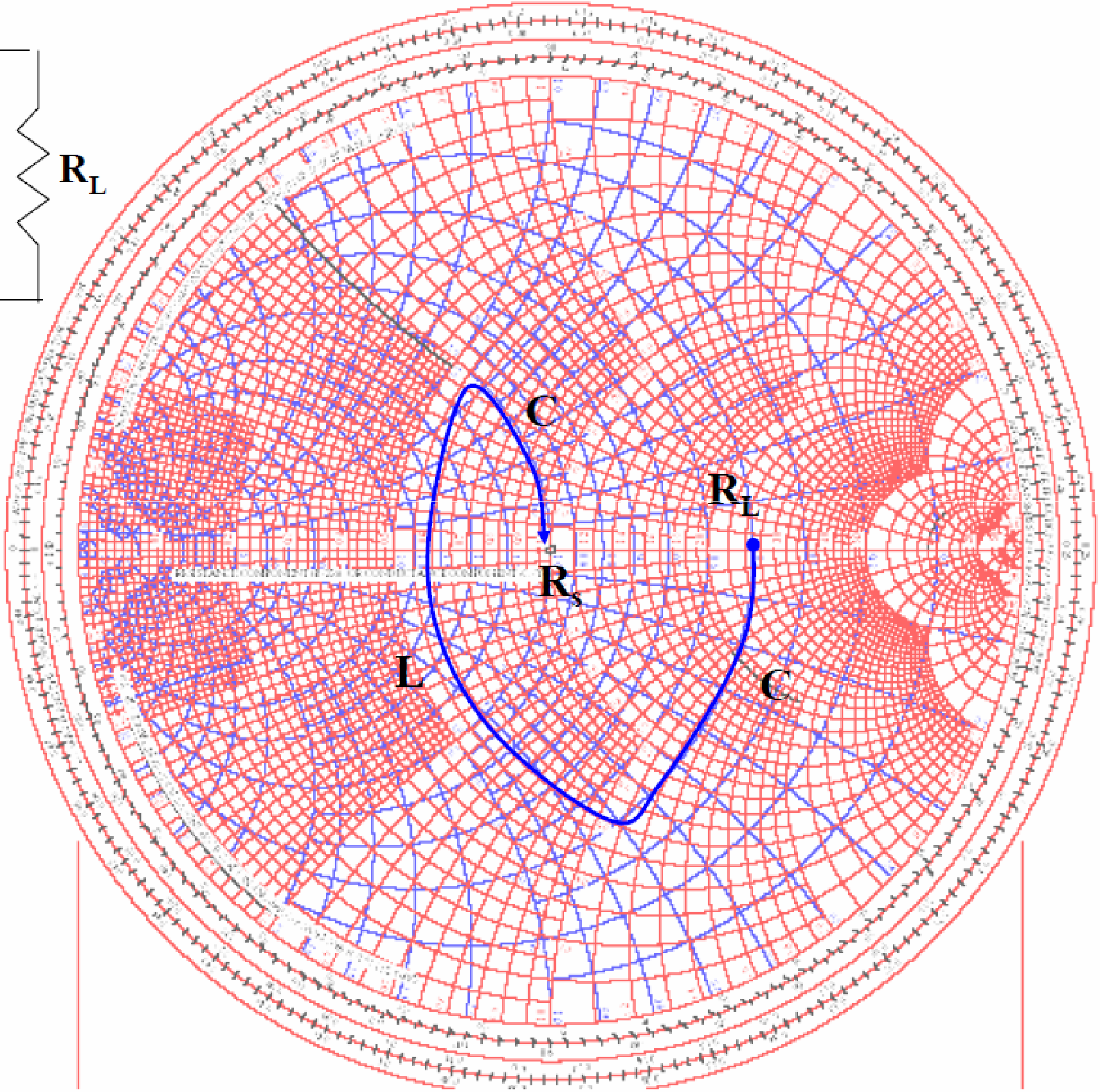


Postopki v Smithovem diagramu 5/8

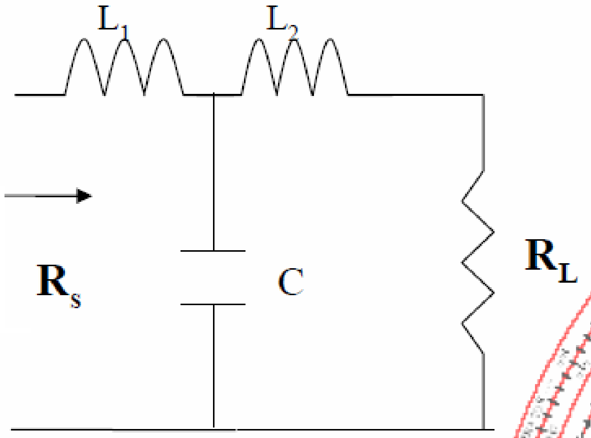


$(R_L > R_s)$

Prilagodilno vezje

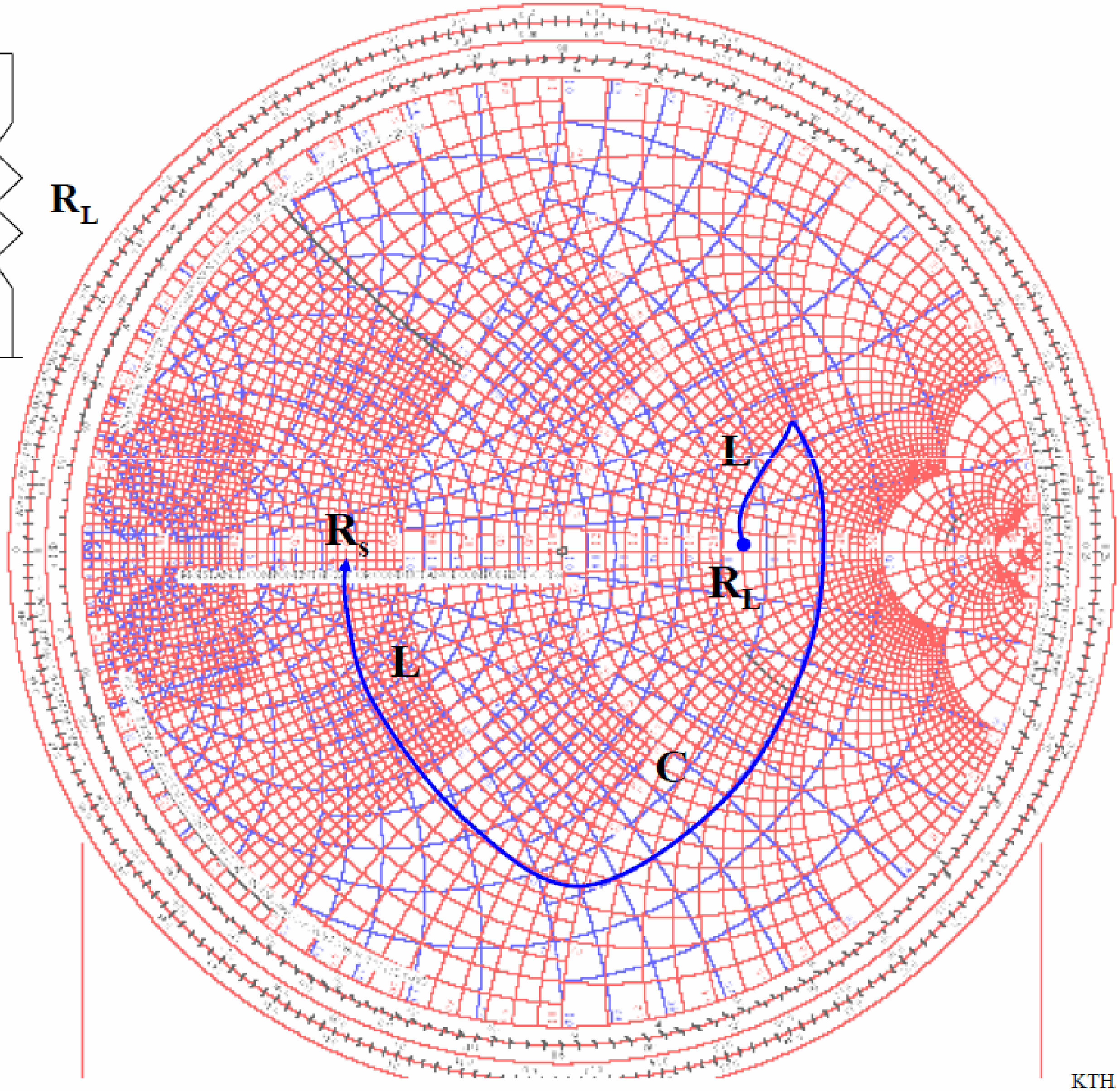


Postopki v Smithovem diagramu 6/8

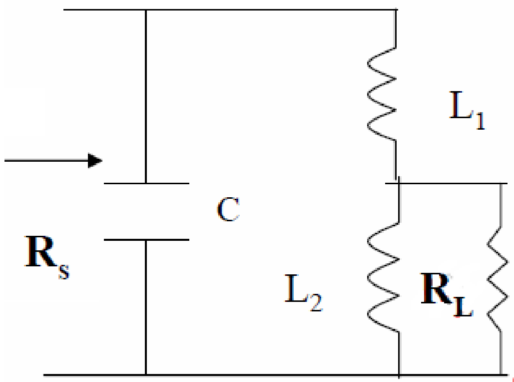


$(R_L > R_s)$

Prilagodilno vezje

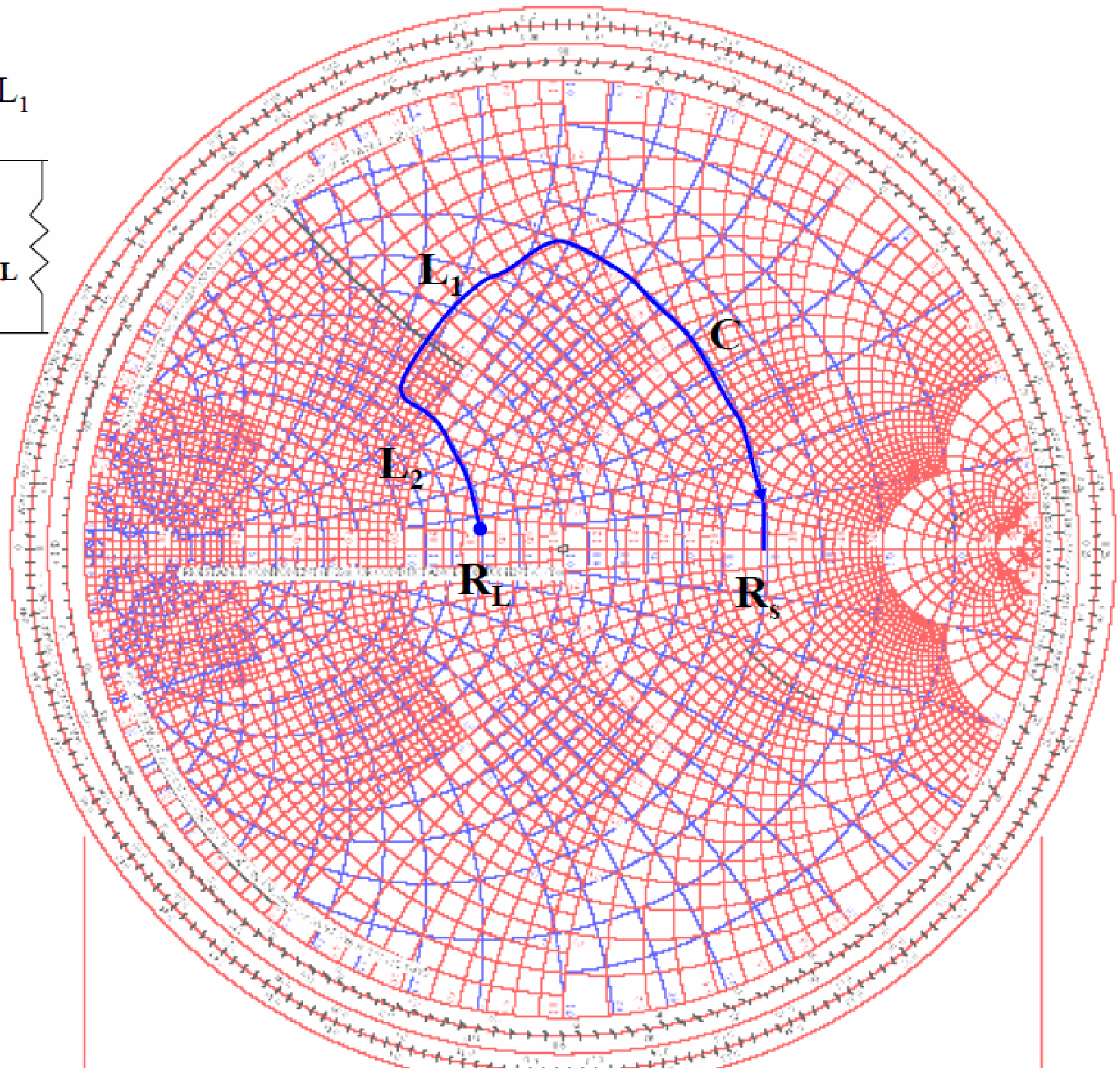


Postopki v Smithovem diagramu 7/8

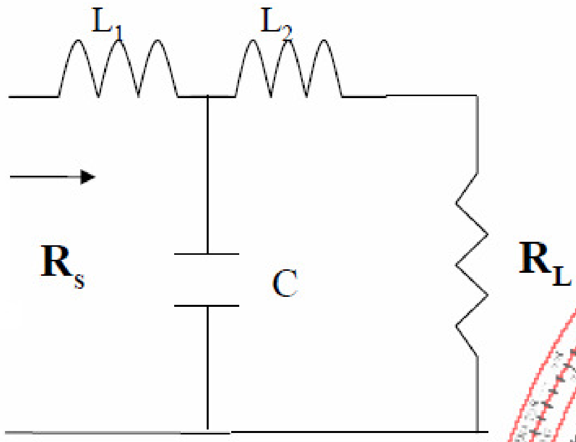


$(R_L < R_s)$

Prilagodilno vezje

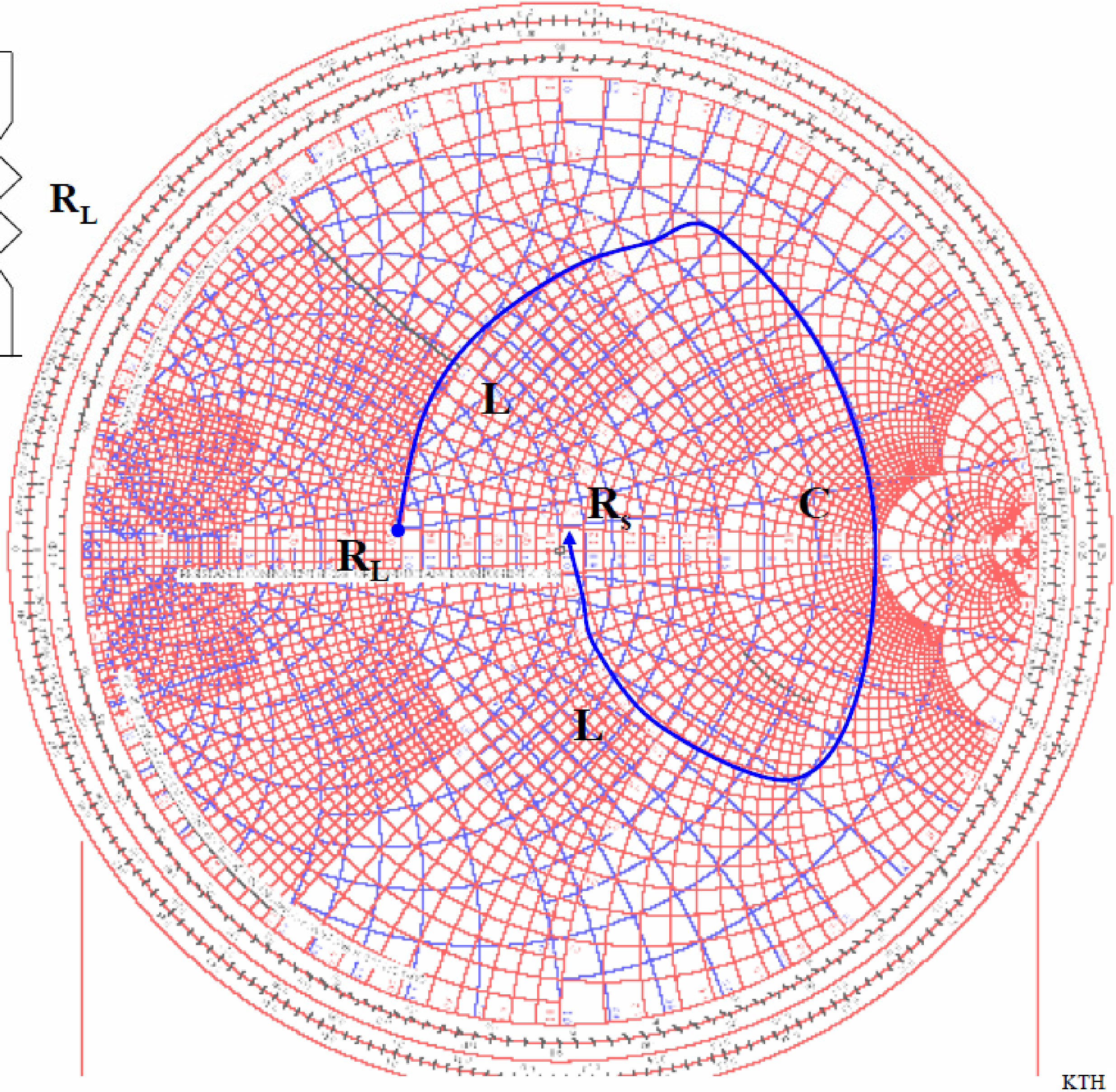


Postopki v Smithovem diagramu 8/8

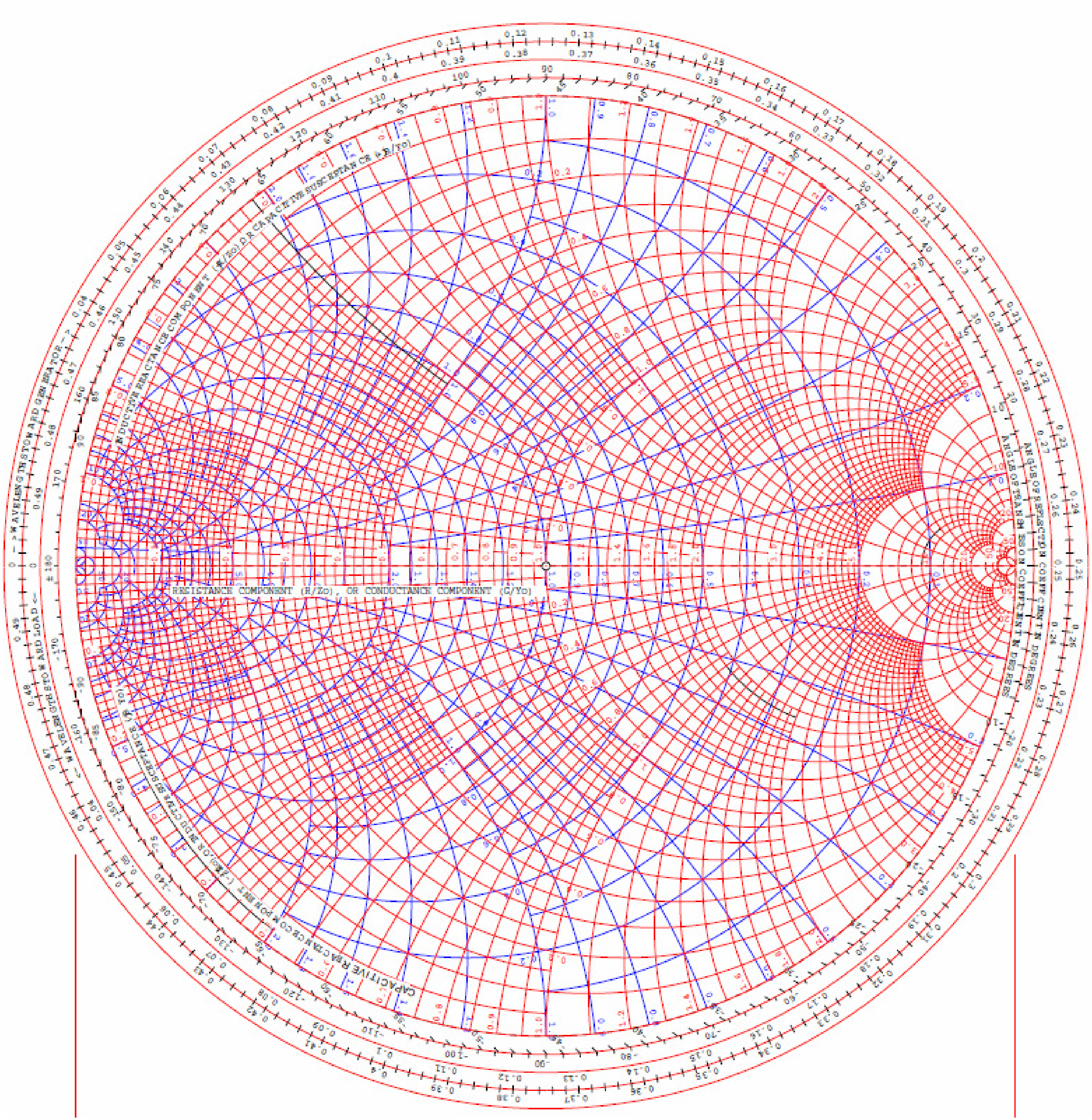


$(R_L < R_s)$

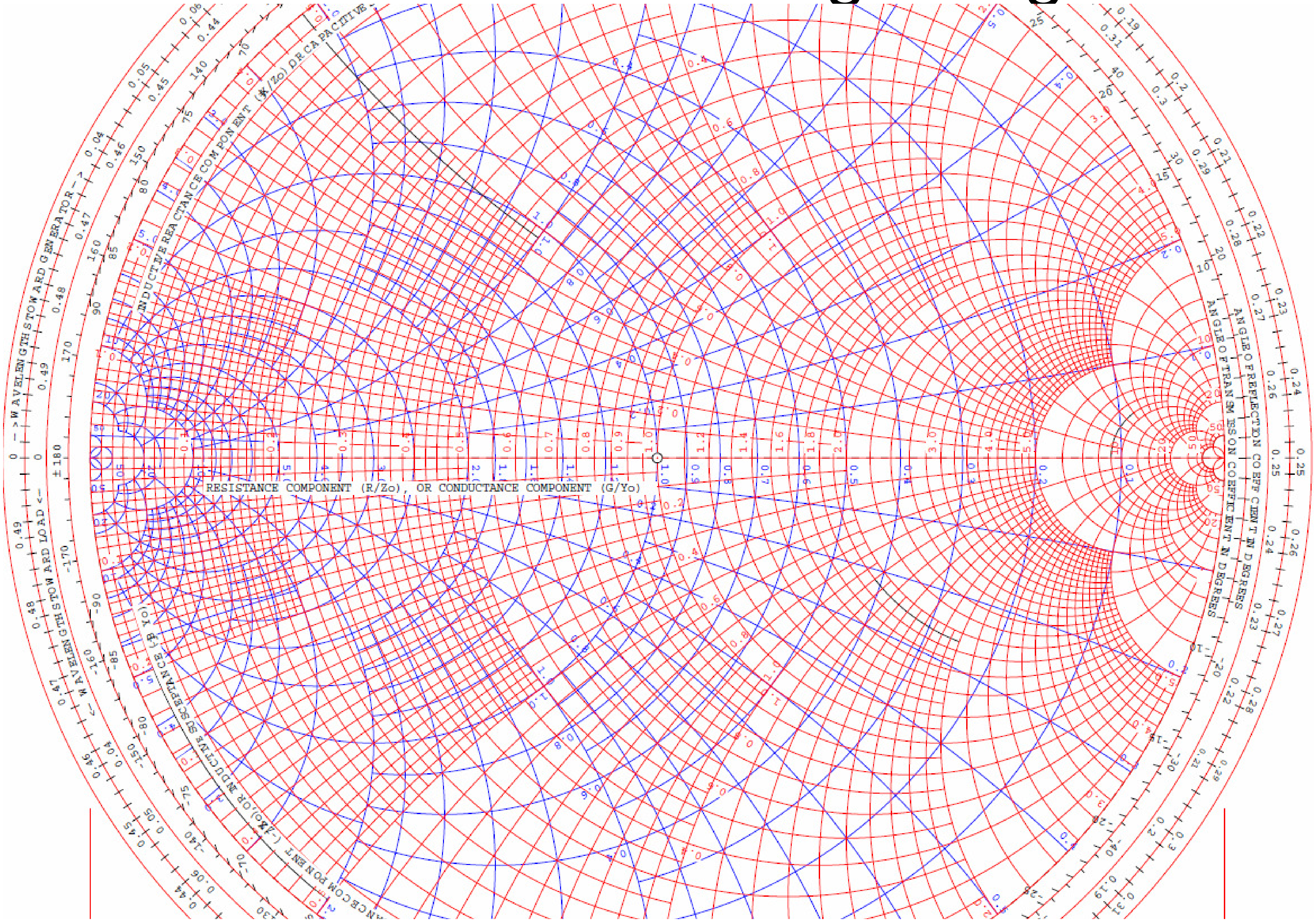
Prilagodilno vezje

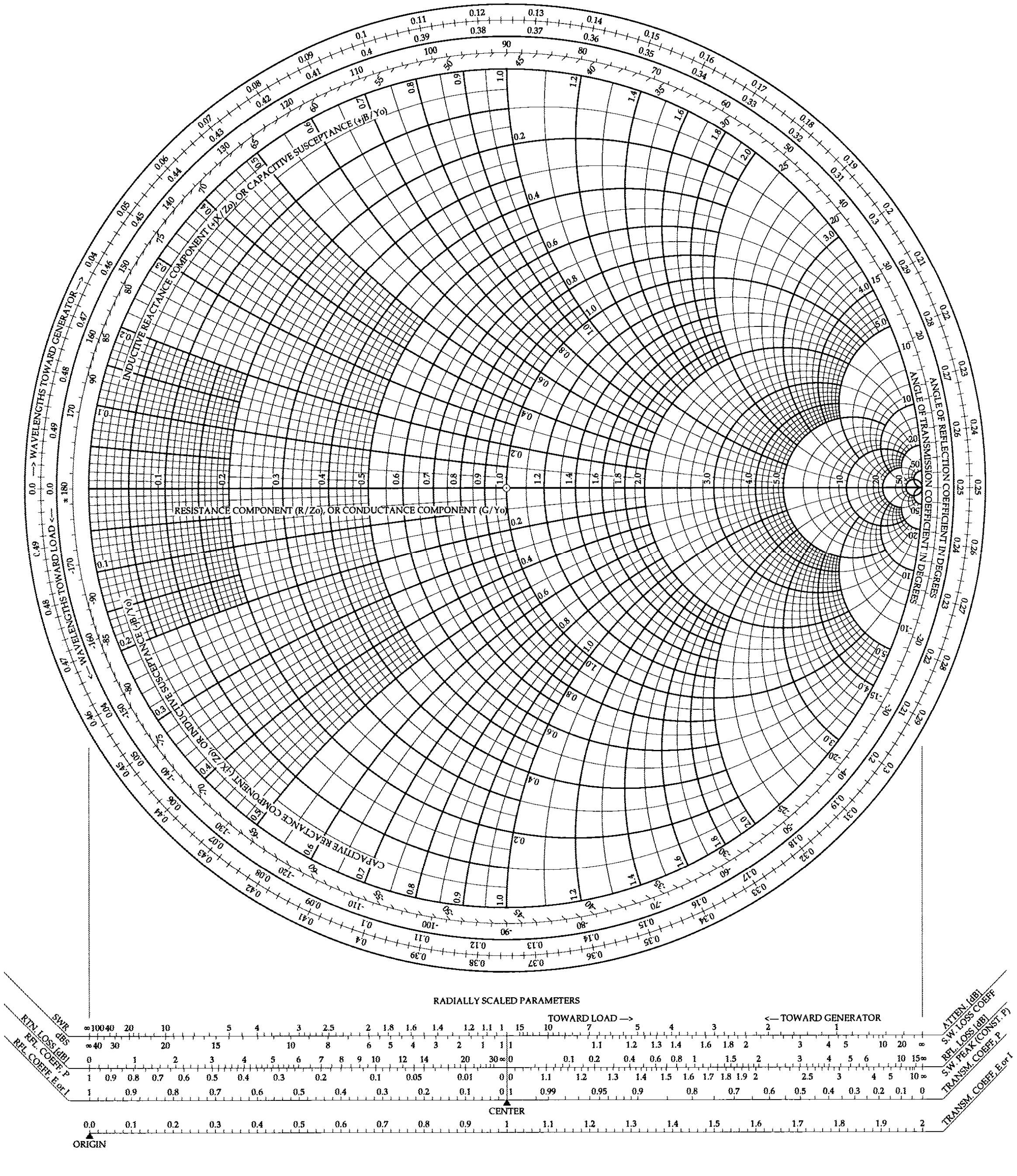


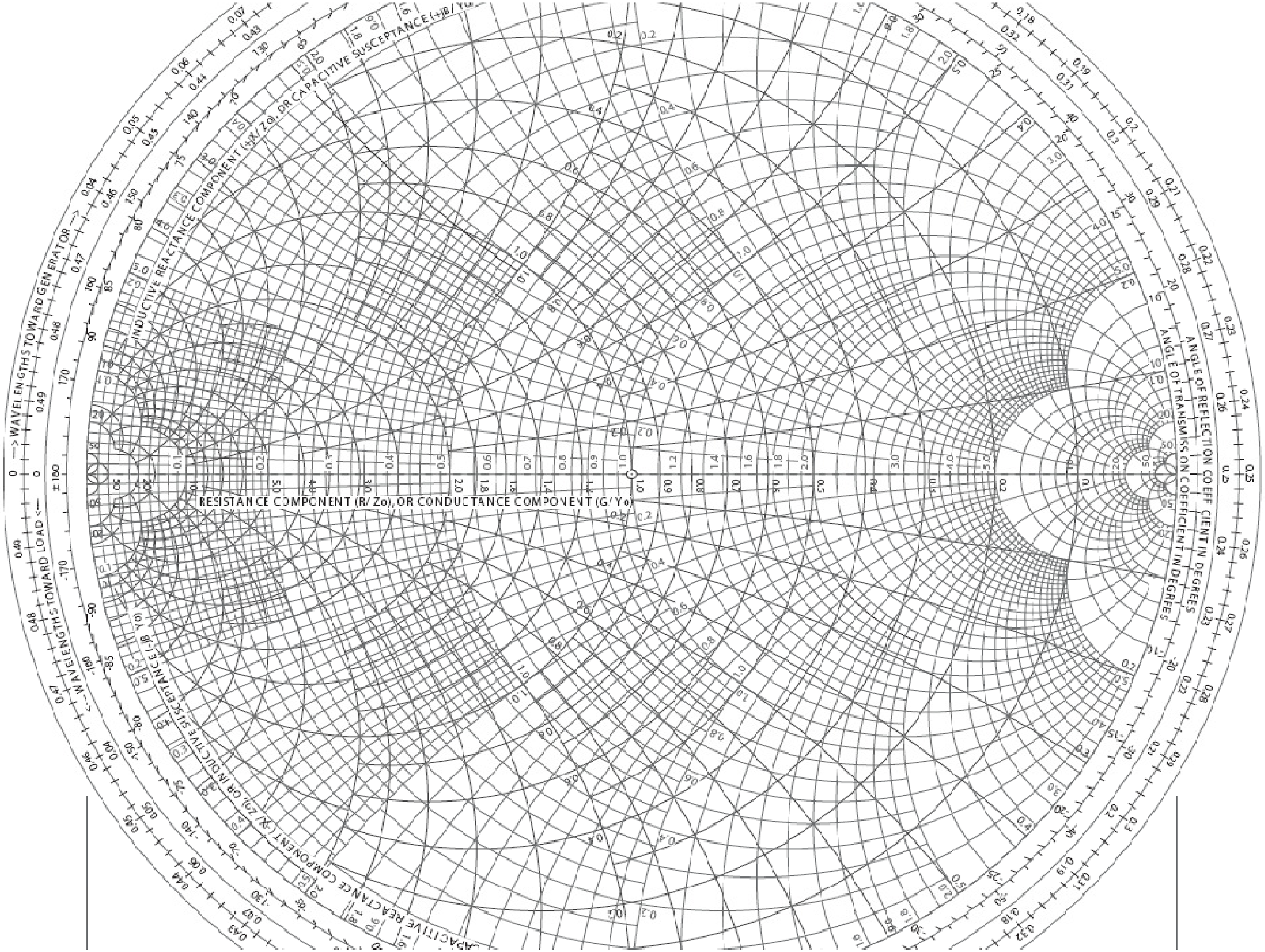
Smithovi diagrami za praktično uporabo


RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)

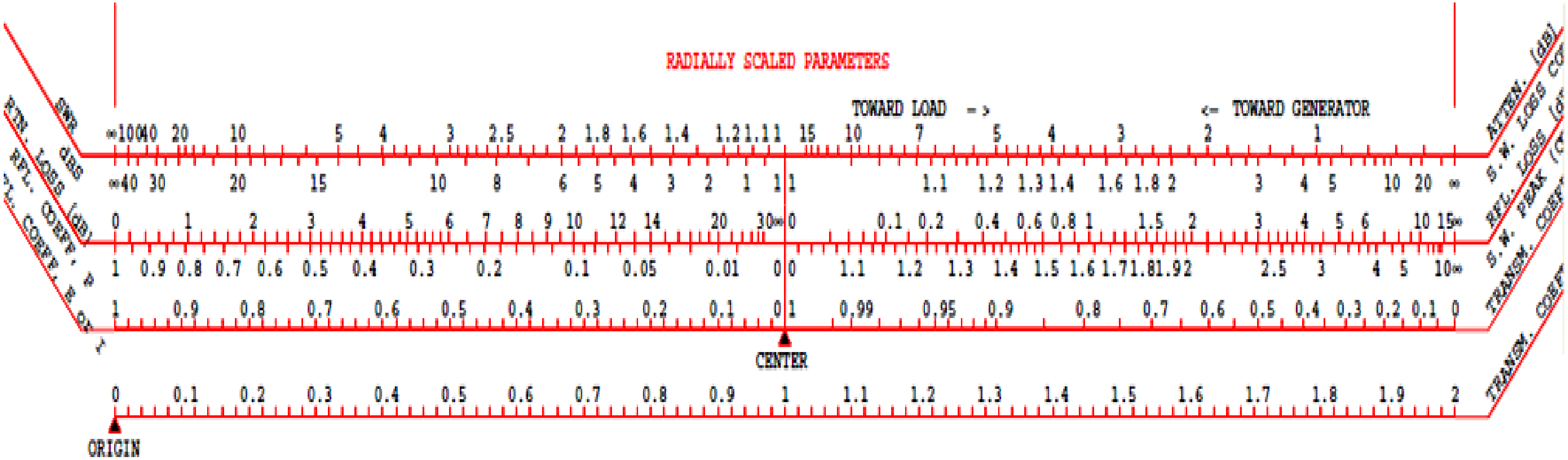
Središčni del Smithovega diagrama




RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
0.0 —> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—
RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF, P
RFL. COEFF, E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF, P
TRANSM. COEFF, E or I
CENTER
ORIGIN


RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
—> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—

Kalkulator


RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
SWR
dBS
RTN. LOSS (dB)
RFL. COEFF. P
RFL. COEFF. E OR I
ATTEN. (dB)
S.W. LOSS COEFF.
RFL. LOSS (dB)
S.W. PEAK (CONST. P)
TRANSM. COEFF. P
TRANSM. COEFF. E OR I
CENTER
ORIGIN

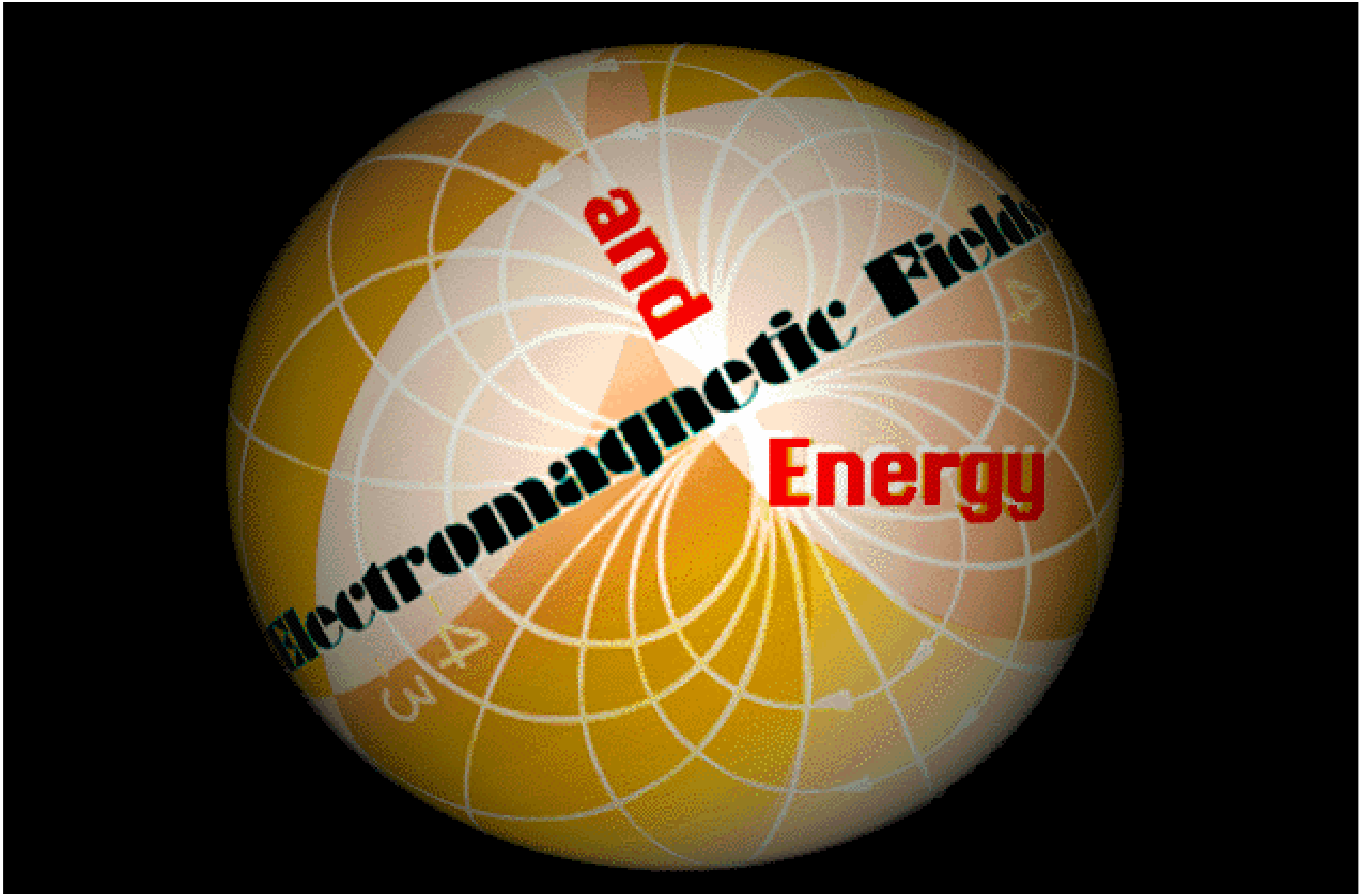
Sklep

1. Kljub novim možnostim, ki jih prinašajo računalniške in optimizacijske metode ter uporaba metod na podlagi matrike S, ostaja Smithov diagram nepogrešljiv pripomoček pri:

- nazorni predstavitvi problema in njegove rešitve
- pri prikazovanju računskih in merilnih rezultatov.

2. Kot impedančni kalkulator je Smithov diagram ohranil pomen dobrega pripomočka za hitro ocenitev rezultatov.

Smithov diagram na krogli



Večantenski sistemi

"Revolucionalna novost, ki dramatično izboljšuje radijsko zvezo v najzahtevnejših pogojih propagacije in mobilnosti".