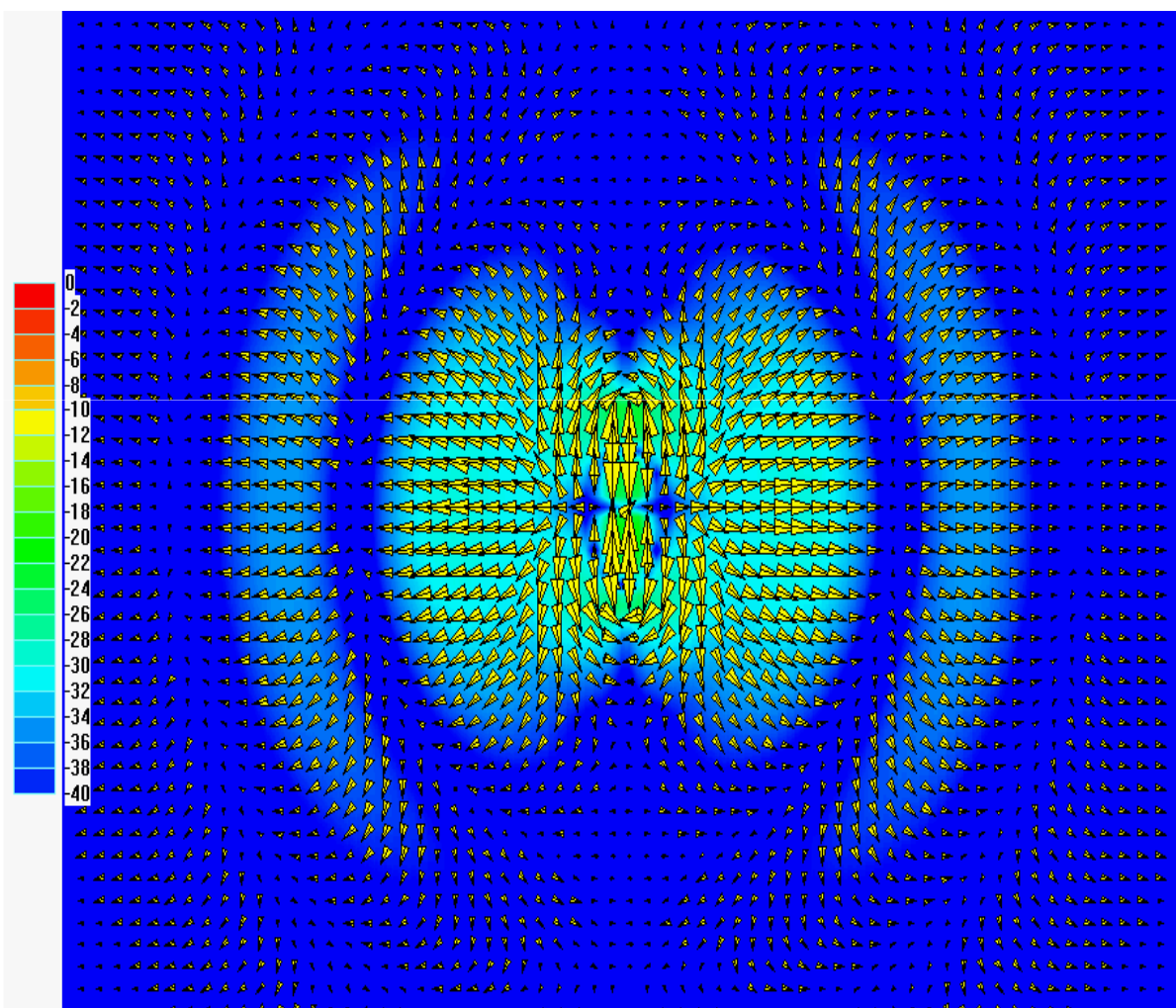


Elementarni viri sevanja



Mobitel d.d.,
Izobraževanje
6. 3. 2009,
predavanje 6

Prof. dr. Jožko
Budin

Elementarni viri sevanja

Vsebina:

- Sevalni pojavi
- Elementarni viri sevanja
 - Električni tokovni element
 - Magnetni tokovni element
- Huygensov vir
- Sevana moč in sevalna upornost
- Smernost, dobitek in efektivna površina

Mobitel d.d. – izobraževanje 6.3.2009, predavanje 6

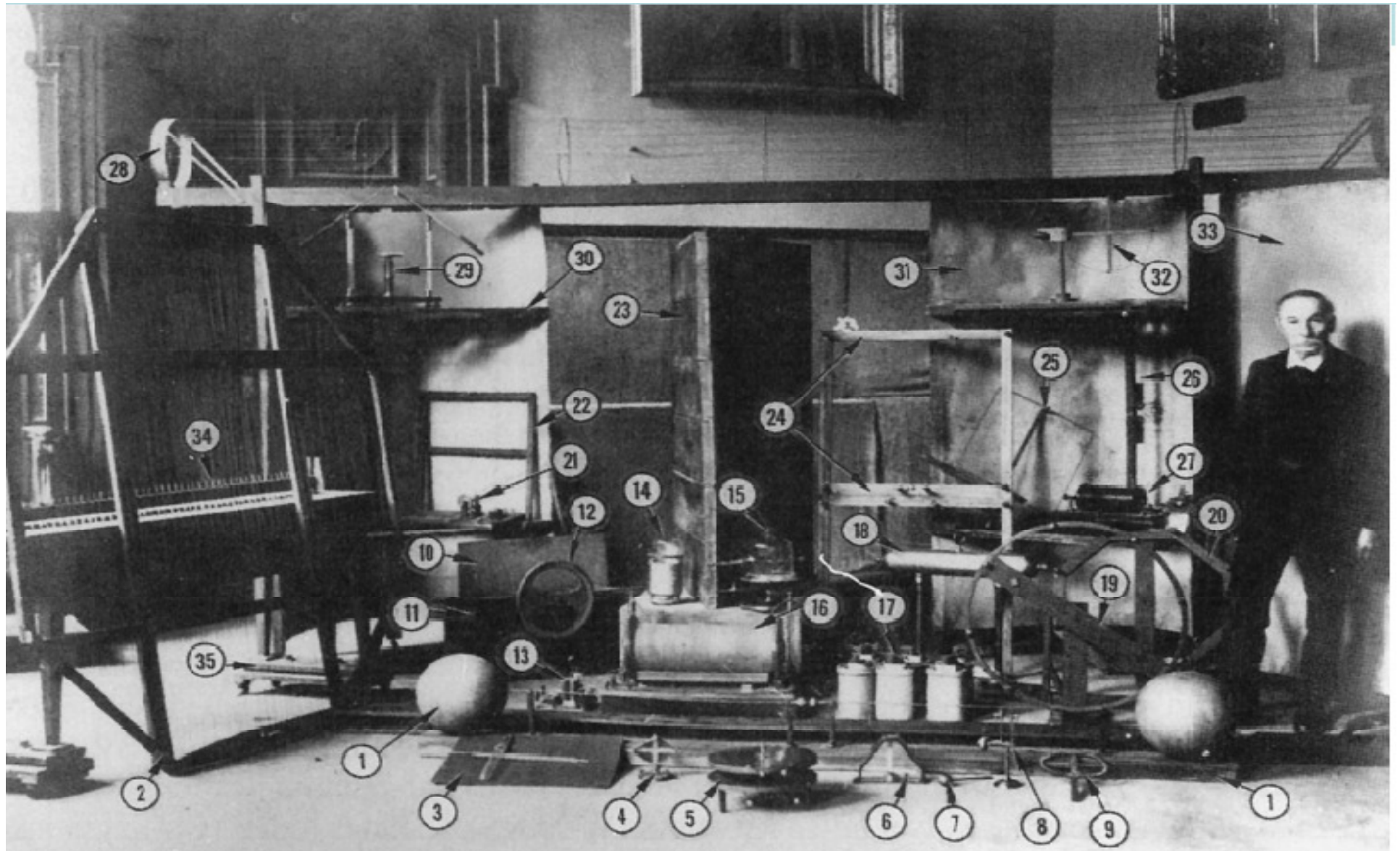
Prof. dr. Jožko Budin

Heinrich Hertz



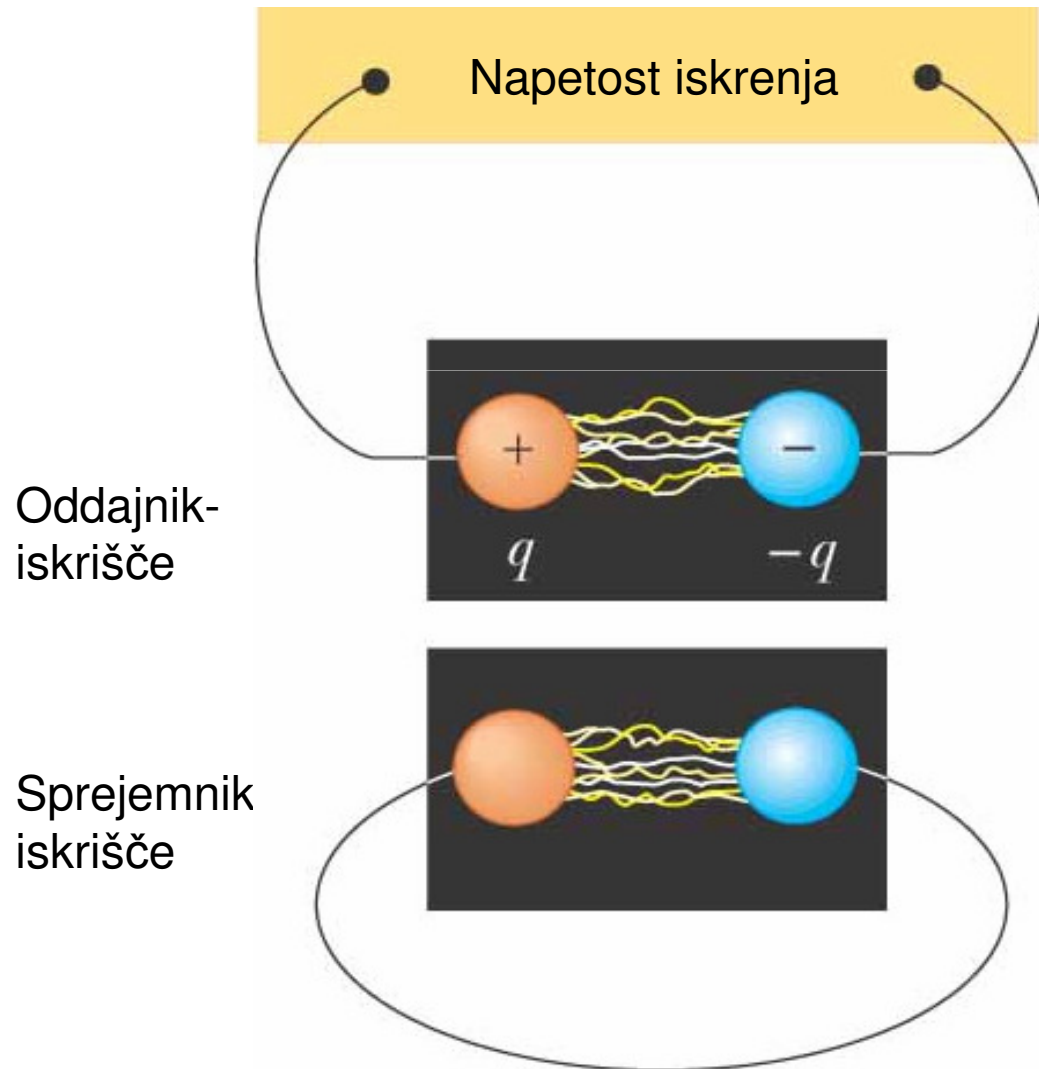
“ Ne mislim, da bodo radijski valovi,
ki sem jih odkril, imeli kakršnokoli
p r a k t i č n o u p o r a b o ” .

1887: eksperimentalno odkritje radijskih valov na $\lambda = 66$ cm.
Polje je polarizirano.



Original apparatus used by Hertz for his electromagnetics experiments. (1) 50 MHz transmitter spark gap and loaded dipole antenna. (2) Parallel wire grid for polarization experiments. (3) Vacuum apparatus for cathode ray experiments. (4) Hot-wire galvanometer. (5) Reiss or Knochenhauer spirals. (6) Rolled-paper galvanometer. (7) metal sphere probe. (8) Reiss spark micrometer. (9) Coaxial transmission line. (10-12) Equipment to demonstrate dielectric polarization effects. (13) Mercury induction coil interrupter. (14) Meidinger cell. (15) Vacuum bell jar. (16) High-voltage induction coil. (17) Bunsen cells. (18) Large-area conductor for charge storage. (19) Circular loop receiving antenna. (20) Eight-sided receiver detector. (21) Rotating mirror and mercury interrupter. (22) Square loop receiving antenna. (23) Equipment for refraction and dielectric constant measurement. (24) Two square loop receiving antennas. (25) Square loop receiving antenna. (26) Transmitter dipole. (27) High-voltage induction coil. (28) Coaxial line. (29) High-voltage discharger. (30) Cylindrical parabolic reflector/receiver. (31) Cylindrical parabolic reflector/transmitter. (32) Circular loop receiving antenna. (33) Planar reflector. (34, 35) Battery of accumulators. Photographed on October 1, 1913 at the Bavarian Academy of Science, Munich, Germany, with Hertz's assistant, Julius Amman.

Hertzov poskus

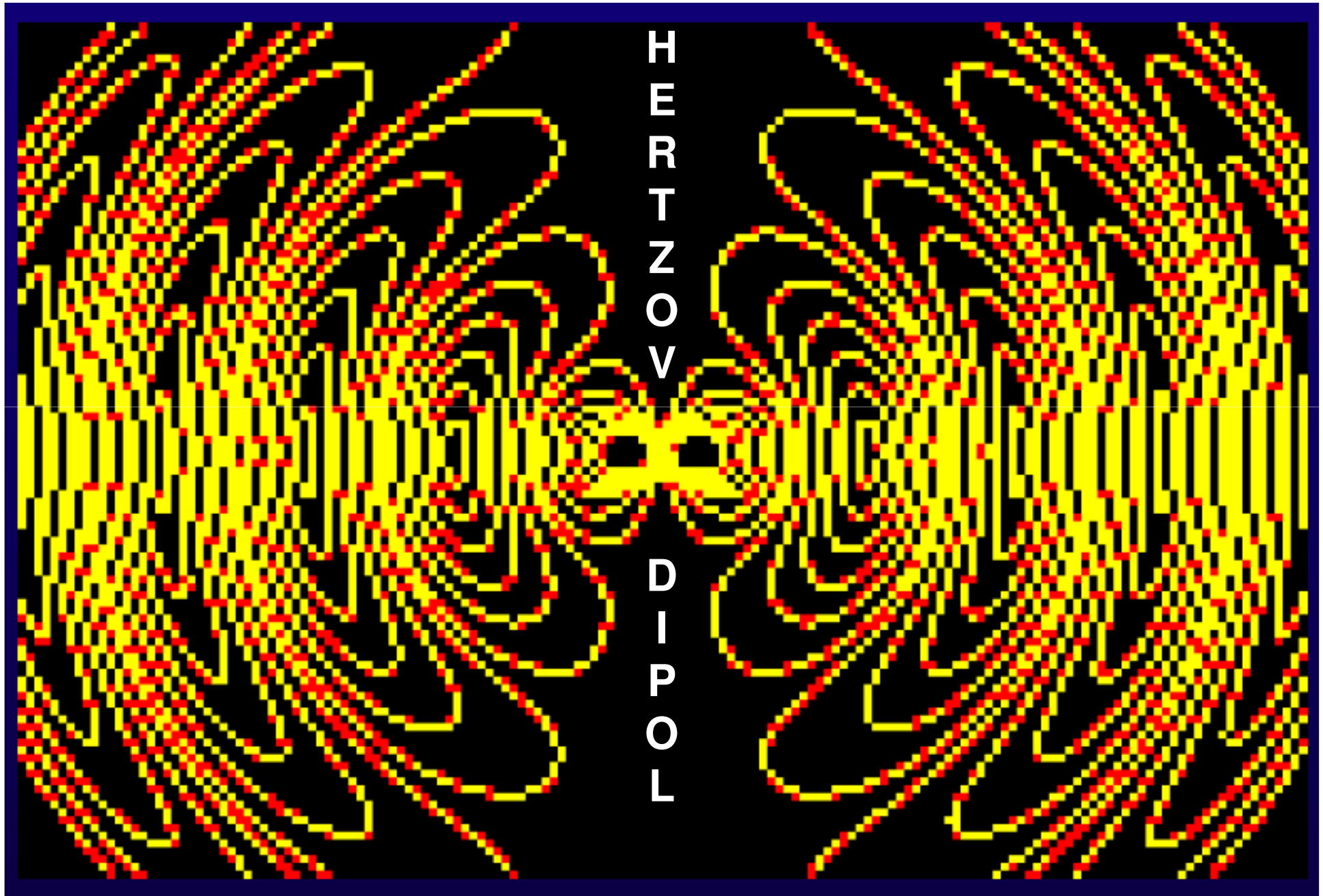


Napetost na indukcijski tuljavi sproži izmenično razelektritev zelo visoke frekvence.

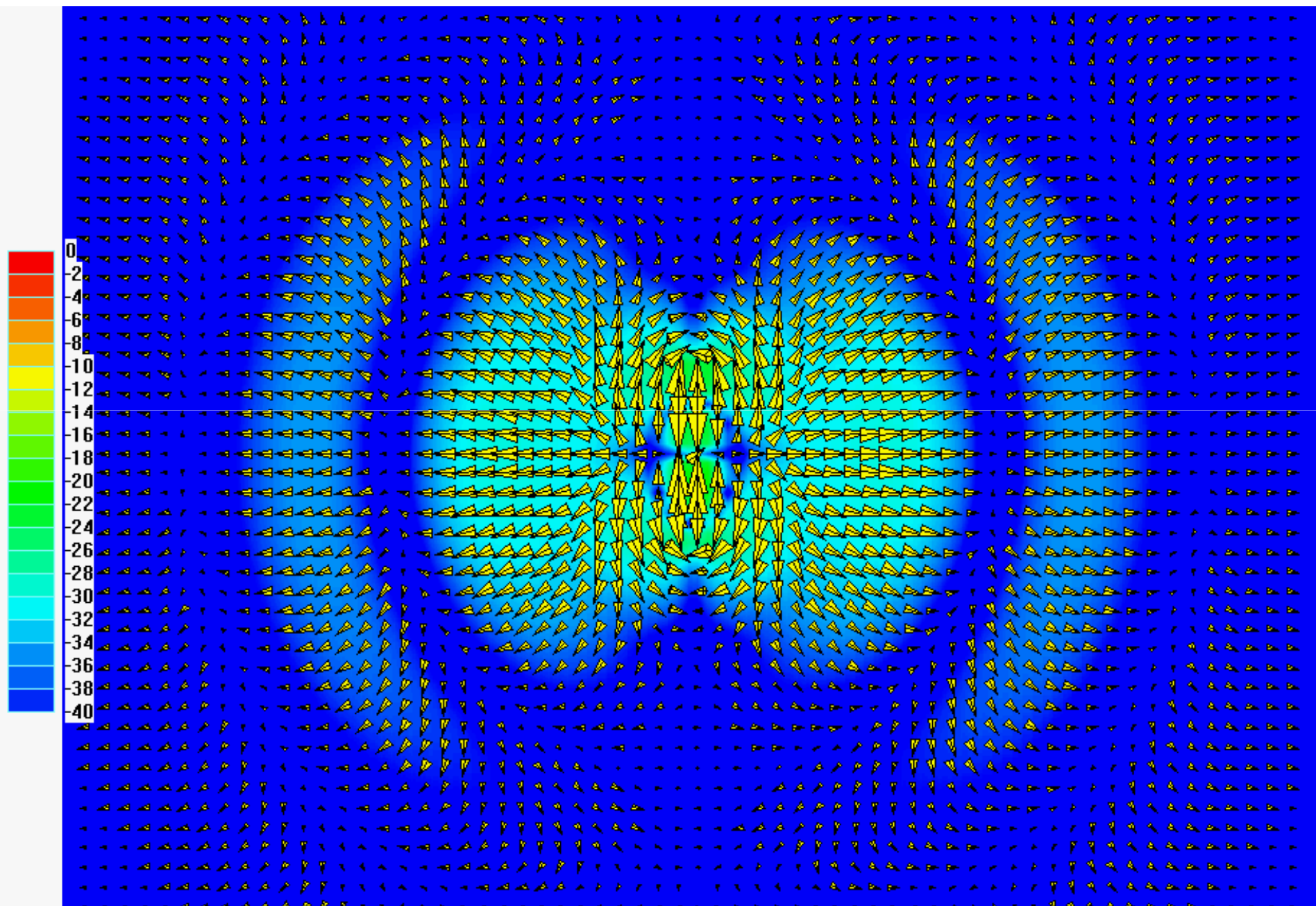
Preskok iskre na oddajnem iskrišču sproži preskok iskre na bližnjem sprejemnem iskrišču.

Hertz je dokazal polarizacijo interferenco, difrakcijo, refleksijo in refrakcijo valov ter hitrost širjenja.

Hertzov dipol



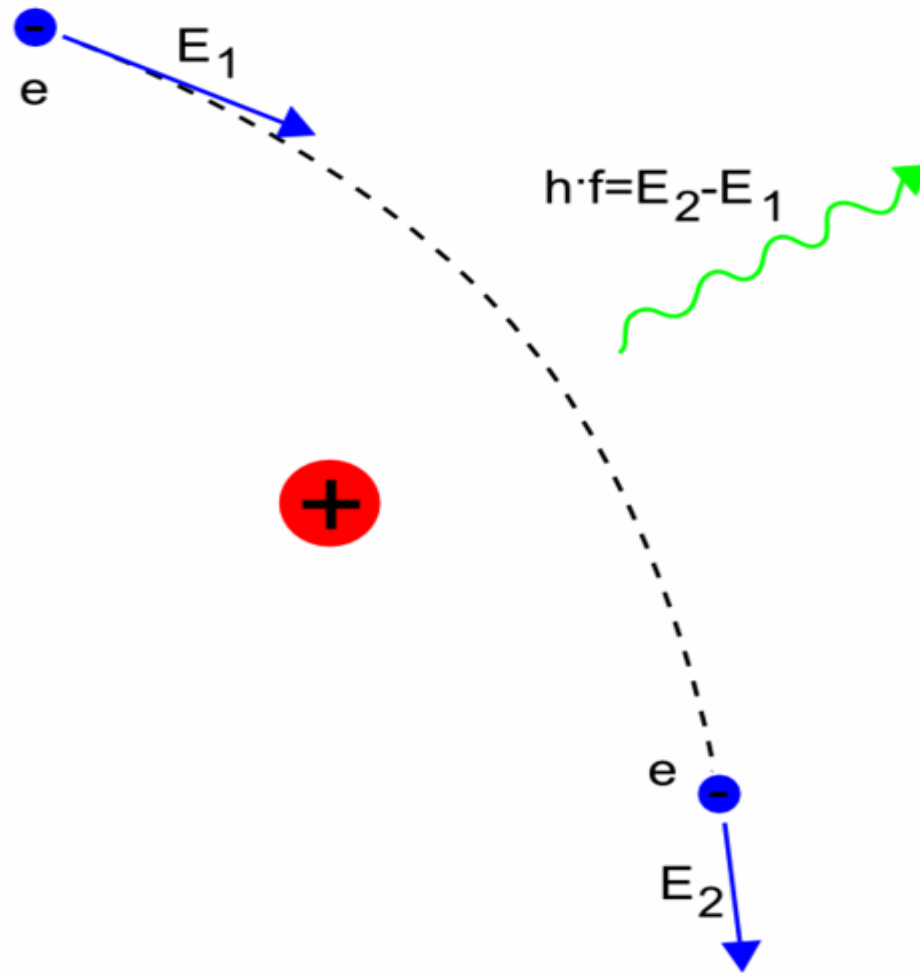
Pretok energije okoli vira



Sevalni pojavi

1. Prehod med energijskimi stanji atoma (Bohrov model)
 - Prehod elektronov med stabilnimi oblami (sevanje plinov)
2. Prehod med energijskimi stanji kvantne jame
 - Prehod elektronov med energijskimi nivoji (kvantni kaskadni laser)
3. Prehod med energijskimi stanji snovi polprevodnika
 - Prehod elektronov s prevodnega v valenčni pas (polprevodniški laser)
4. Prehod med energijskimi stanji molekule
 - Vibracijsko - rotacijski energijski pasovi (maser, laser)
 - Primer trdih Roentgenskih žarkov
5. Pospešeno gibanje naelektrenih delcev
 - Enakomerno kroženje
 - Oscilatorno (izmenično) gibanje elektronov – izmenični tok, antena
6. Kombinirano gibanje
 - Sinhrotronsko sevanje

Zavorno sevanje



Sevanje elektrona pri preskoku



Sevanje elektrona pri zaviranju



Sevanje kot naravni pojav

Kaj ga povzroča:

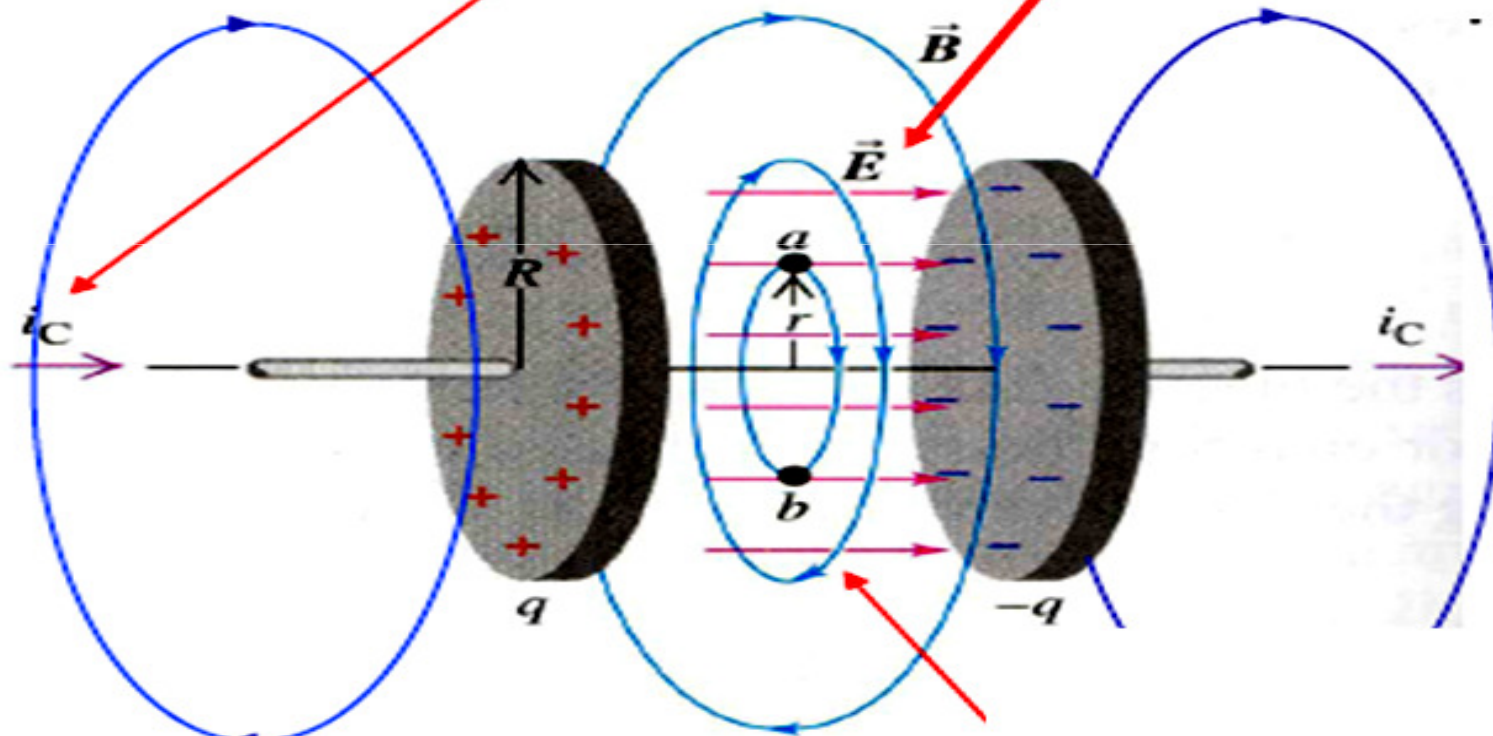
- Naelektreni delci, ki se gibljejo pospešeno, oddajajo energijo v prostor (izmenični konduktivni tok J (v kovini), izmenični konvektivni tok J v prostoru (ionosfera))
- Poljski tok $j\omega\epsilon E$ in $-j\omega\mu H$ v prostoru (ekvivalenca polja in tokov)

Kakšno je:

- Sevano polje je tisti del polja okoli antene, ki se od vira osamosvoji in potuje nepovratno v prostor
- Sevano polje ima v lokalnih razsežnostih lastnosti TEM vala poljubne polarizacije
- Sevano polje izhaja iz navideznega faznega središča antene
- S porazdelitvijo sevanega polja po smereh je določen smerni (sevalni) diagram antene. Daje porazdelitev amplitude in faze sevanega polja od smeri
- Elektromagnetno sevanje je neizotropno, spreminja se s smerjo v prostor. Antena s krogelnim smernim diagramom ne obstaja.

Kontinuiteta toka

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \int_S \vec{J} \cdot d\vec{A} + \mu_0 \epsilon_0 \frac{d}{dt} \int_S \vec{E} \cdot d\vec{A}$$



Magnetno polje
konduktivnega toka
toka v vodniku

Magnetno polje
poljskega toka
v kondenzatorju

Magnetno polje
konduktivnega toka
v vodniku

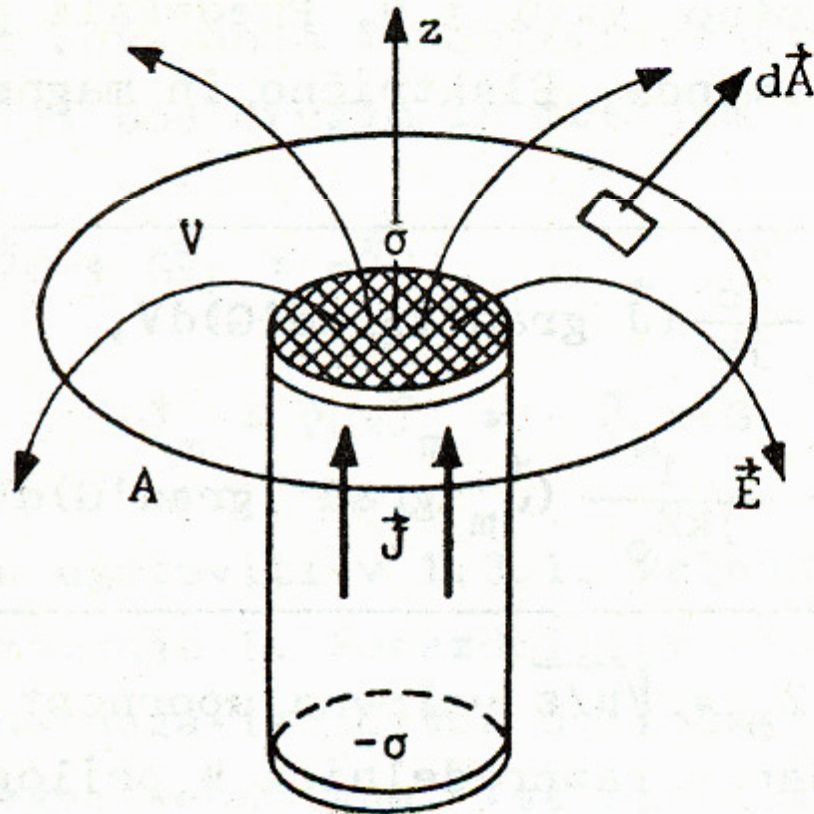
Tokovi

- **Konduktivni tok:**
gostota $\mathbf{J} = \gamma \mathbf{E}$ (A/m²), kovina
- **Konvektivni tok:**
gostota $\mathbf{J} = \rho \mathbf{v}$ (A/m²), ionosfera
- **Poljski tok:**
gostota $\mathbf{J} = j\omega \mathbf{D}$ (A/m²), dielektrik

Tokovi vseh vrst povzročajo enake učinke.

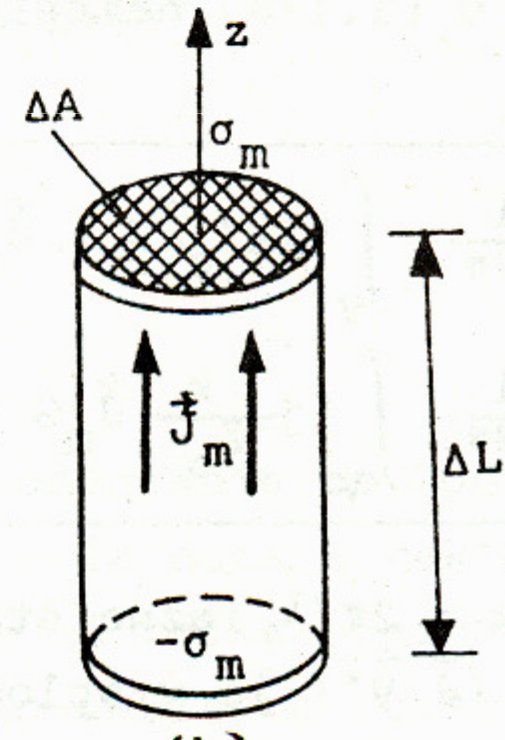
Električni in magnetni tokovni element

Stebriček električnega toka



Deluje kot tokovni generator

Stebriček navideznega magnetnega toka



Deluje kot napetostni generator

Elementarni viri sevanja

1. Realni (obstoječi) linearni viri sevanja:

- Elementarni električni dipol (dolžinski element električnega toka), $I\Delta L$. Ima pomen vsiljenega toga (tokovni generator v vezjih)
- Elementarni magnetni dipol (elementarna tokovna zanka), $I\Delta A$

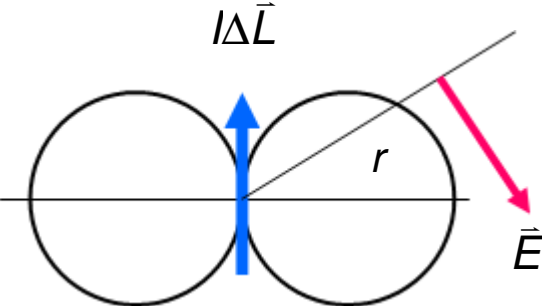
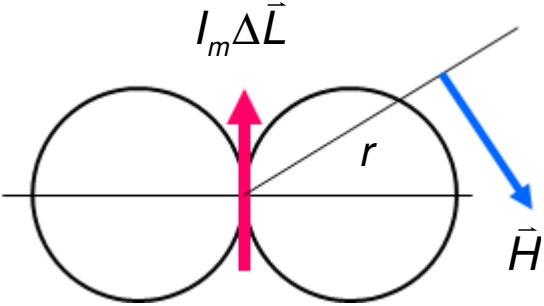
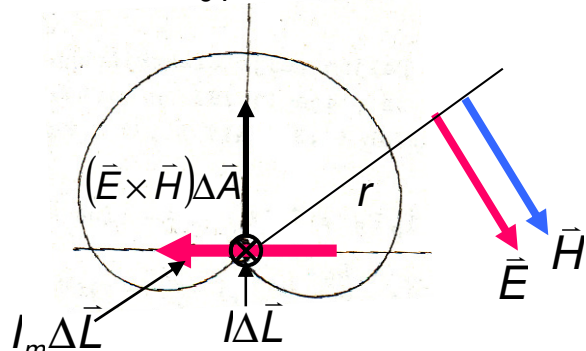
2. Virtualni (navidezni) linearni viri sevanja

- Elementarni magnetni dipol (dolžinski element navideznega magnetnega toka), $I_m\Delta L$. Ima pomen vsiljene napetosti (napetostnega generatorja v vezjih)
- Elementarni električni dipol (elementarna zanka navideznega magnetnega toka), $I_m\Delta A$

3. Realni ploskovni vir sevanja (Huygensov vir)

- $\mathbf{E} \times \mathbf{H} \cdot \Delta \mathbf{A}$
- Prekrižani elementarni električni/magnetni dipol, $I\Delta L \times I_m\Delta L$

Elementarni viri sevanja – antenski gradniki

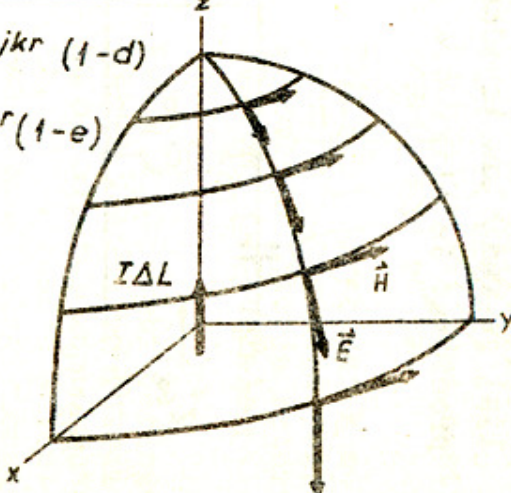
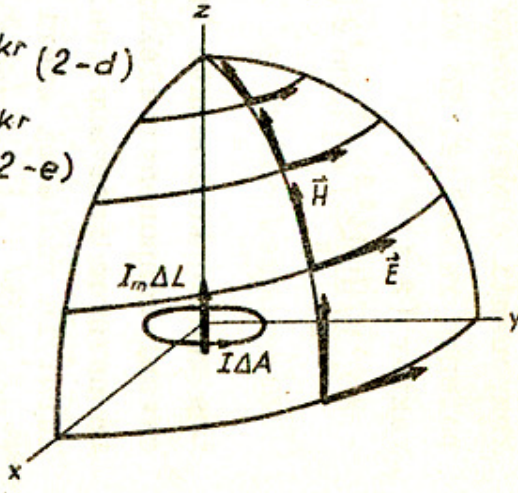
Električni tokovni element	Magnetni tokovni element	Elementarni ploskovni vir
Elementarni električni dipol	Elementarni magnetni dipol	Huygensov vir
Polje TM_{01} – sferični val	Polje TE_{01} – sferični val	Polje $TM_{01} + TE_{01}$
 <p>Komponente: $\frac{1}{r^3}, \frac{1}{r^2}, \frac{1}{r}$</p>	 <p>Komponente: $\frac{1}{r^3}, \frac{1}{r^2}, \frac{1}{r}$</p>	 <p>Komponente: $\frac{1}{r^3}, \frac{1}{r^2}, \frac{1}{r}$</p>
Ekvivalenca: vrtinec \vec{J}_m	Ekvivalenca: vrtinec \vec{J}	Ekvivalenca: tokovni križ
$I\Delta L = \frac{k}{jZ_0} I_m \Delta A$	$I_m \Delta L = jkZ_0 I \Delta A$	$\vec{J} = \vec{1}_n \times \vec{H}$ $\vec{J}_m = \vec{E} \times \vec{1}_n$ $\frac{E}{H} = Z_0$
Gradnik kovinskih anten	Gradnik režastih anten	Gradnik odprtinskih anten

Lastnosti:

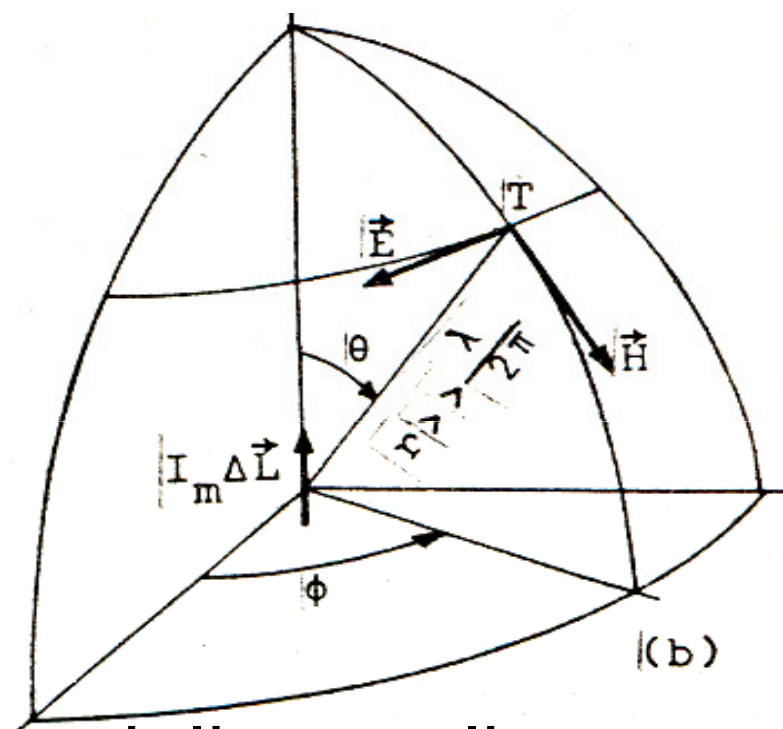
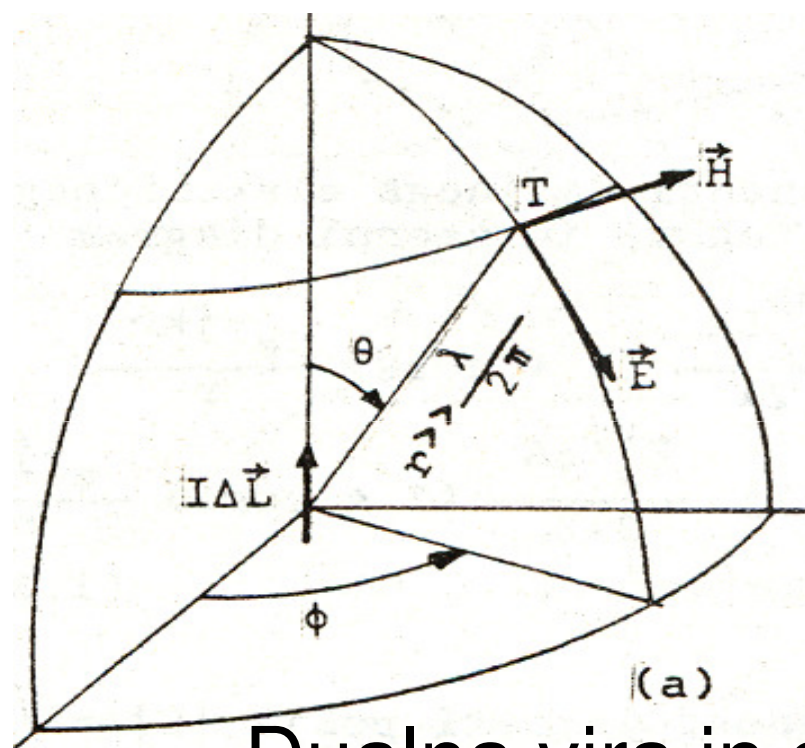
- sevanje elementarnih virov je neizotropno, izotropnih elektromagnetnih virov v naravi ni.
- sevajo le komponente, ki upadajo z razdaljo kot $1/r$.

Polje dipola

TABELA 4.2. ELEMENTARNI ELEKTRIČNI IN ELEMENTARNI MAGNETNI DIPOL

Električni dipol $I\Delta L$	Magnetni dipol $I_m\Delta L = jk\eta I\Delta A$
$E_r = \eta k^2 \frac{I\Delta L}{4\pi} \left[\frac{2}{(kr)^2} - \frac{2j}{(kr)^3} \right] \cos\theta e^{-jkr} \quad (1-a)$ $E_\theta = \eta k^2 \frac{I\Delta L}{4\pi} \left[\frac{j}{kr} + \frac{1}{(kr)^2} - \frac{j}{(kr)^3} \right] \sin\theta e^{-jkr} \quad (1-b)$ $E_\phi = H_r = H_\theta = 0$ $H_\phi = k^2 \frac{I\Delta L}{4\pi} \left[\frac{j}{kr} + \frac{1}{(kr)^2} \right] \sin\theta e^{-jkr} \quad (1-c)$	$E_r = E_\theta = H_\phi = 0$ $E_\phi = -k^2 \frac{I_m\Delta L}{4\pi} \left[\frac{j}{kr} + \frac{1}{(kr)^2} \right] \sin\theta e^{-jkr} \quad (2-a)$ $H_r = k^2 \frac{I_m\Delta L}{4\pi\eta} \left[\frac{2}{(kr)^2} - \frac{2j}{(kr)^3} \right] \cos\theta e^{-jkr} \quad (2-b)$ $H_\theta = k^2 \frac{I_m\Delta L}{4\pi\eta} \left[\frac{j}{kr} + \frac{1}{(kr)^2} - \frac{j}{(kr)^3} \right] \sin\theta e^{-jkr} \quad (2-c)$
<p>Daljnje polje</p>	
$\frac{E_\theta}{H_\phi} = j\eta \frac{I\Delta L}{2\lambda r} \sin\theta e^{-jkr} \quad (1-d)$ $H_\phi = j \frac{I\Delta L}{2\lambda r} \sin\theta e^{-jkr} \quad (1-e)$ $E_r = E_\phi = 0$ $H_r = H_\theta = 0$ $\frac{E_\theta}{H_\phi} = \eta$ 	$E_\phi = -j \frac{I_m\Delta L}{2\lambda r} \sin\theta e^{-jkr} \quad (2-d)$ $H_\theta = j \frac{I_m\Delta L}{2\eta\lambda r} \sin\theta e^{-jkr} \quad (2-e)$ $E_r = E_\theta = 0$ $H_r = H_\phi = 0$ $\frac{E_\phi}{H_\theta} = \eta$ $\frac{I_m\Delta L}{I\Delta A} = jk\eta$ 

Električni in magnetni tokovni element – polje v daljavi $r \gg \lambda/2\pi$



Dualna vira in dualno daljne polje

$$\vec{E} = \hat{r}_\theta E_\theta = \hat{r}_\theta j Z_0 \frac{I \Delta L}{2\lambda} \sin\theta \frac{e^{-jkr}}{r},$$

$$\vec{H} = \hat{r}_\phi \frac{E_\theta}{Z_0} = \hat{r}_\phi j \frac{I \Delta L}{2\lambda} \sin\theta \frac{e^{-jkr}}{r},$$

$$\vec{E} = \hat{r}_\phi E_\phi = -\hat{r}_\phi j \frac{I_m \Delta L}{2\lambda} \sin\theta \frac{e^{-jkr}}{r},$$

$$\vec{H} = -\hat{r}_\theta \frac{E_\phi}{Z_0} = \hat{r}_\theta j \frac{I_m \Delta L}{2\lambda Z_0} \sin\theta \frac{e^{-jkr}}{r}.$$

Tokovni element – odvisnost komponent polja od razdalje in smeri v prostor

$$E_r = \eta \frac{I_o \ell}{2\pi r^2} \cos \theta \left[1 + \frac{1}{jkr} \right] e^{-jkr}$$

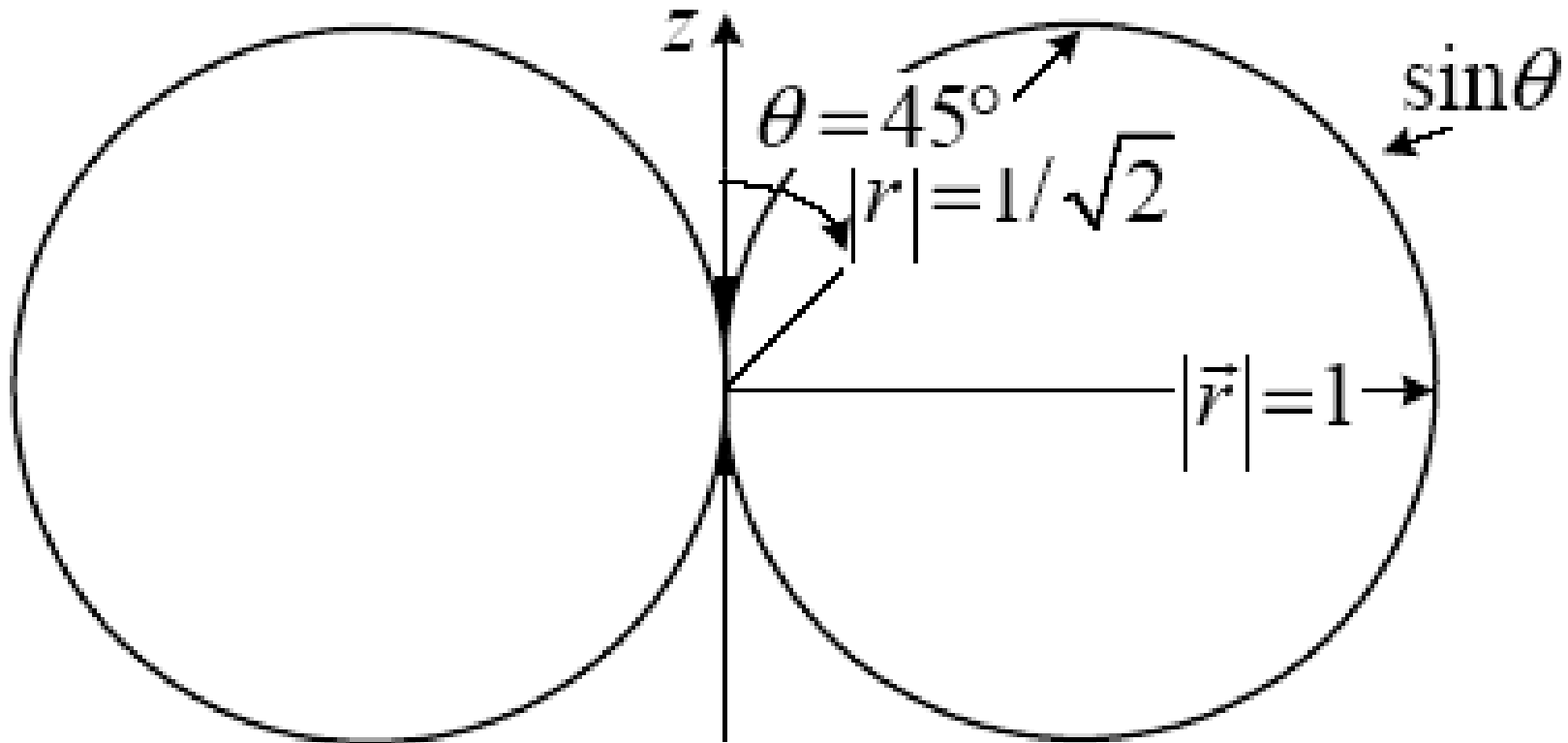
$$E_\theta = j\eta \frac{kI_o \ell}{4\pi r} \sin \theta \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2} \right] e^{-jkr}$$

$$E_\phi = H_r = H_\theta = 0$$

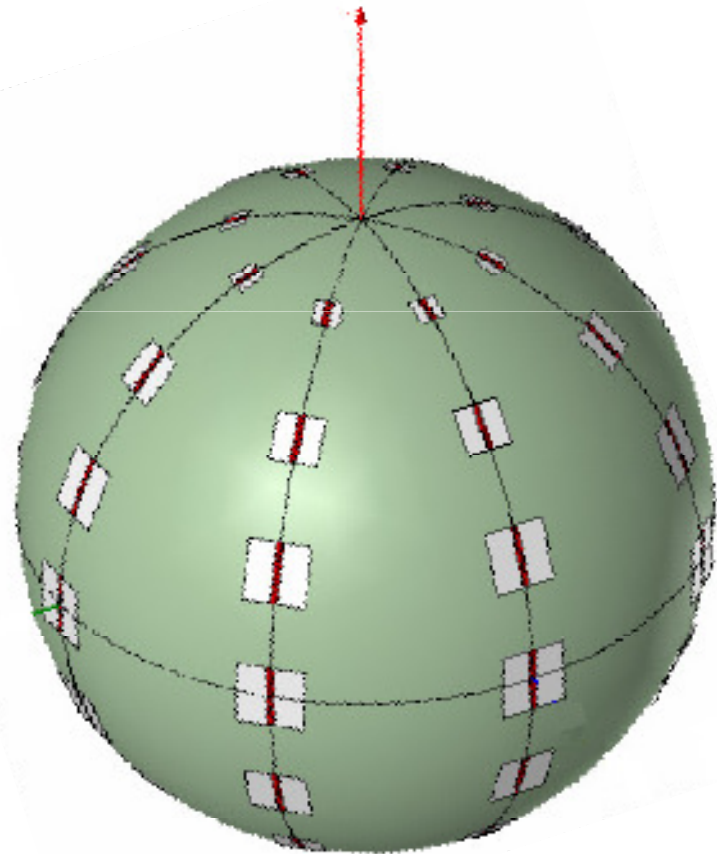
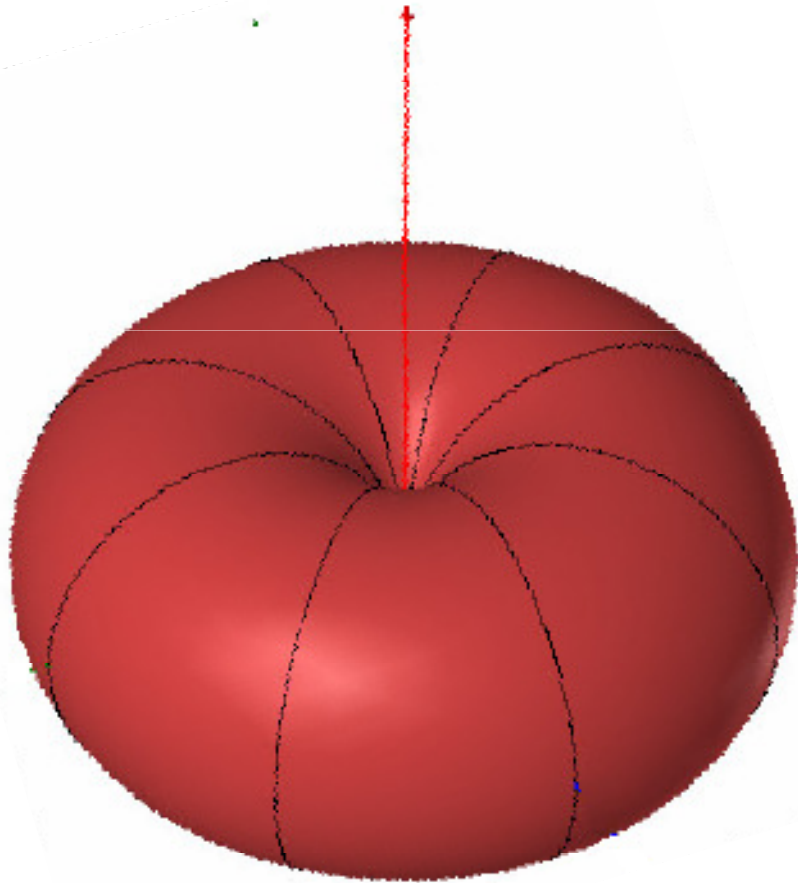
$$H_\phi = j \frac{kI_o \ell}{4\pi r} \left[1 + \frac{1}{jkr} \right] \sin \theta e^{-jkr}$$

Smerni diagram tokovnega elementa

Ravnina elevacije $\varphi = \text{konst.}$
tokovni element v smeri z



Prostorski smerni diagram in slika električnega polja tokovnega elementa



Elementarna tokovna zanka – odvisnost komponent polja

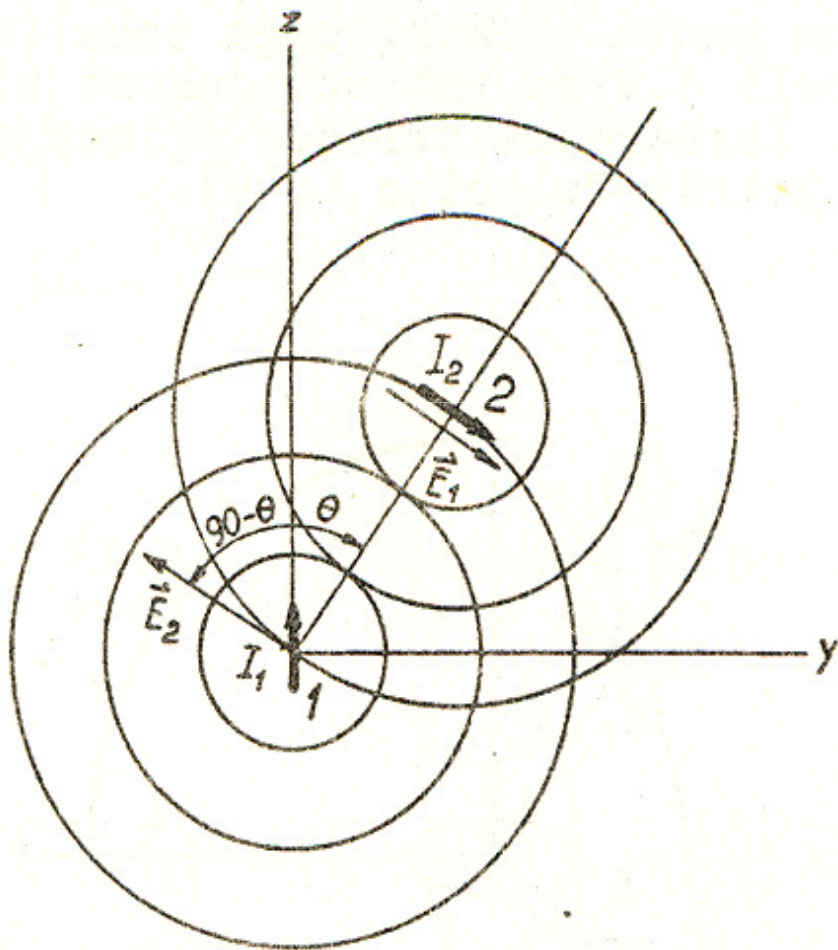
$$E_r = E_\theta = H_\phi = 0$$

$$E_\phi = \eta \frac{(ka)^2 I_o \sin \theta}{4r} \left[1 + \frac{1}{jkr} \right] e^{-jkr}$$

$$H_r = j \frac{ka^2 I_o \cos \theta}{2r^2} \left[1 + \frac{1}{jkr} \right] e^{-jkr}$$

$$H_\theta = -\frac{(ka)^2 I_o \sin \theta}{4r} \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2} \right] e^{-jkr}$$

Pojasnitev odvisnosti polj od razdalje r in polarnega kota θ



1. Površinska gostota moči $S = E^2/2Z_0$ upada kot r^{-2} , zato upada E kot r^{-1} .
2. Za vira in polji v točkah 1 in 2 velja recipročnost $I_1 \Delta l_1 E_2 \sin \theta = I_2 \Delta l_2 E_1$. Odtod je E_1 v smeri pod kotom θ sorazmerna s $\sin \theta$.
3. Magnetno polje sledi iz električnega po $E/H = Z_0$.

Določitev razmerja D/A

Iz recipročnosti izhaja $G_1/A_1 = G_2/A_2$, torej je enako za vse antene. Izračunajmo ga za tokovni element $|\Delta l|$:

$$G = 3/2$$

$$A = P/S$$

$$P = (E \cdot \Delta l)^2 / 8R, \quad R = 80\pi^2 (\Delta l / \lambda)^2$$

$$S = E^2 / (2Z_0), \quad Z_0 = 120 \pi$$

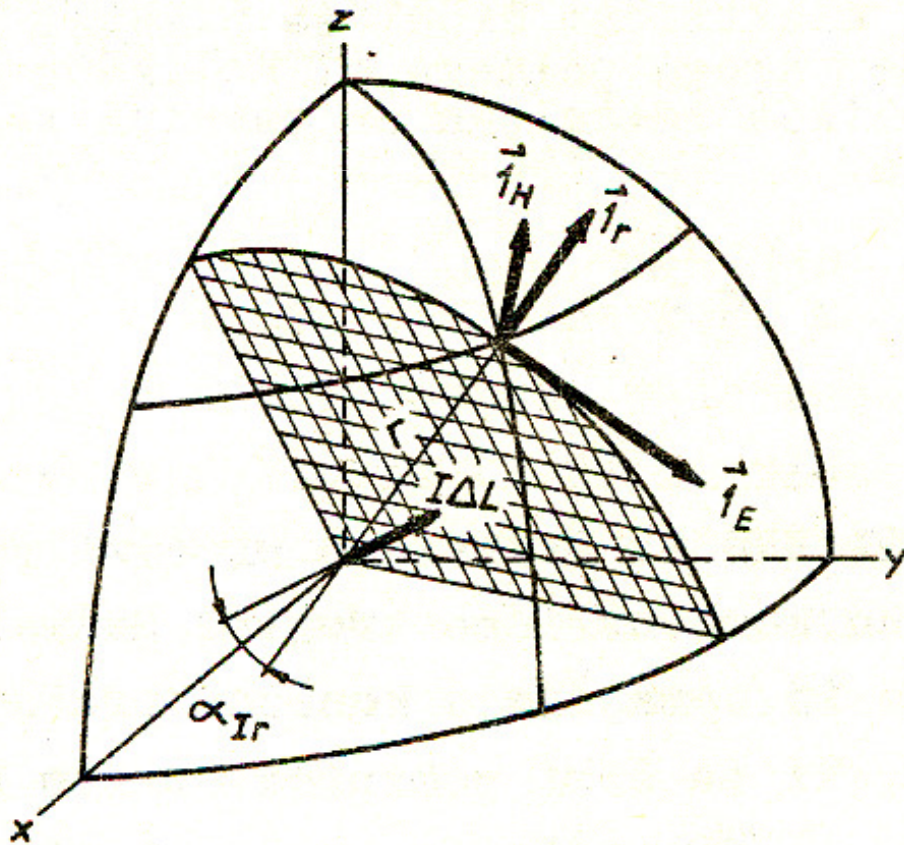
Sledi:

$$G/A = 4\pi/\lambda^2, \text{ veljavno za vse antene.}$$

Elementarni dipol v splošni legi

$$\vec{E} = \vec{1}_E E = (\vec{1}_I \times \vec{1}_r) \times \vec{1}_r j\eta \frac{I\Delta L}{2\lambda r} e^{-jkr},$$

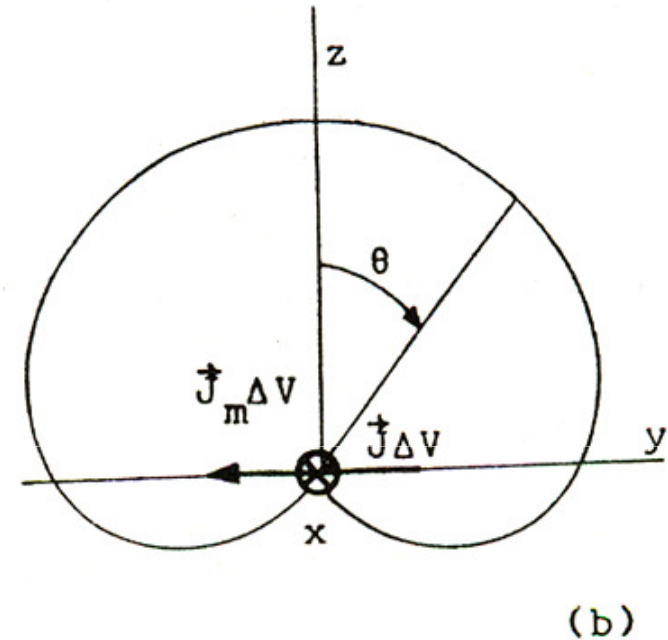
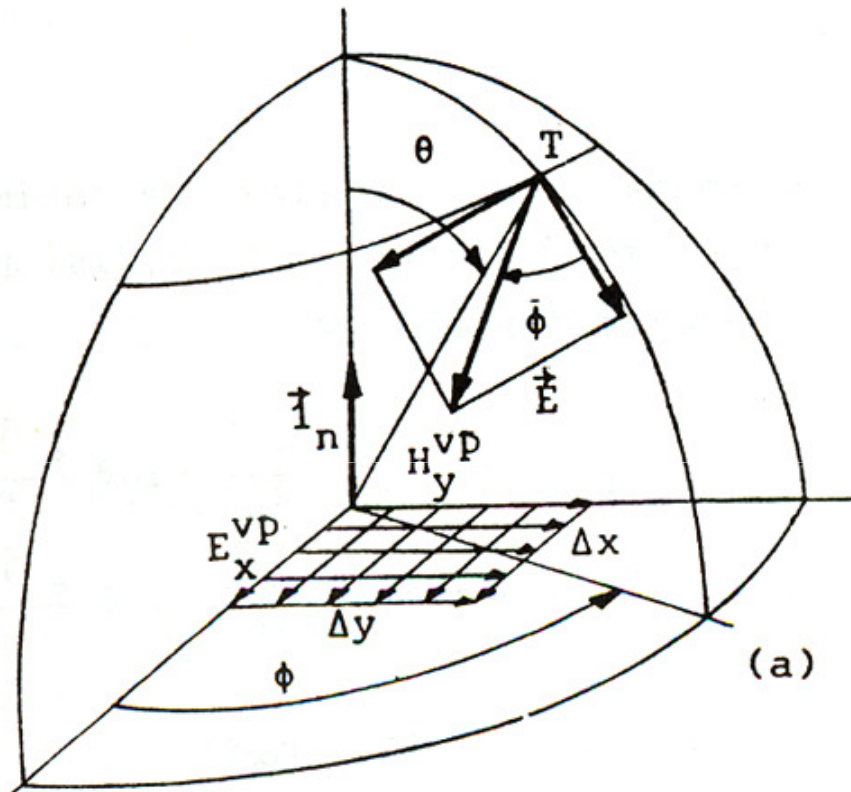
$$\vec{H} = \vec{1}_H H = (\vec{1}_r \times \vec{1}_E) \frac{E}{\eta} = (\vec{1}_I \times \vec{1}_r) j \frac{I\Delta L}{2\lambda r} e^{-jkr}.$$



$$\vec{1}_H = \frac{\vec{1}_I \times \vec{1}_r}{\sin\alpha_{Ir}}.$$

$$\vec{1}_E = \frac{\vec{1}_I \times \vec{1}_r}{\sin\alpha_{Ir}} \times \vec{1}_r.$$

Daljno polje Huygensovega vira



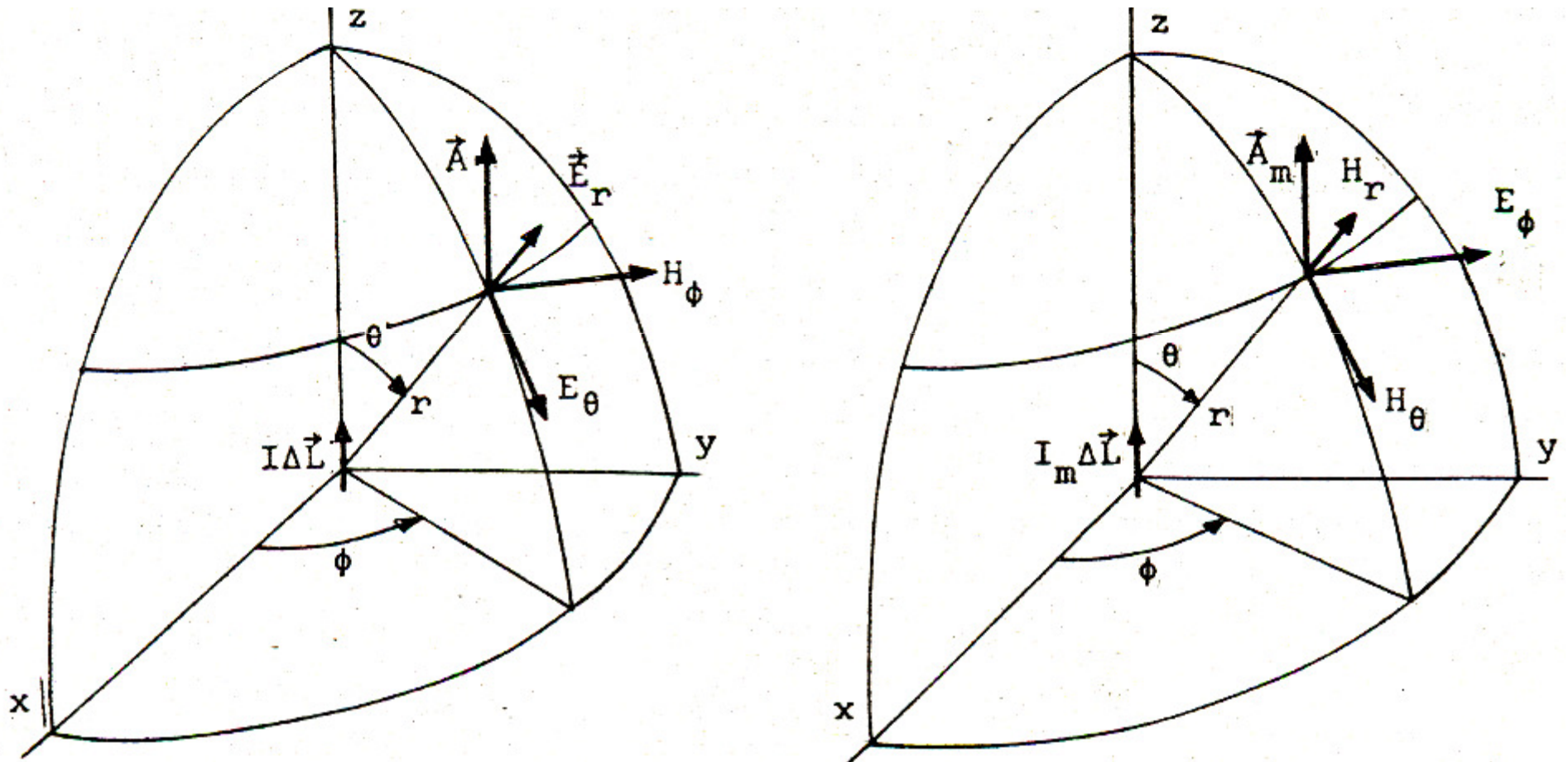
$$\vec{E} = \hat{i}_\theta E_\theta + \hat{i}_\phi E_\phi = (\hat{i}_\theta \cos\phi - \hat{i}_\phi \sin\phi) j \frac{E_x^{VP} \Delta A}{2\lambda} (1 + \cos\theta) \frac{e^{-jkr}}{r},$$

$$\vec{H} = \hat{i}_\theta \left(-\frac{E_\phi}{Z_0}\right) + \hat{i}_\phi \frac{E_\theta}{Z_0} = (\hat{i}_\theta \sin\phi + \hat{i}_\phi \cos\phi) j \frac{E_x^{VP} \Delta A}{2\lambda Z_0} (1 + \cos\theta) \frac{e^{-jkr}}{r}.$$

Dodatek

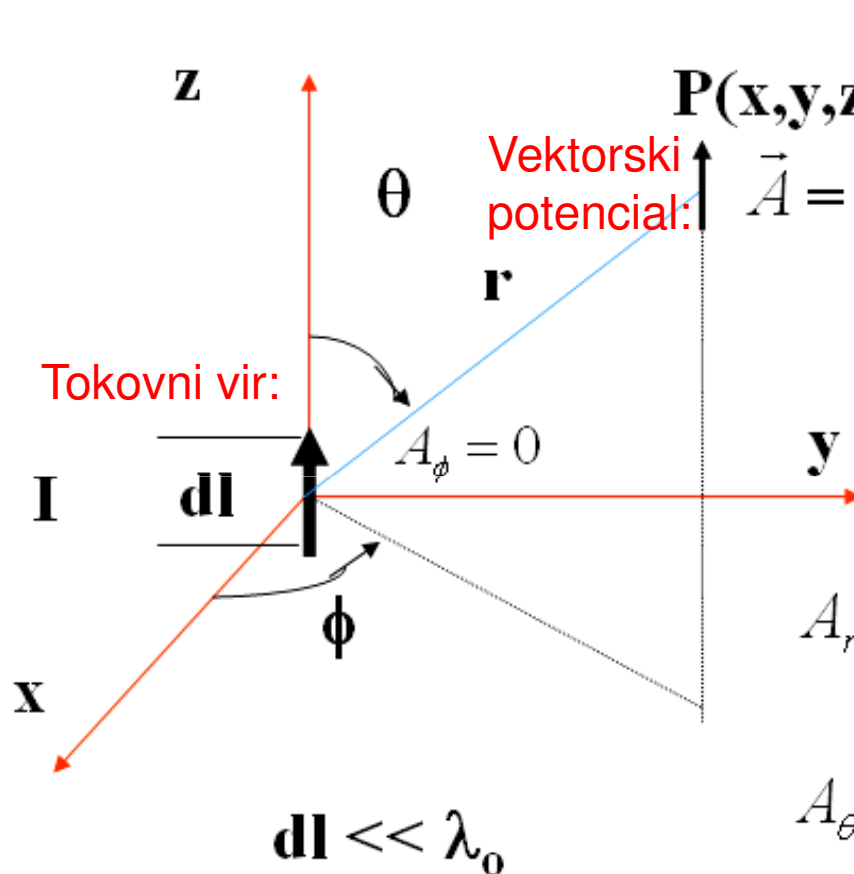
IZRAČUNI

Električni in magnetni tokovni element – potencial in polje v prostoru



Dualna vira, dualna potenciala in dualni polji

Tokovni element – vektorski potencial



$$A_z = \int_v \frac{\mu_0 \vec{J}(x', y', z') e^{-jk_0 R}}{4\pi R} dv$$

$$= \frac{\mu_0 I(dl) e^{-jk_0 r}}{4\pi r}$$

Sferične komponente potenciala:

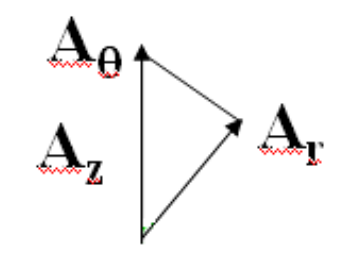
$$A_r = A_z \cos \theta = \frac{\mu_0 I(dl) e^{-jk_0 r}}{4\pi r} \cos \theta$$

$$A_\theta = -A_z \sin \theta = -\frac{\mu_0 I(dl) e^{-jk_0 r}}{4\pi r} \sin \theta$$

$$A_\phi = 0$$

$$\vec{A} = \hat{a}_x A_x + \hat{a}_y A_y + \hat{a}_z A_z$$

$$= \hat{a}_R A_R + \hat{a}_\theta A_\theta + \hat{a}_\phi A_\phi$$



Tokovni element – magnetno polje

$$\vec{H} = \frac{1}{\mu_o} \nabla \times \vec{A}$$

$$H_r = \frac{1}{\mu_o r \sin \theta} \left[\frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial}{\partial \phi} A_\theta \right] = 0$$

$$H_\theta = \frac{1}{\mu_o r} \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \phi} A_r - \frac{\partial}{\partial r} (r A_\phi) \right] = 0$$

$$H_\phi = \frac{1}{\mu_o r} \left[\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial}{\partial \theta} (A_r) \right]$$

Tokovni element – električno polje

$$\vec{E} = \frac{1}{j\omega\epsilon_o} \nabla \times \vec{H}$$

$$E_r = \frac{1}{j\omega\epsilon_o} \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (H_\phi \sin \theta) - \frac{\partial H_\theta}{\partial \phi} \right]$$

$$= -\frac{Idl}{4\pi} \eta_o k_o^2 2 \cos \theta \left[\frac{1}{(jk_o r)^2} + \frac{1}{(jk_o r)^3} \right] e^{-jk_o r}$$

$$E_\theta = \frac{1}{j\omega\epsilon_o} \frac{1}{r} \left[-\frac{\partial}{\partial r} (rH_\phi) \right]$$

$$= -\frac{Idl}{4\pi} \eta_o k_o^2 \sin \theta \left[\frac{1}{jk_o r} + \frac{1}{(jk_o r)^2} + \frac{1}{(jk_o r)^3} \right] e^{-jk_o r}$$

Tokovni element – komponente polja

$$E_{\phi} = \frac{1}{j\omega\epsilon_0} \frac{1}{r} \left[\frac{\partial}{\partial r} (rH_{\theta}) - \frac{\partial H_r}{\partial \theta} \right] = 0$$

$$H_{\phi} = -\frac{Idl}{4\pi} k_0^2 \sin \theta \left[\frac{1}{jk_0 r} + \frac{1}{(jk_0 r)^2} + \frac{0}{(jk_0 r)^3} \right] e^{-jk_0 r}$$

$$\frac{E_{\theta}}{\eta_0} = -\frac{Idl}{4\pi} k_0^2 \sin \theta \left[\frac{1}{jk_0 r} + \frac{1}{(jk_0 r)^2} + \frac{1}{(jk_0 r)^3} \right] e^{-jk_0 r}$$

$$\frac{E_r}{\eta_0} = -\frac{Idl}{4\pi} k_0^2 2 \cos \theta \left[\frac{0}{jk_0 r} + \frac{1}{(jk_0 r)^2} + \frac{1}{(jk_0 r)^3} \right] e^{-jk_0 r}$$

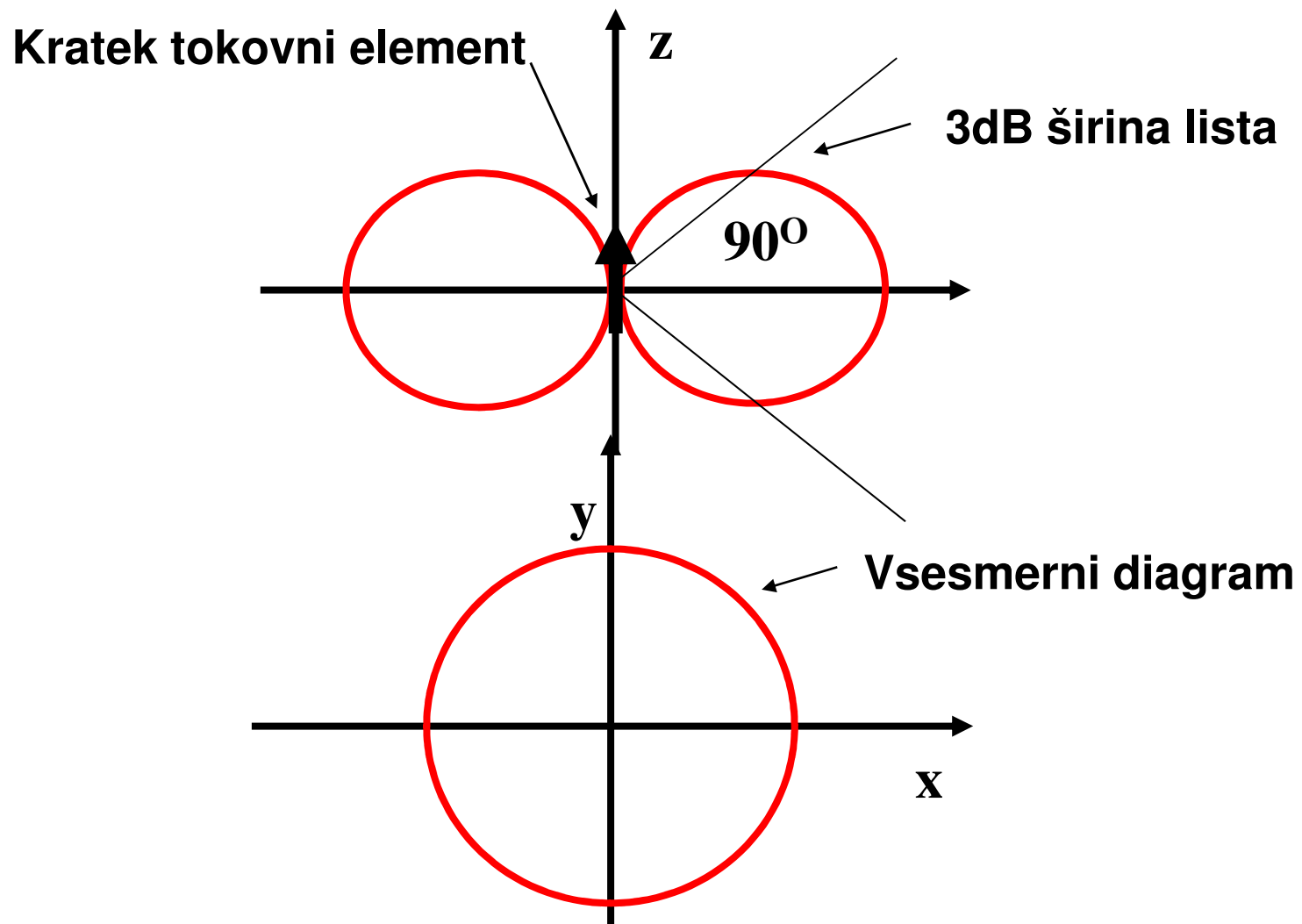
Tokovni element – daljno polje

$$H_{\phi} = j \frac{Idl}{4\pi} \left(\frac{e^{-jk_o r}}{r} \right) k_o \sin \theta$$

$$E_{\theta} = j \frac{Idl}{4\pi} \left(\frac{e^{-jk_o r}}{r} \right) \eta_o k_o \sin \theta$$

$$\therefore \frac{E_{\theta}}{H_{\phi}} = \eta_o = 120 \pi$$

Smerni diagram



Tokovni element - smernost

$$\begin{aligned} D &= \frac{|E_{\max}|^2}{\frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} |E(\theta, \phi)|^2 \sin \theta d\theta d\phi} \\ &= \frac{|E_{\max}|^2}{\frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} |\sin \theta|^2 \sin \theta d\theta d\phi} = \frac{1}{\frac{1}{4\pi} 2\pi \int_0^{\pi} \sin^3 \theta d\theta} \\ &= \frac{1}{\frac{1}{2} \times \frac{4}{3}} = 1.5 \end{aligned}$$

Tokovni element – sevana moč

$$\begin{aligned}\vec{P} &= \frac{1}{2} \operatorname{Re} [\vec{E} \times \vec{H}^*] = \frac{1}{2} \operatorname{Re} [E_\theta H_\phi^*] \hat{r} \\ &= \frac{1}{2} \operatorname{Re} [\eta_o H_\phi H_\phi^*] \hat{r} = \frac{1}{2} \eta_o |H_\phi|^2 \hat{r}\end{aligned}$$

$$P = \iint \vec{P} \cdot d\vec{s}$$

$$= \frac{1}{2} \eta_o \int_0^{2\pi} \int_0^\pi |H_\phi|^2 r^2 \sin \theta d\theta d\phi$$

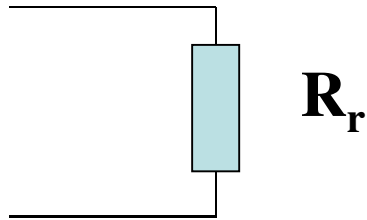
$$= \frac{1}{2} \eta_o \left(\frac{I dl k_o}{4\pi} \right)^2 \int_0^{2\pi} \int_0^\pi \sin^3 \theta d\theta d\phi$$

$$|H_\phi| = \frac{I dl}{4\pi r} k_o \sin \theta$$

Tokovni element – sevana moč

$$\begin{aligned} P &= \eta_o \frac{(Idlk_o)^2}{16 \pi} \int_0^\pi \sin^3 \theta d\theta = \eta_o \frac{(Idlk_o)^2}{16 \pi} \int_0^\pi -\sin^2 \theta d \cos \theta \\ &= \eta_o \frac{(Idlk_o)^2}{16 \pi} \int_0^\pi (\cos^2 - 1) d \cos \theta \\ &= \eta_o \frac{(Idlk_o)^2}{16 \pi} \left(\frac{\cos^3 \theta}{3} - \cos \theta \right)_0^\pi \\ &= \eta_o \frac{(Idlk_o)^2}{16 \pi} \left(-\frac{1}{3} + 1 - \frac{1}{3} + 1 \right) \\ &= \eta_o \frac{(Idlk_o)^2}{12 \pi} \end{aligned}$$

Tokovni element – sevalna upornost



$$P = \frac{1}{2} I^2 R_r$$

$$= \eta_o \frac{(I d l k_o)^2}{12 \pi}$$

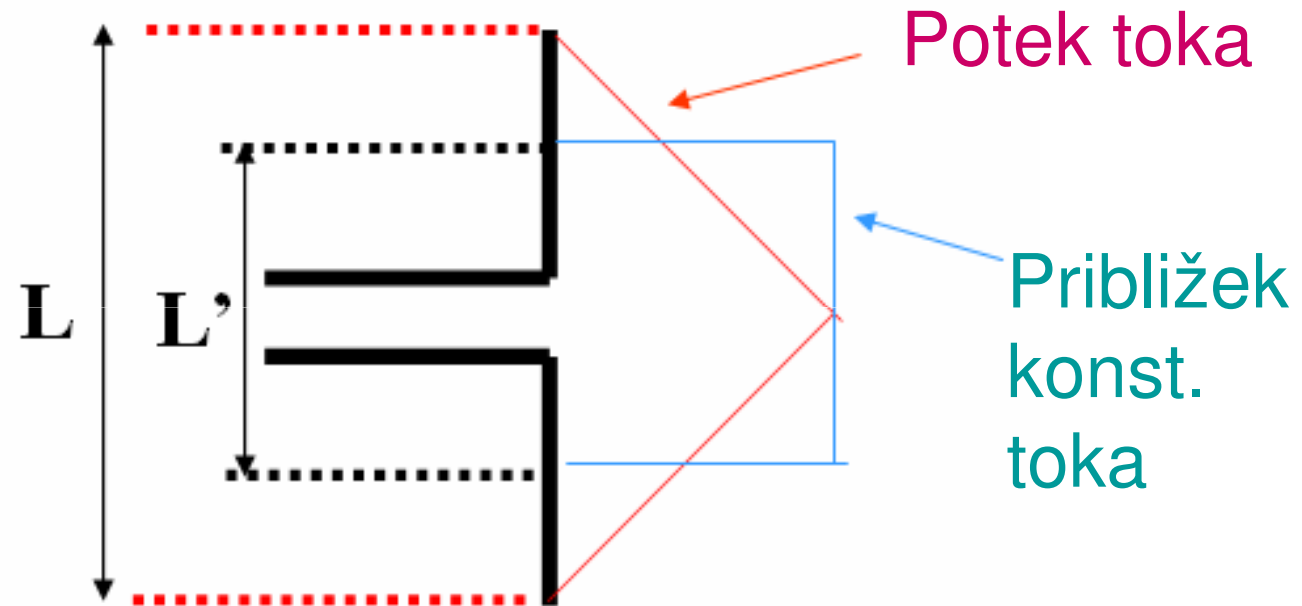
$$R_r = \eta_o \frac{(d l k_o)^2}{6 \pi}$$

$$= \frac{120 \pi}{6 \pi} \left(\frac{2 \pi}{\lambda_o} d l \right)^2$$

$$= 80 \pi^2 \left(\frac{d l}{\lambda_o} \right)^2$$

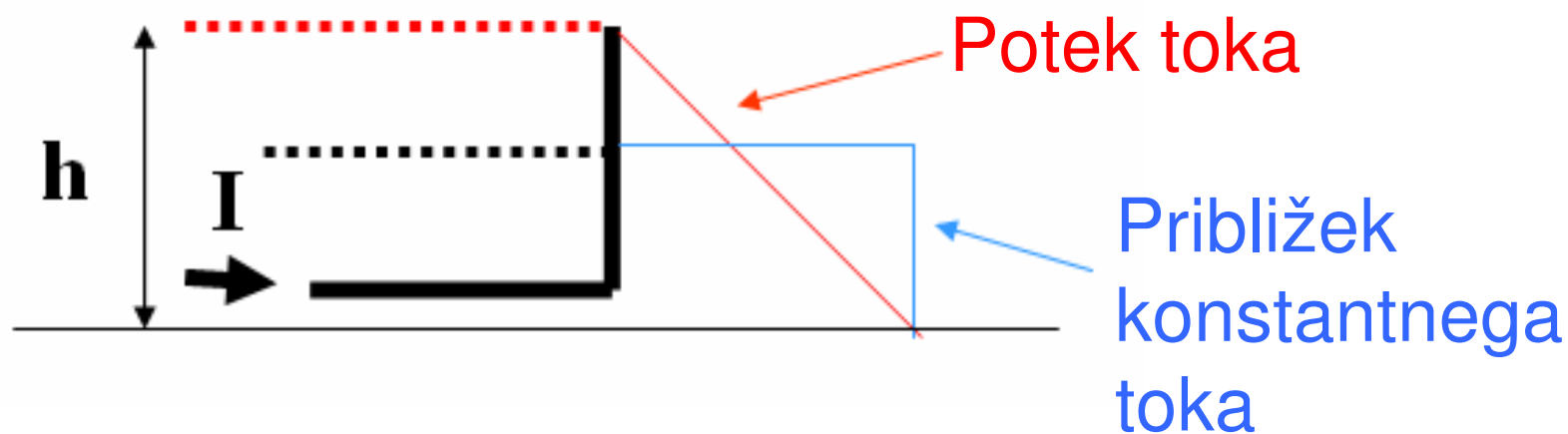
Kratek dipol

Približek tokovnega elementa



$$R_r = 80\pi^2 \left(\frac{dl}{\lambda_0} \right)^2 = 80\pi^2 \left(\frac{L}{2\lambda_0} \right)^2 = 20\pi^2 \left(\frac{L}{\lambda_0} \right)^2$$

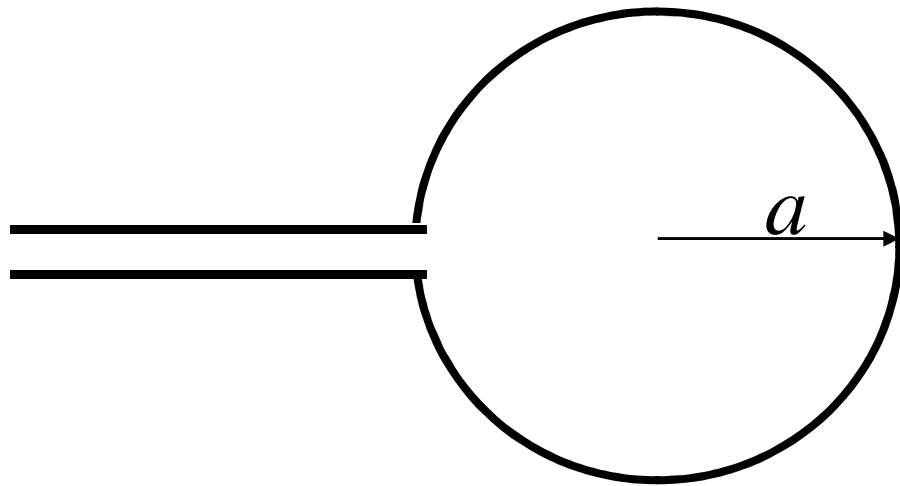
Približek tokovnega monopola



$$R_r = 10\pi^2 \left(\frac{L}{\lambda_o} \right)^2 = 10\pi^2 \left(\frac{2h}{\lambda_o} \right)^2 = 40\pi^2 \left(\frac{h}{\lambda_o} \right)^2$$

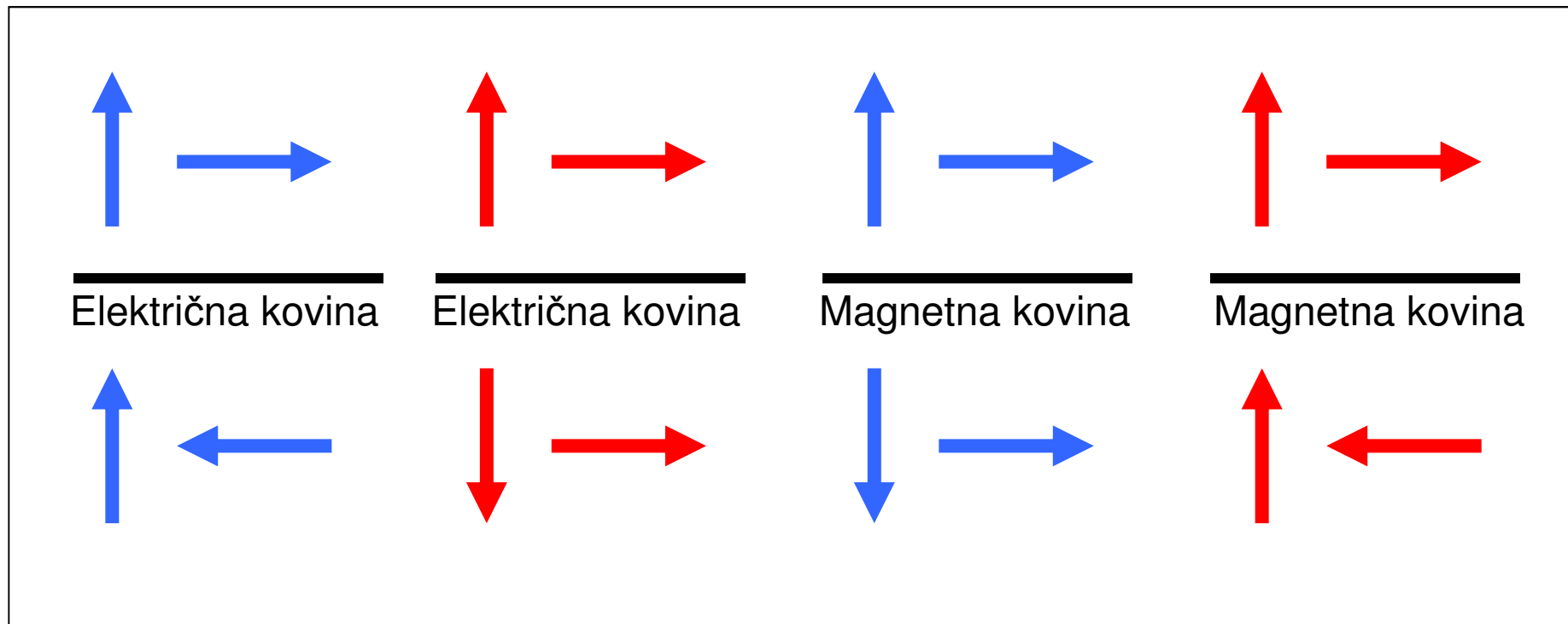
$$h = 0.1 \lambda_o, R_r = 0.38 \Omega$$



Elementarna tokovna zanka



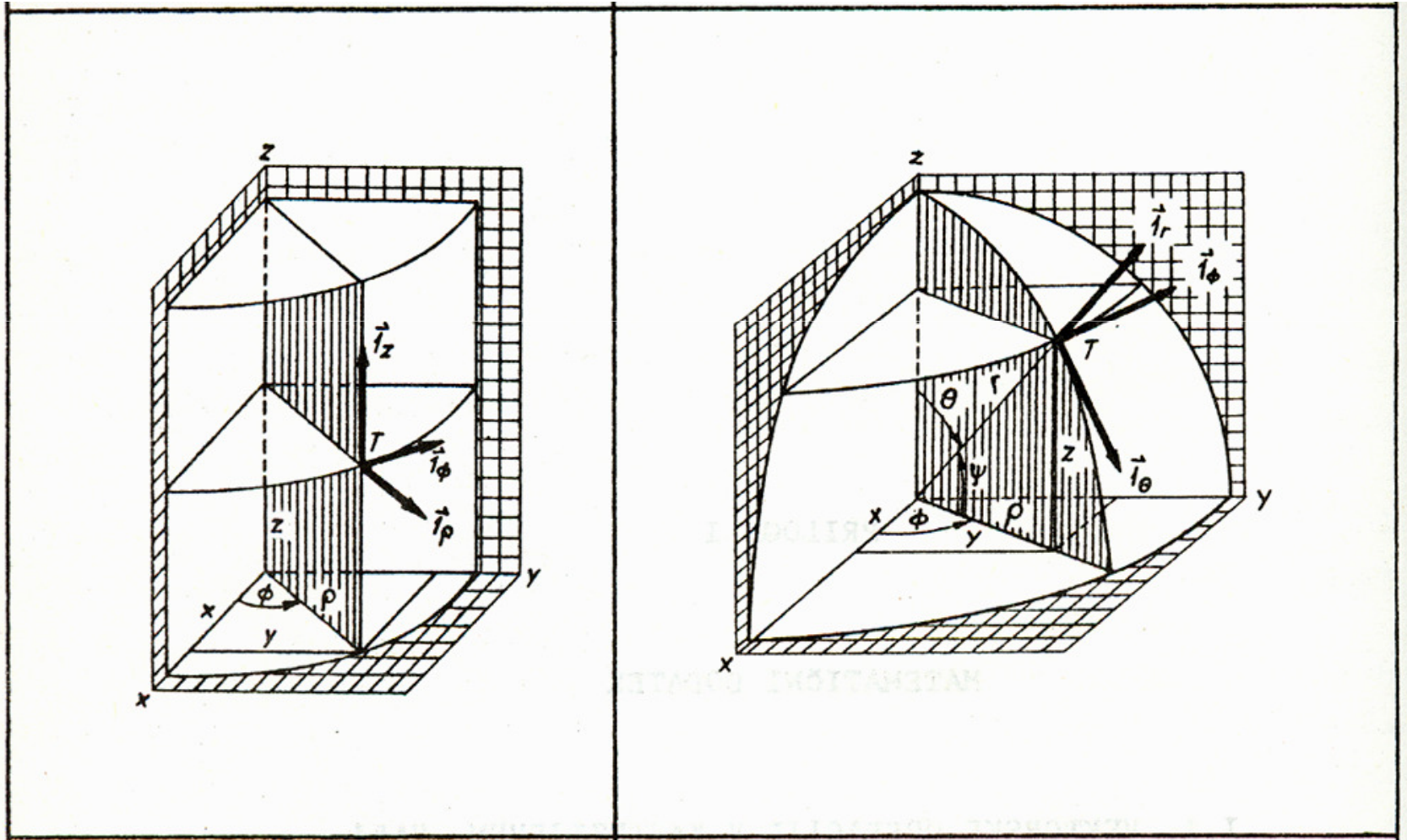
$$R_r = 20\pi^2 \left(\frac{2\pi a}{\lambda_o} \right)^2$$

Pravila zrcaljenja električnih in magnetnih tokov



 Električni tok
 Navidezni magnetni tok

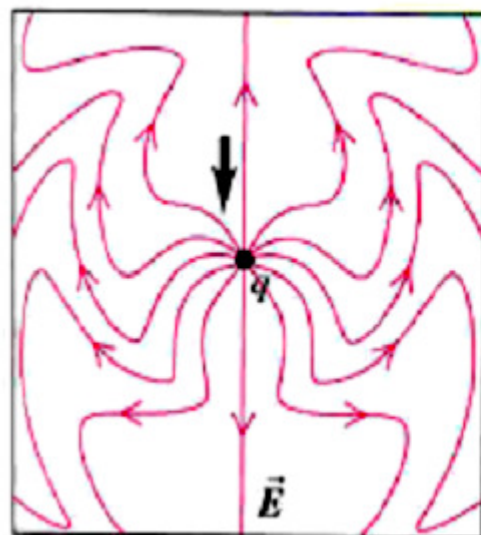
Smerni vektorji



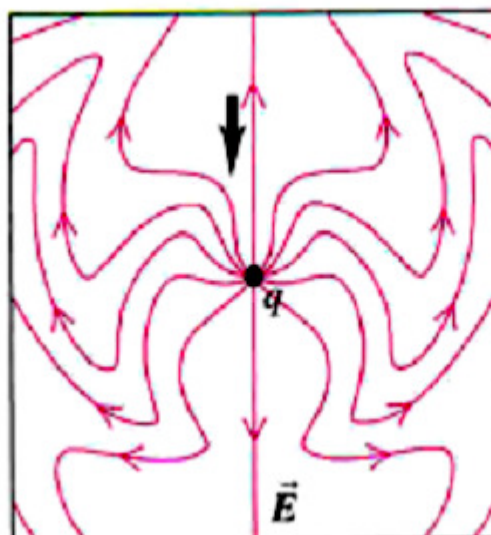
Smerni vektorji

$\hat{i}_x = \hat{i}_\rho \cos\phi - \hat{i}_\phi \sin\phi$ $\hat{i}_y = \hat{i}_\rho \sin\phi + \hat{i}_\phi \cos\phi$ $\hat{i}_z = \hat{i}_z$	$\hat{i}_x = \hat{i}_r \sin\theta \cos\phi + \hat{i}_\theta \cos\theta \cos\phi - \hat{i}_\phi \sin\phi$ $\hat{i}_y = \hat{i}_r \sin\theta \sin\phi + \hat{i}_\theta \cos\theta \sin\phi + \hat{i}_\phi \cos\phi$ $\hat{i}_z = \hat{i}_r \cos\theta - \hat{i}_\theta \sin\theta$
$\hat{i}_\rho = \hat{i}_x \cos\phi + \hat{i}_y \sin\phi$ $\hat{i}_\phi = -\hat{i}_x \sin\phi + \hat{i}_y \cos\phi$ $\hat{i}_z = \hat{i}_z$	$\hat{i}_r = \hat{i}_x \sin\theta \cos\phi + \hat{i}_y \sin\theta \sin\phi + \hat{i}_z \cos\theta$ $\hat{i}_\theta = \hat{i}_x \cos\theta \cos\phi + \hat{i}_y \cos\theta \sin\phi - \hat{i}_z \sin\theta$ $\hat{i}_\phi = -\hat{i}_x \sin\phi + \hat{i}_y \cos\phi$
	$\cos\theta_x = \frac{x}{r} = \sin\theta \cos\phi$ $\cos\theta_y = \frac{y}{r} = \sin\theta \sin\phi$ $\cos\theta_z = \frac{z}{r} = \cos\theta$

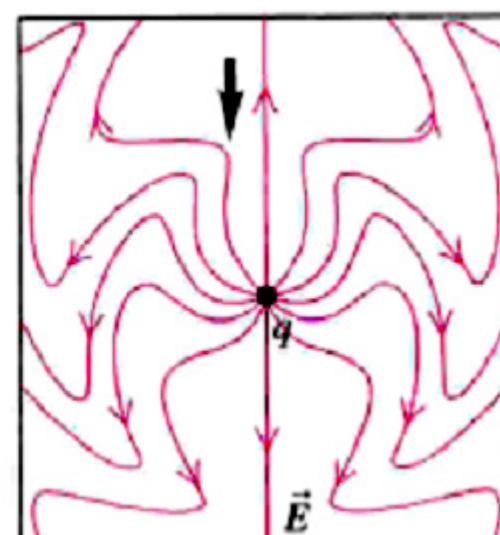
Električno polje oscilirajočega naboja



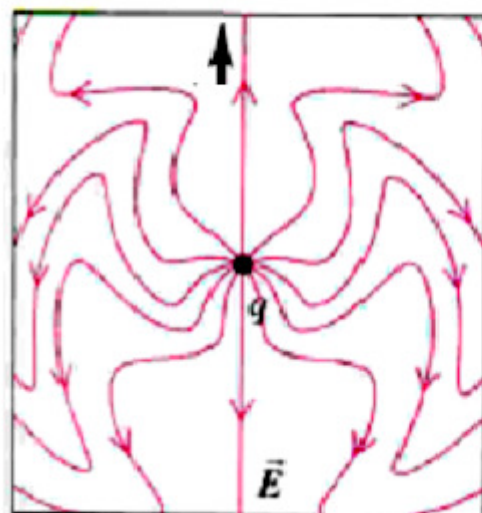
$t = 0$



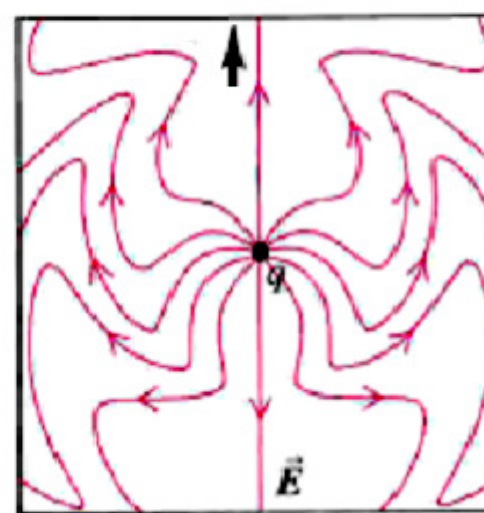
$t = T/4$



$t = T/2$



$t = 3T/4$



$t = T$